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TECHNICAL NOTE

Analysis of Hydraulic Jump Characteristics in U-Shaped Channel

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ABSTRACT: Typical supercritical flow characteristics like, sequent depth ratio, relative jump height, relative energy loss, efficiency of jump, relative prejump depth, relative postjump depth, relative length of the roller and jump in the U-shaped channel are experimentally studied. Based on the experimental findings, physical theories for the variance in these characteristics concerning the Froude number are presented. Empirical models are developed considering the influence of inflow Froude number varying between 4 and 20 and Reynolds number between 1,638,009 and 3,394,784. Some models were also validated and yielded satisfactory results with good R^2 values. For comparison and a deeper comprehension of hydraulic jump characteristics, computational multivariate statistical techniques like Principal Component Analysis (PCA) and Factor Analysis (FA) are applied. These techniques are used to identify patterns in an effort to explain the variation in a sizable set of closely related jump characteristics. Verifiers for the different principal components were analyzed, and verifactor VF1 (with 64 %) had strong positive loadings on the sequent depth ratio, the relative jump height, the relative energy loss, the relative length of the roller and jump, while VF2 (with 33%) had strong positive loadings on the relative prejump depth and the relative postjump depth and moderate positive loading on the efficiency of jump. These statistical methods proved to be valuable tools for identifying the key characteristics of a phenomenon and its relative significance.

Keywords: Super Critical Flow, U-Shaped Channel, Empirical Modeling, Principal Component Analysis, Factor Analysis.

1. Introduction

When the flow in a U-shaped channel changed from super-critical to sub-critical, there is a hydraulic jump formation. This phenomenon is vital for ensuring the stability of the channel and preventing undesirable consequences of erosion, scouring, and dissipating excess momentum.

U-shaped channel beds were found to be more effective in reducing the jump length and sequent depth over other shaped channels (Hager, 1987, 1989). Experimental observations by different authors were focused on surface profiles, shear stress, and jump length. Even their results proved that different shaped beds (including corrugated) are insignificant on supercritical flow properties for a lower range of Froude numbers. When the channel cannot expand laterally or vertically (Gandhi et al., 2024) and the tail water depth is insufficient to provide good jumps, a U-shaped channel may be one of the alternatives for hydraulic jump

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formation in energy dissipation.

Additionally, it replaces the additional appurtenance arrangement to reduce the basin length (Bushra and Afzal, 2006; Jayant and Jhamnani, 2023). Along with experimental and dimensionless approaches, methods of multivariate statistics (including Principal Component Analysis (PCA) and Factor Analysis (FA)) are also useful to analyse possible factors which influence the hydraulic jump characteristics (Bagheri et al., 2023; Ogarekpe et al., 2022). Belanger (1849) and Bidone (1819) conducted the first substantial empirical study on the hydraulic jump. Using the momentum principle, they put up a theoretical explanation for the subsequent depth ratio. Hager (1987) published the initial experimental findings regarding free surfaces without dimensions. Gandhi and Singh (2016) discovered that as the Froude number increases, the length and subsequent depth of a classical hydraulic jump also increase.

Studies on the hydrodynamics of flow in a U-shaped channel were carried out by different authors (Lin et al., 2021; Hu and Zhang, 2018; Parsaie et al., 2022; Shi and Jin, 2022). For specific instances of jump formation over spillways, empirical modeling on energy dissipation has been carried out by Ghaderi et al. (2020). Azimi and Shabanlou (2019) simulated three-dimensional hydraulic jumps in this typical-shaped channel by focusing on variations of flow over the free surface using the volume of fluid method. The values of Mean Absolute Percentage Error (MAPE), Root Mean Square Error (RMSE), and Coefficient of Determination (R^2) are calculated as 7.62, 0.022, and 0.99, respectively, for sequent depth, relative length of jump, and roller.

Under the assumption that flow is two-dimensional, Bushra and Afzal (2006) used the Reynolds equations to study the turbulent behavior of the hydraulic jump to predict the subsequent depth ratio. In order to forecast the changes in the flow-free surface, it is suggested to use either the smoothed particle hydrodynamics model to simulate the hydraulic jump as a two-dimensional flow or a numerical technique (i.e, volume of fluid technique). By analyzing the answers for sequent depth, roller length, and jump length, Bushra and Afzal (2006) have given the reason that the product of the depth-averaged axial velocity gradient and constant eddy viscosity gives the depth-averaged effective normal Reynolds stress.

According to the research works of Hager (1987 and 1989), the analysis for the hydraulic jump in a U-shaped channel has restrictions and boundary requirements. In a U-shaped channel, where the lower half has a semicircular section with a diameter D, and the upper half has a constant width that is proportional to the diameter, Bushra and Afzal (2006) expanded the model even further. Their results for the sequential depth ratio and relative length of jump for different non-dimensional upstream flow depths ranging from 0.1 to 0.5 were in good agreement with those of Hager (1989). Stahl and Hager (1999) have put forward the empirical relations for sequent depth ratio and length of jump and found nearly 20 % deviations from experimental results.

Some authors (Wang et al., 2023; Torkamanzad et al., 2019) stated that tail water level often affects the location and height of jump formation. By ensuring that the flow width is consistent amongst the various flow areas, non-symmetric flow can be stabilized. Hager (1987) and (1989) studied the sequent depth ratio, assuming that the channel bottom consists of a semicircular section has constant width at the top with 4% error.

Circular closed channel has been considered by Gargano and Hager (2002), Hager (1989), Maryami et al. (2021) and Stahl and Hager (1999). The analysis for estimating the length of the jump is scarce according to many sources, such as Gandhi (2024) and Yousefi et al. (2019). In the present study, hydraulic jump characteristics of U-shaped beds were analyzed and compared to identify the most effective conditions of the channel for reducing jump length and other flow characteristics. These were tested for a range of Froude numbers from 4 to 20.

In order to cover the gaps in the literature, the present study focuses on developing and validating empirical models for distinct jump characteristics. Further, different multivariate statistical techniques, namely PCA/FA, were applied to offer a better understanding of different hydraulic jump characteristics in a U-shaped channel. These methods allow for identifying the possible factor that influences the phenomenon most.

Many authors (Bagheri et al., 2023; Ogarekpe et al., 2022; Singh et al., 2004) stated that PCA describes the entire experimental data set with minimal loss of original information and offers information on the most meaningful parameters. Most importantly, models so developed can be used directly in the field for analysing hydraulic characteristics of flow over the spillway. This study is of practical applications to dams and spillways structures, where supercritical flow of high velocity in the channel is used to dissipate excess kinetic energy through hydraulic jump formation.

2. Experimentation and Data Acquisition

All the experiments for the aforementioned eight characteristics, including the sequent depth ratio (Y_2/Y_1), the relative jump height (h_j/Y_1), the relative energy loss (E_L/E_1), the efficiency of jump (E_2/E_1), the relative prejump depth (Y_1/E_1), the relative postjump depth (Y_2/E_2), the relative length of the roller (L_r/Y_1) and jump (L_j/Y_2) for Froude number varying between 4 and 20 and Reynolds number between 1,638,009 and 3,394,784 were conducted in a U-shaped channel. Parameters of the abovementioned relations are as follows: Y_1: is the prejump depth, Y_2: is the post jump depth, h_j: is the actual height of the hydraulic jump, E_1: is the specific energy before the jump, E_2: is the specific energy after the jump, L_r: is the length of roller, L_j: is the length of jump, and E_L: is the energy loss.

The experimental setup consists of six basic segments, namely, inlet tank, stilling basin, sharp-edge vertical regulating gates (both at upstream and downstream ends), main channel having perspex bottom and side wall, discharge tank, and discharge channel with proper instrumentation for measurement and control. The depth of flow across the breadth and length of the channel can be measured at various places from the entrance gate using pointer gauges. The least count of the pointer gauge is 1 mm.

Through a connecting pipe with a diameter of 10 cm and a regulating valve, water was able to enter the 0.43 × 31 × 0.80 m^3 constant head input tank, which was designed to minimize variation caused by changes in flow. Discharge is primarily regulated from the minimum to maximum capacity of the inlet tank to obtain the different data at various Froude numbers. Experiments were conducted in a U-shaped channel for number of runs at different discharges using sharp-edged controlling gates (at both upstream and downstream), and the same parameters were recorded for each run (Table 1).

Channels measuring 2.8 m in length, 0.3 m in width, and 0.4 m in height were used for the experiments. The generation of eddies and rollers is minimized by making the bottom surface flat for flow. The design of the side walls makes it easy to monitor the beginning and end of the jump and roller lengths during the experiments. At the very top of the side walls are affixed parallel rails that allow the pointer gauge to measure depth at various points throughout the length and width of the channel. A total of thirty-five sets of runs were made in extracting experimental data with a horizontal bed. Proper care would be taken to locate the end of the jump by finding the immediate downstream positions of the roller where the jump meets the downstream water surface. Tail water depth is ensured using downstream regulating gates installed in the setup. With the help of the upstream gate, the discharge is changed and allowed jump to form by adjusting the downstream gate. Once the jump position gets established at a particular discharge, all the measurement was taken.

Table 1. Different flow characteristics and its range for F_{r1} = 4 to 20 and R_{e1} = 16,38,009 to 33,94,784

Measured parameters	Y_1, Y_2, Q, L_j, L_r
Varied Parameters	F_{r1}, R_{e1}
Sequent depth ratio 'Y_2/Y_1'	1.5 - 5
Relative jump height 'h_j/Y_1'	0.7 - 3.5
Relative energy loss 'E_L/E_1'	0.5 - 0.95
Efficiency of jump 'E_2/E_1'	0.06 - 0.45
Relative prejump depth 'Y_1/E_1'	0.005 - 0.1
Relative postjump depth 'Y_2/E_2'	0.2 - 0.5
Relative length of roller 'L_r/Y_1'	4.5 - 22
Relative length of jump 'L_j/Y_2'	3 - 6

During the recording process, the author paid close attention to the occurrence of surface rollers and extreme turbulence. Achieving the lowest water losses, symmetric flow, depths at three locations throughout the main channel, and discharge at the downstream end have been the major design factors in order to get accurate and dependable experimental data. A schematic and a sectional view of the experimental setup are shown in Figures 1 and 2, respectively.

3. Development of Empirical Models

Several empirical models for flow properties are described in the literature (Bushra and Afzal, 2006; Rajaratnam and Subramanya, 1968), which describe that Reynolds number is a key factor in determining flow behavior. The efficacy of these approaches is well defined in identifying the drag influence on hydraulic jump phenomena and was investigated by Hu and Zhang (2018). Parameters Y_1, Y_2, V_1, V_2, L_r, L_j, E_L, υ, g, ρ, and ε are the variables that are involved and impact the flow characteristics in a U-shaped channel. As indicated in Eq. (1), they can be expressed as functions of both dependent and independent variables.

$$f\,(Y_1, Y_2, V_1, V_2, L_r, L_j, E_L, \mu, g, \rho, \varepsilon) = 0 \qquad (1)$$

where Y_1: is the prejump depth, Y_2: is the post jump depth, V_1: is the prejump velocity, V_2: is the post jump velocity, L_r: is the length of roller, L_j: is the length of jump, E_L: is the energy loss, ρ: is the density of water (kg/m^3), υ: is the kinematic viscosity (m^2/s), ε: is the surface roughness (m) and μ: is the dynamic viscosity of water (Ns/m^2). The dimensionless groups can be represented as Eq. (2).

$$\left(\frac{Y_2}{Y_1}, \frac{h_j}{Y_1}, \frac{E_L}{E_1}, \frac{E_2}{E_1}, \frac{Y_1}{E_1}, \frac{Y_2}{E_2}, \frac{L_r}{Y_1}, \frac{L_j}{Y_2}, \frac{V_1^2}{gY_1}, \frac{\rho V_1 Y_1}{\mu}, \frac{\varepsilon}{Y_1}\right) = 0 \qquad (2)$$

All variables involved in the phenomenon are considered and represented as a function in Eq. (1) as per the dimensional approach. Different dimensionless groups (parameters) are formed using these variables as mentioned in Eq. (2). Among this, Y_2/Y_1, h_j/Y_1, E_L/E_1, E_2/E_1, Y_1/E_1, Y_2/E_2, L_r/Y_1 and L_j/Y_2 are dependent parameters as they depend upon varying velocity (discharge), and F_{r1} and R_{e1} are independent parameters. A relationship between the approach Froude number and the incoming Reynolds number is found to hold for all eight hydraulic jump characteristics in a U-shaped channel.

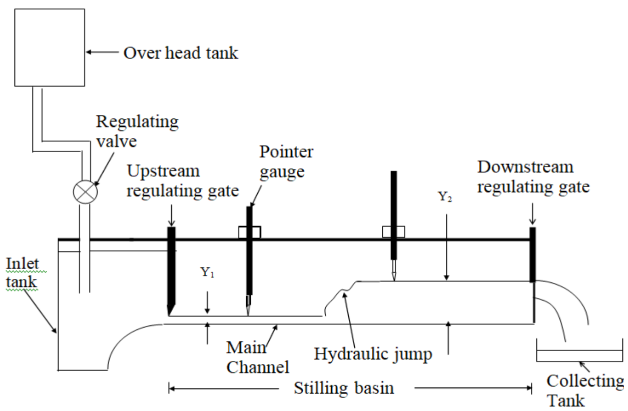


Fig. 1. Schematic diagram of experimental setup

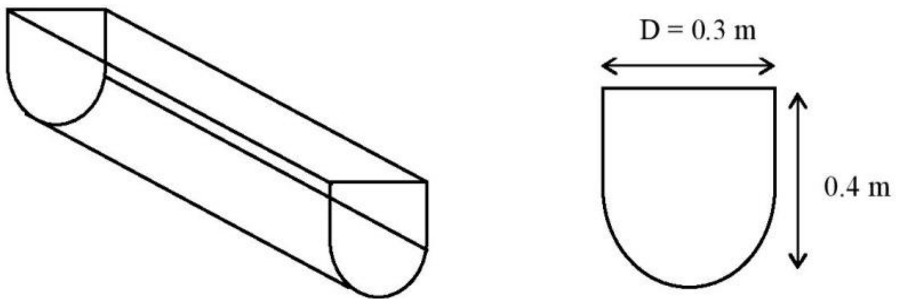
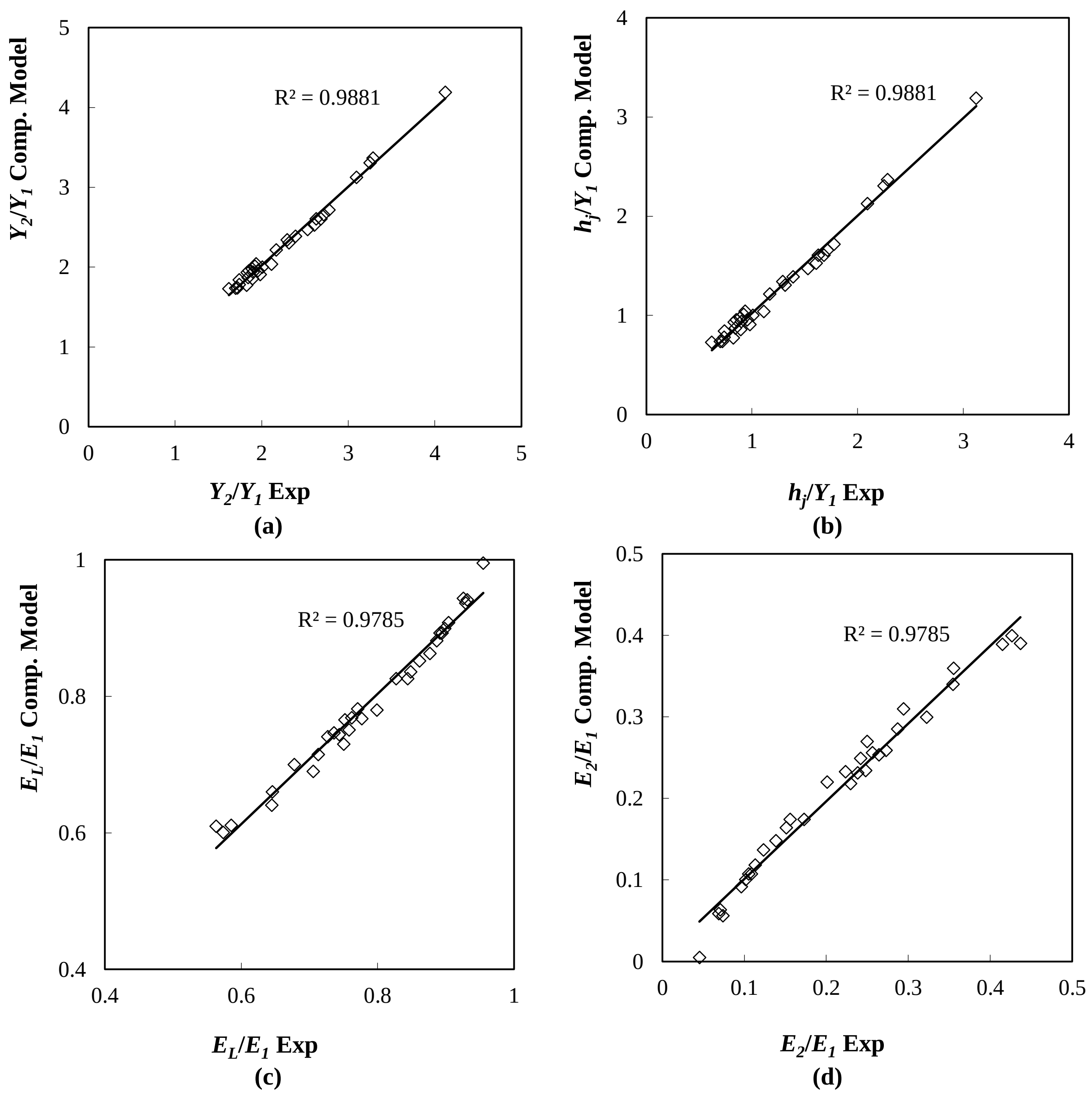


Fig. 2. Sectional view of a U-shaped channel

As an example, the sequent depth ratio in terms of dynamic viscosity can be expressed as Eq. (3).

$$\frac{Y_2}{Y_1} = f\left(\frac{V_1^2}{gY_1}, \frac{\rho V_1 Y_1}{\mu}\right) \quad (3)$$

The remaining flow parameters can be represented in a manner analogous to Eq. (3). The influence of surface roughness was not taken into the account when creating these groupings because of experimental constraints. Here, from Eq. (4) to Eq. (11), developed empirical models for all flow characteristics are provided. Figures 3a to 3h display the best-fitting model along with its R^2 value.



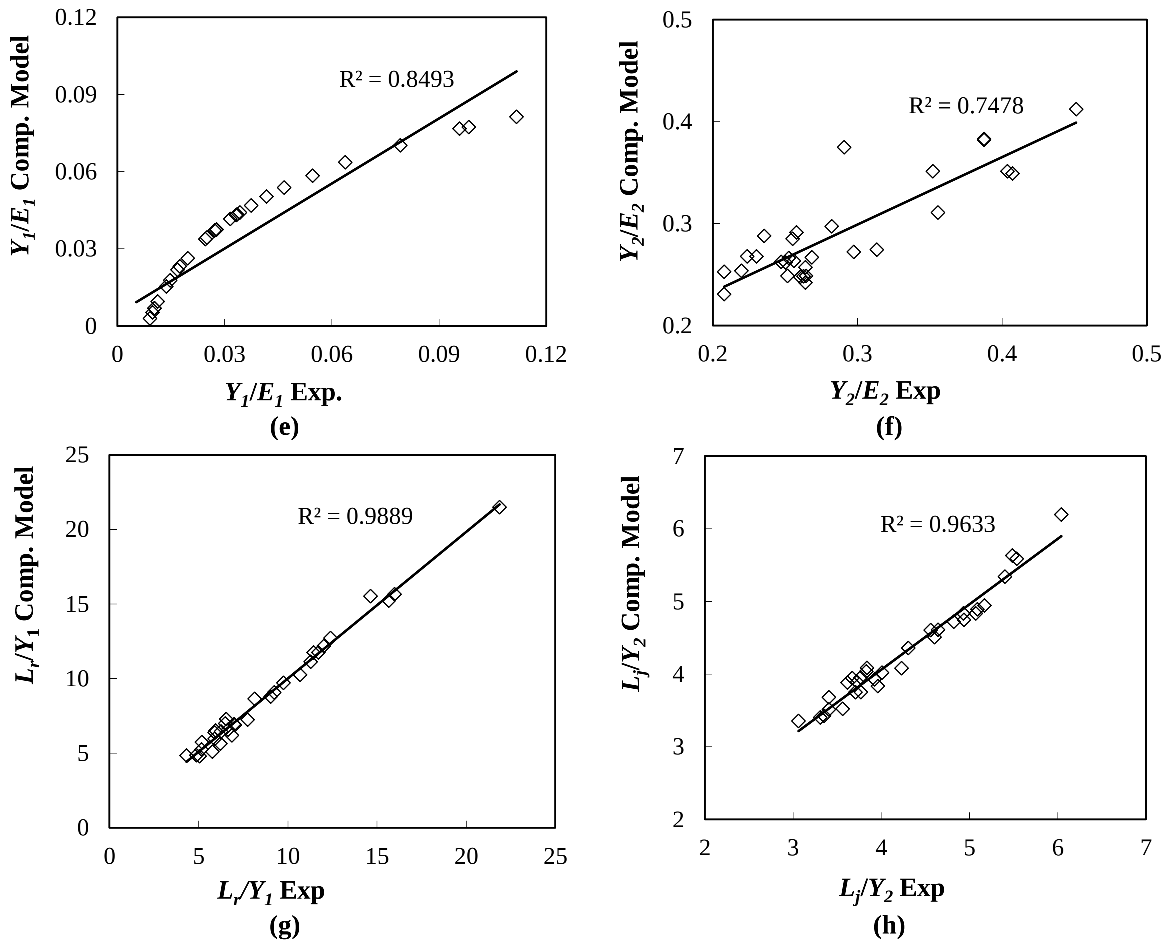


Fig. 3. Linear fit of empirical models (Eqs. (4) to (11)): a) Sequent depth ratio; b) Relative height of jump; c) Relative energy loss; d) Efficiency of jump; e) Relative prejump depth; f) Relative post jump depth; g) Relative length of roller; and h) Relative length of jump

Table 2. R^2 values for developed empirical models

Y_2/Y_1	h_j/Y_1	E_L/E_1	E_2/E_1
0.9881	0.9881	0.9785	0.9785
Y_1/E_1	Y_2/E_2	L_r/Y_1	L_j/Y_2
0.8493	0.7478	0.9889	0.9633

$$\frac{Y_2}{Y_1} = 17549\left(\frac{F_{r1}^2}{R_{e1}}\right) + 1.6 \tag{4}$$

$$\frac{h_j}{Y_1} = 17549\left(\frac{F_{r1}^2}{R_{e1}}\right) + 0.6 \tag{5}$$

$$\frac{E_L}{E_1} = 13.13\left(\frac{F_{r1}^{0.02}}{R_{e1}^{0.004}}\right) - 12 \tag{6}$$

$$\frac{E_2}{E_1} = -13.13\left(\frac{F_{r1}^{0.02}}{R_{e1}^{0.004}}\right) + 13 \tag{7}$$

$$\frac{Y_1}{E_1} = -1.9\left(\frac{F_{r1}^{0.03}}{R_{e1}^{0.001}}\right) + 2 \tag{8}$$

$$\frac{Y_2}{E_2} = 340\left(\frac{F_{r1}^{0.05}}{R_{e1}^{0.4}}\right) - 1 \tag{9}$$

$$\frac{L_r}{Y_1} = 74.5\left(\frac{F_{r1}^2}{R_{e1}^{0.5}}\right) + 4 \tag{10}$$

$$\frac{L_j}{Y_2} = 611\left(\frac{F_{r1}^{0.8}}{R_{e1}^{0.5}}\right) + 2 \tag{11}$$

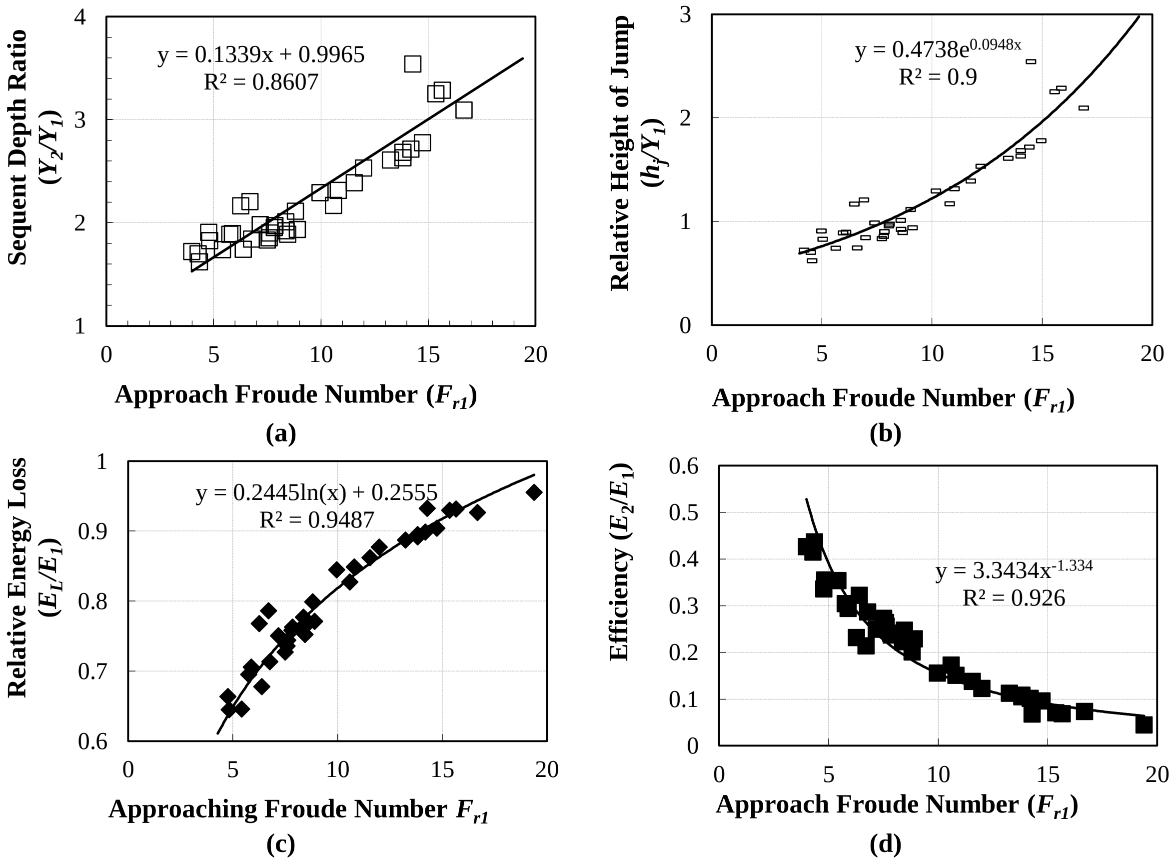
4. Data Analysis

Literature reveals that there are only a few studies reported for the analysis of hydraulic jump characteristics, which involve the prediction of jump length and its profile (Rajaratnam and Subramanya, 1968; Yousefi et al., 2019). In order to observe the influence of all the eight hydraulic jump characteristics in a U-shaped channel, the experimental results were used (Table 1).

The variations of hydraulic jump characteristics are shown in Figures 4a to 4h. Figure 4a shows a linear variation of sequent depth ratio (Y_2/Y_1) against the approach Froude number (F_{r1}) varied between 4 and 20. It shows that approximately 75 % of data lying within the range of ± 10% of the best-fit line drawn with R^2 value of 0.86, which shows scattering of data points due to the development of high surface rollers, causing inaccuracy in the measurements (Hu and Zhang, 2018). Similar results were also obtained by Bushra and Afzal (2006) and Hager (1989), but the observations by these researchers are slightly different because they performed the experiments on a completely filled U-shaped channel, while in the present study, no such condition was maintained as the U-shaped channel length was not completely filled.

In Figure 4b, relative jump height (h_j/Y_1) varies non-linearly and tends to increase as the approach Froude number (F_{r1}) increases from 4 to 20. Hager (1989) also found a similar pattern for the relative jump height versus Froude number, although they took into account different experimental flow circumstances. Approximately 65% of the data points fall within a 10% margin of error of the best-fit line, as can be seen in this image. Because of the extremely turbulent flow and the development of surface rollers, approximately 35% of the data points are scattered. Figure 4c shows a nonlinear increasing trend of relative energy loss (E_L/E_1) against approach Froude number (F_{r1}) ranging from 4 to 20. Approximately 90 % of data points are lying within ± 10 % of the best-fit line drawn. R^2 value of 0.95 indicates the best logarithmic relationship between relative energy losses against approach Froude number.

Figure 4d shows a nonlinear decreasing trend of variation of efficiency of jump (E_2/E_1) against approach Froude number (F_{r1}) varied between 4 and 20. In Figure 4, about 75% data are seen lying within the range of ±10% limit of the best fit curve drawn for observed values, the remaining 25% data shows either non-uniformity or unsymmetrical high surface roller formed during the flow. Subramanya (2019) also observed a similar trend for the efficiency of jump in his study. Figure 4e shows a nonlinear power fit of the experimental data for relative pre-jump depth (Y_1/E_1) and the approach Froude number (Fr_1) ranging between 4 and 20. Good fitting of data (R^2 = 0.99) shows a strong relationship between relative pre jump depth and approach Froude number. Figure 4f shows a nonlinear variation of relative post-jump depth (Y_2/E_2) against approach Froude number (F_{r1}) with a poor power fit of data points with a least value of R^2 = 0.04.



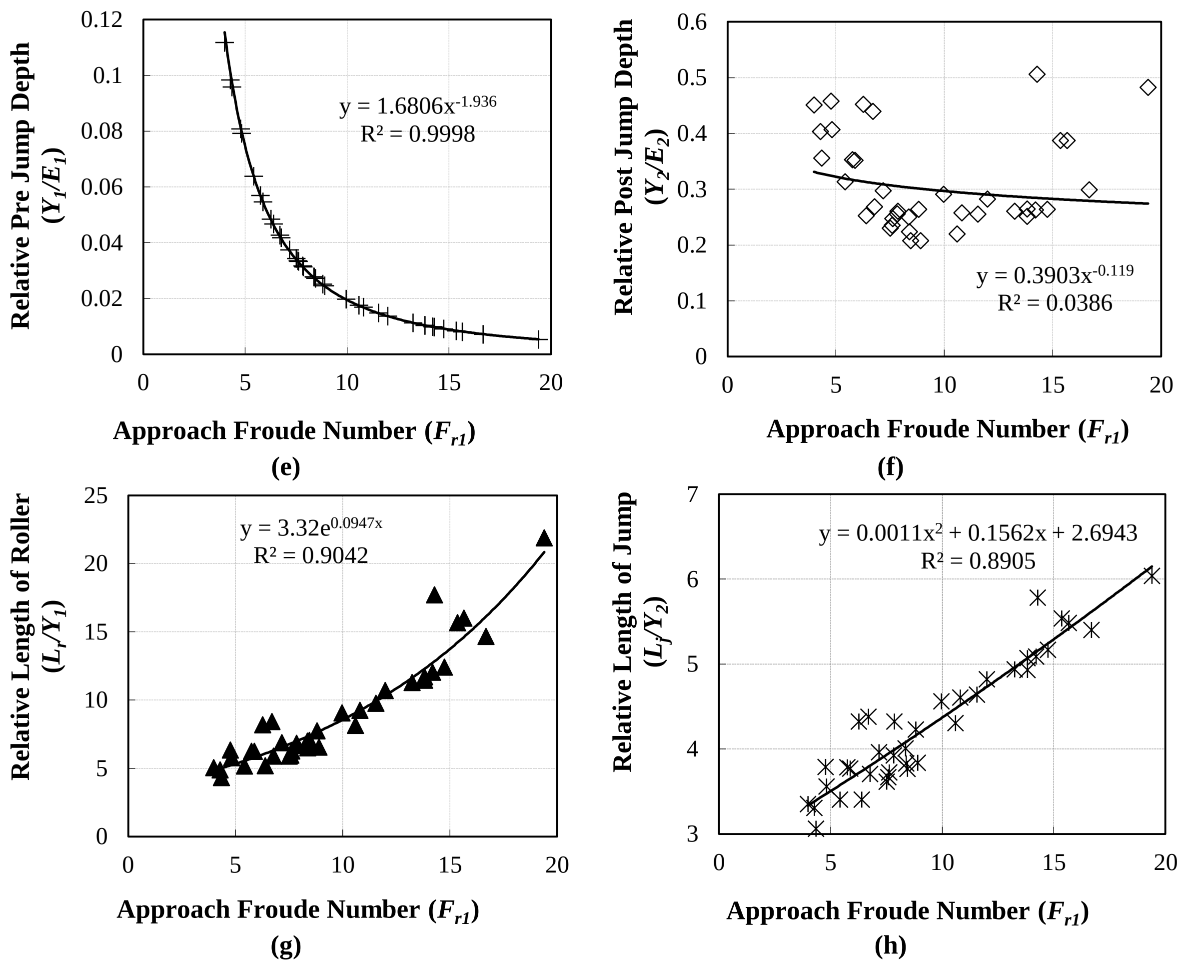


Fig. 4. Variation of hydraulic jump characteristics: a) Sequent depth ratio; b) Relative height of jump; c) Relative energy loss; d) Efficiency of jump; e) Relative prejump depth; f) Relative post jump depth; g) Relative length of roller; and h) Relative length of jump

Large deviation of data points indicates that relative post-jump depth is poorly related with approach Froude number between 4 and 20 under present channel conditions. Other researchers (Hager, 1989; Stahl and Hager, 1999) have studied the relative length of jump on a U-shaped channel under different channel conditions and observed deviations. Figure 4g shows a non-linear increment of relative length of roller (L_r/Y_1) against Froude number (F_{r1}) ranging from 4 to 20 with R^2 value of 0.9. It also shows that nearly 65% of data are seen lying within the range of ± 10% of the best fit curve drawn nearly 35% data is found scattered. The reason to this effect can be attributed to difficulties faced in accurate judgment of position of the starting and end of the roller.

Figure 4h shows a nonlinear variation of relative length of the jump (L_j/Y_2) against Froude number (F_{r1}) from 4 to 20 with R^2 value of 0.9. Nearly 75% of the data points lying within the range of ± 10% of the best fit polynomial drawn, and nearly 25% data are seen scattered lying outside the range of ± 10%, this again can be attributed due to high surface turbulence. Similar results have been reported by Achour and Debabeche (2003) using sill in a U-shaped channel by fixing Y_1 and varying the approach Froude number.

5. Testing and Comparison

Figure 5 shows the comparison of sequent depth ratio (Y_2/Y_1) against approach Froude number (F_{r1}) for U-shaped channel between the values predicted from Eq. (4) with the experimental data of Achour and Debabeche (2003) and Hager (1989) for U-shaped channel and Hager (1989) and Stahl and Hager (1999) for circular channel showing some deviation among each other.

This can be attributed due to the reason that Achour and Debabeche (2003) experimental data holds good for a channel provided with a sill, as they studied the channel provided with a sill and assumed that the dimension of the sill plays a significant role in the formation of a hydraulic jump. On the other hand, Hager (1989) and Stahl and Hager (1999) presented their data for circular channels with the condition that $0.1 < Y_1 < 0.8$ and the downstream depth is completely filled.

Hager (1989) has conducted his experiment with the condition that downstream depth $Y_2 = 1$ for various values of upstream flow depth $0.1 < Y_1 < 0.8$ at all approach Froude numbers and the semicircular part remained filled, whereas present experimentation is conducted for different upstream depth Y_1 at all approach Froude number F_{r1} with different depth at semicircular portion of the channel. Moreover, the obtained value of Y_2/Y_1 from the present model lying between the experimental results of other authors and can be assumed to be approximately correct.

Figure 6 shows the comparison of values obtained for relative height of jump (h_j/Y_1) using present model (Eq. (5)) with the experimental data of Bushra and Afzal (2006), Achour and Debabeche (2003) and Hager (1989) for U-shaped channel and Hager (1989) and Stahl and Hager (1999) for circular channel, which showing some deviation among each other. This again can be attributed to the same reason as explained above. Hence, it is concluded that the results of different authors for the present channel varying from each other due to dependence on experimental conditions.

However, results obtained from Eq. (5) may be accepted for the present channel condition. Figure 7 shows the comparison of the relative length of jump (L_j/Y_2) for the values obtained from the present model (Eq. (11)) with the experimental data of Achour and Debabeche (2003) and Hager (1989) for the U-shaped channel, showing some deviation among each other. This can be attributed to experimental data holds good (Achour and Debabeche, 2003) for a channel provided with a sill, as their study is based on the significant role of the dimension of the sill on the relative length of the jump. Hager (1989) has conducted his experiment by keeping downstream depth $Y_2 = 1$ (constant) for various value of upstream flow depth $0.1 < Y_1 < 0.8$ at all approach Froude number F_{r1} with semicircular part remained filled, whereas present experiment is conducted for different upstream depth Y_1 at all approach Froude number F_{r1} with different depth at semicircular portion of the channel. Moreover, the obtained value of L_j/Y_2 from the present model Eq. (11) lying between the experimental results of Achour and Debabeche (2003) and Hager (1989), which may be assumed approximately correct to the present channel condition for the prediction of the relative length of jump.

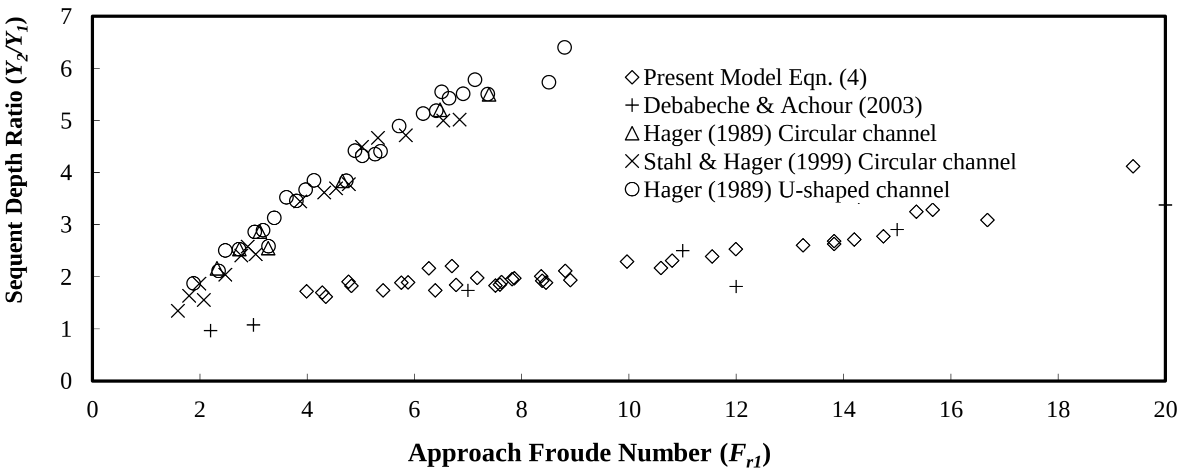


Fig. 5. Comparison between the obtained values from the present model (Eq. (4)) with different authors' data

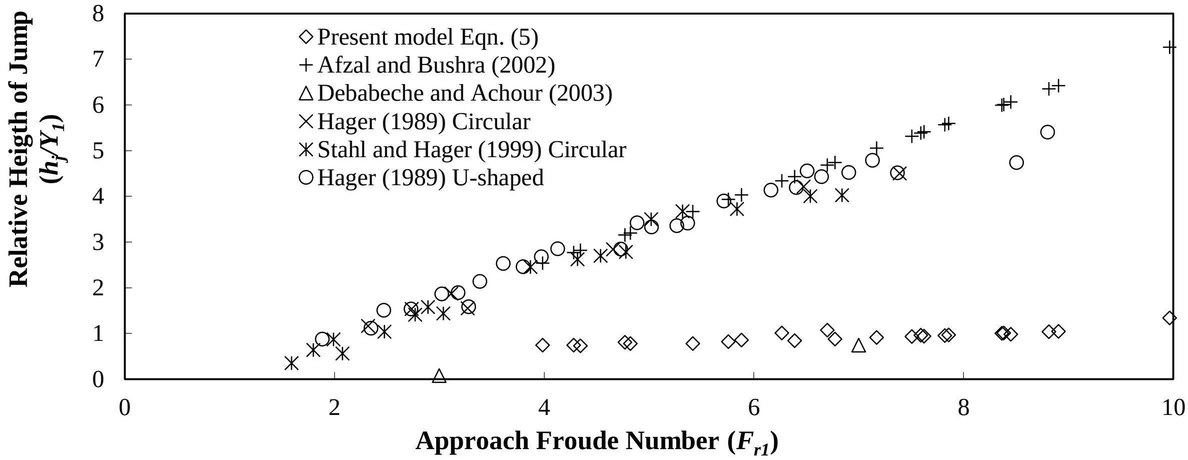


Fig. 6. Comparison between the obtained values from the present model (Eq. (5)) with different authors' data

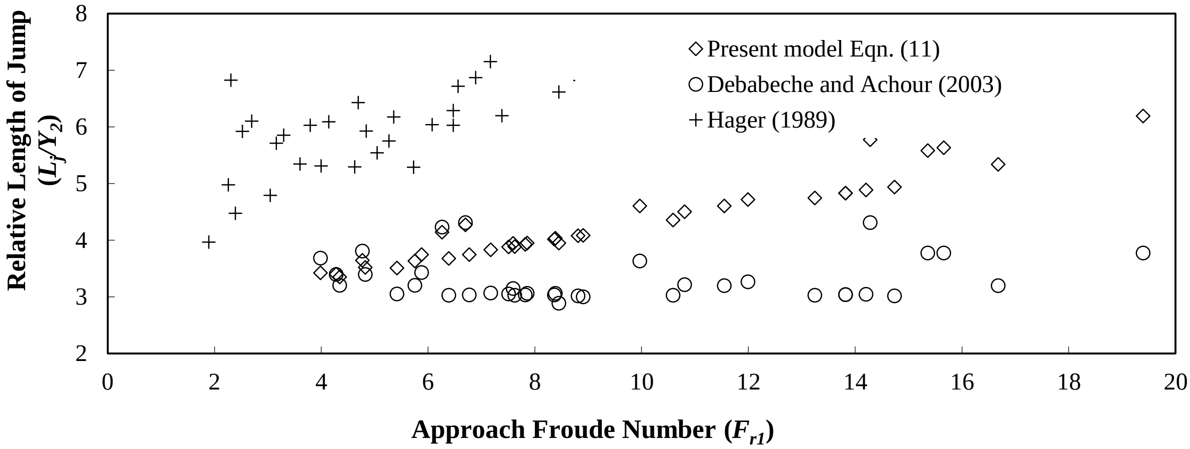


Fig. 7. Comparison of values obtained using Eq. (11) with experimental values of other authors

The predictions are well supported by the experimental results of Hager (1989) and Stahl and Hager (1999). Figure 8 shows the comparison of the relative length of jump varying against approach Froude number (L_j/Y_1). It compares the values obtained from Eqs. (4) and (11) with the value obtained using Bretz (1987) and Rajaratnam and Subramanya (1968) models. The results obtained from the present model, Eqs. (4) and (11), lying below the results of other authors, which may be attributed to different experimental and channel conditions.

Bradley and Peterka (1957) proposed their model based on experimental results for prismatic channels. In their study, the length of the surface roller was given more preference then length of jump, i.e., the length of roller $L_r = 4.5Y_2$ and the length of jump is approximately $L_j = 1.3L_r$.

Bretz (1987) has given his empirical model for $4 < F_{r1} < 12$ with an average deviation of + 5 Y_1. It is therefore concluded that the results obtained from different authors are varying, some have proposed length of jump is independent of channel shape (Bushra and Afzal, 2006; Stahl and Hager, 1999) proposed length of jump is order of length of roller, hence the presented model may be accepted for the prediction of the relative length of jump in U-shaped horizontal channel. Due to the limited availability of results for validation of models, namely E_L/E_1, E_2/E_1, Y_1/E_1, Y_2/E_2, and L_r/Y_1 presented in Section 6 were not tested. Nevertheless, the models described in Eqs. (6) to (10) can still be used to forecast the hydraulic jump characteristics with a high degree of accuracy, as they show a good R^2 value (provided in Table 2).

6. Principal Component Analysis (PCA)/ Factor Analysis (FA)

A better understanding of hydraulic jump features under diverse discharge conditions can be achieved by applying multivariate statistical approaches, such as PCA and FA. According to Bagheri et al. (2023) and Singh et al. (2004), PCA can help identify the most important factors that characterize complete data sets, allowing for data reduction with minimal loss of original information. An effective pattern detection method that decomposes a large number of interrelated variables (Y_2/Y_1, h_j/Y_1, E_L/E_1, E_2/E_1, Y_1/E_1, Y_2/E_2, L_r/Y_1, L_j/Y_2, F_{r1}, R_{e1}, ε/Y_1) into their major components to explain their variance.

According to the calculations done by Ogarekpe et al. (2022), the PCA can be represented mathematically by Eq. (12).

$$Z_{ji} = a_{i1}x_{1j} + a_{i2}x_{2j} + a_{i3}x_{3j} + ...a_{im}x_{mj} \quad (12)$$

where a: is the component loading, i: is the component number, j: is the sample number, m: is the total number of variables (involved in hydraulic jump characteristics), x: is the measured value of the variable, and Z: is the component score.

FA is a technique that is used to reduce a large number of variables (Y_2/Y_1, h_j/Y_1, E_L/E_1, E_2/E_1, Y_1/E_1, Y_2/E_2, L_r/Y_1, L_j/Y_2, F_{r1}, R_{e1}, ε/Y_1) into fewer number of factors according to Ogarekpe et al. (2022). By spinning the axis, it extracts a new set of variables called Verifactors (VFs) and further minimizes the influence of variables that were not significant in PCA. The FA is represented by Eq. (13).

$$Z_{ji} = a_{f1}f_{1i} + a_{f2}f_{2i} + a_{f3}f_{3i} + ...+ a_{fm}f_{mi} + e_{fi} \quad (13)$$

where a: is the factor loading, e: is the residual term accounting for errors or other sources of variation, f: is the factor score, i: is the sample number, j: is the variable number, m: is the total number of factors, and Z: is the measured value of a variable.

Both PCA and FA use similar equations to represent their respective methods. The only difference between the two is that FA uses a combination of factors to represent the measured variable, whereas PCA uses a linear combination of measured variables. According to Helena et al. (2000) and Bagheri et al. (2023), VFs in FA can contain hypothetical, latent variables that cannot be seen. The author ran PCA/FA on the correlation matrix of the observed data set.

The Variance and inter-variable relationships are quantified in the correlation coefficient matrix. PCA/FA was applied to the data matrix of the U-shaped channel (Table 3). The results obtained are shown in Tables 3 and 4. This tables reveal two principal components explaining 64.239% and 33.491% of the total variance. Using the procedure mentioned above, the verifactors for the different principal components are shown in Table 4. The verifactor VF1 has a strong positive loading on Y_2/Y_1, h_j/Y_1, E_L/E_1, L_r/Y_1, and L_j/Y_2, while VF2 has strong positive loadings on Y_1/E_1 and Y_2/E_2 and moderate strong positive loading on E_2/E_1. The results indicate that all eight jump variables analyzed in this study have a substantial impact on hydraulic jump in U-shaped channels. Hence, all the jump features play a crucial part in steering the hydraulic jump phenomena.

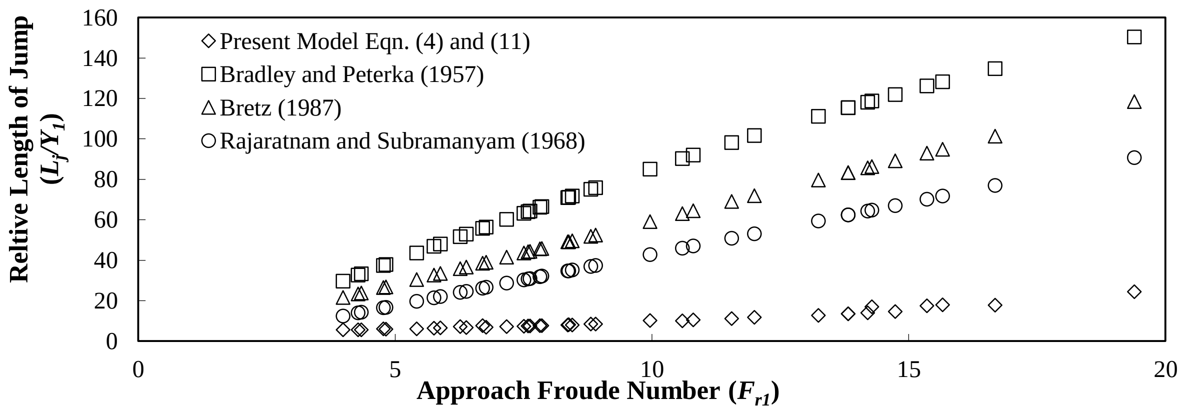


Fig. 8. Comparison of the obtained relative length of jump using Eqs. (4) and (11) with different authors

Table 3. Variance explained by two principal components

Rotation sums of squared loadings			
Component	**Total**	**% of Variance**	**Cumulative (%)**
1	7.709	64.239	64.239
2	4.019	33.491	97.730
Hydraulic jump characteristics			
Component	**Strong +ve loading**		**Moderate +ve loading**
1	Y_2/Y_1, h_j/Y_1, E_L/E_1, E_2/E_1, L_r/Y_1, L_j/Y_2		---
2	Y_1/E_1, Y_2/E_2		E_2/E_1

Table 4. Rotated component matrix for two principal components

Characteristics	Component	
	VF 1	**VF 2**
Y_2/Y_1	0.980	-0.185
h_j/Y_1	0.980	-0.185
E_L/E_1	0.782	-0.599
E_2/E_1	-0.782	0.599
Y_1/E_1	-0.541	0.826
Y_2/E_2	0.497	0.844
L_r/Y_1	0.980	-0.188
L_j/Y_2	0.931	-0.327

7. Conclusions

Data analysis was carried out for all the eight hydraulic jump characteristics using experimental results. The obtained functional relationship is subjected to empirical modeling with a high coefficient of determination (R^2).

Comparisons of these models were also made to explore the merit of the empirical models for specific characteristics using the available data from literature (Bradley and Peterka, 1957; Bretz, 1987; Bushra and Afzal, 2006; Achour and Debabeche, 2003; Hager, 1989; Rajaratnam and Subramanya, 1968; Stahl and Hager, 1999). Also, the existing literature models for the U-shaped channel are tested for the fitness of the present experimental results. Characteristics Y_2/Y_1, h_j/Y_1, L_j/Y_2, L_j/Y_1 were tested, validated, and compared with the results of Bushra and Afzal (2006), Achour and Debabeche (2003), Hager (1989) and Stahl and Hager (1999) for different channel conditions. The obtained values of Y_2/Y_1, h_j/Y_1, and L_j/Y_2 from the present models lying between experimental results of Achour and Debabeche (2003) and Hager (1989), proving their efficacy (Figures 5 to 7). Eqs. (4) and (11) models were validated for L_j/Y_1 with the experimental values of Bradley and Peterka (1957), Bretz (1987) and Rajaratnam and Subramanya (1968), and it shows little deviation from their values because their results holds for prismatic channel based on surface roller.

Due to different assumptions, it can be stated based on a similar trend that L_j/Y_1 model can be used. The results obtained from PCA/FA analysis can be successfully used to identify the principal hydraulic jump characteristics. Methods of multivariate statistical analysis can be used as valuable tools for identifying the key characteristics in a phenomenon and its relative significance. This method requires a huge database too for acquiring the most preferable and definite results. It is recommended to acquire a huge database for applications of PCA/FA for a better representation of key hydraulic jump characteristics.

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9. Declaration

No generative artificial intelligence tools were used in the preparation, writing, or image creation of this manuscript.

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