





Probabilistic Analysis of Braced Excavation for Box Drain Construction in Cuttack

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ABSTRACT: This study presents a probabilistic failure analysis of a Braced Excavation System (BES) for a 9.0 m deep box drain in Cuttack, Odisha, using Monte Carlo Simulation (MCS) and Subset Simulation (SS) methods. The drain spans 3.0 km through highly plastic clayey soil with low shear strength, requiring excavation and braced techniques. Geotechnical field exploration suggests that the soil along the 3.0 km long stretch of the proposed box drain site has a very wide variation. A probabilistic analysis is conducted to ascertain the risk involved in the designed BES. In probabilistic analysis of the BES, cohesion and unit weight of soil are treated as lognormal distributed random variables. Spatially correlated random fields along the depth are generated using the Pearson correlation matrix and the Markov correlation function. MCS with 10000 samples has been run to conduct the probabilistic response of the BES. The paper also presents the results of SS, an advanced version of MCS that enables rapid probabilistic analysis. Furthermore, the results obtained using MCS and SS are compared to establish the relative superiority of SS over MCS.

Keywords: Braced Cut, Probabilistic Analysis, Random Field, Cross-Correlation, Monte Carlo Simulation, Subset Simulation.

1. Introduction

Underground infrastructure construction is increasingly being considered in densely populated urban areas to optimise land utilisation. Braced Excavation Systems (BESs) are often used in places with deep deposits of soft clay, and excavation is required to reach the desired foundation level or for other purposes (Farzi et al., 2018). Braced excavation is a method where deep excavations with straight

vertical faces are laterally supported by a sheeting and bracing system until the structure is built. The design of braced cuts involves two distinct but interrelated features, namely, stability of excavation, ground movement, control of water into the excavation, effect of adjoining structures, and so on. In deep excavations, creating a safe slope on the excavation face is generally not feasible due to high cost, unavailability of space, etc. Therefore, the excavation is temporarily supported by

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sheets/walls and struts, which are removed individually when their requirements cease.

One crucial aspect of BES design is to ensure the structural integrity and safety of the bracing system. To calculate the stresses exerted on the struts in braced excavations, empirical methods like as the Apparent Pressure Diagram (APD) are widely used.

Various researchers (Peck, 1969; Terzaghi et al., 1996; Zhang et al., 2021; Dan and Sahu, 2022; Han et al., 2023) advocated the use of the APD to assess the amount and distribution of prop loads in braced excavations for different soil types, such as sands, strong-fissured clays, and soft to medium clays. The uncertainties associated with the variation of soil properties along the depth can negatively affect the stability and safety of the BES. Therefore, it is of utmost importance to consider these uncertainties during the design and analysis of the BES.

Several researchers (Sekhavatian and Janalizadeh Choobbasti, 2018a; He et al., 2020; Zhang and Liu, 2022; Zhao et al., 2022; Zhang et al., 2024) have used different approaches for Probabilistic Analysis (PA) for braced cut. Luo et al. (2012) used an equivalent variance technique to consider the spatial variability so that PA using the first-order random field technique of basal heave braced excavation failure can be accessed. It was noticed that the First Order Reliability Method (FORM) (Zhang et al., 2019) and the Monte Carlo Simulation (MCS) technique produce almost similar results. Luo and Das (2016) used the system-level probabilistic serviceability assessment approach for braced excavations in clays. The authors used MCS to generate multiple realizations of the random variables, and evaluated the system-level reliability index (β).

Qi and Zhou (2017) investigated braced excavation in a three-step procedure. First, a braced excavation's finite element model was created, taking bracing systems into account as well as the impacts of the interaction between the soil and the structure. Also, a PA was conducted in

order to account for the uncertainties associated with back analysis. Chowdhury (2017) presented a Reliability Analysis (RA) of excavation-induced basal heave, concentrating on determining the stability of deep excavations while taking ground movements at the excavation's base into consideration to assess the excavation system's vulnerability.

Sekhavatian and Janalizadeh Choobbasti (2018b) presented a deep excavation RA utilising the response surface and MCS methodologies. Luo et al. (2018) performed finite element modelling to predict the excavation-induced wall and ground movement and concluded that the system probability of serviceability failure is greater than or equal to that for each single failure mode. It is a known fact that soil parameters are correlated with each other, and it is necessary to determine the relative contributions of these factors on the overall stability of the system.

Cho and Park (2010) highlighted the importance of considering spatial variability cross-correlation for analyzing and designing strip footings. The choice of the cross-correlation coefficient between the soil's parameters is an important factor for the correct estimation of the probability of failure (P_f) of any geotechnical structure. Javankhoshdel and Bathurst (2016) investigated the influence of cross-correlation between soil parameters on the P_f for simple cohesive and c - ϕ soil slopes.

Nguyen and Chowdhury (1985) identified the possible correlation between random values of the shear strength parameter that can affect the likelihood of failure for slopes. Recently, Ahmad et al. (2024) conducted a probabilistic analysis, particularly in railway embankment, using Subset Simulation (SS) by incorporating machine learning techniques. Sabri et al. (2024) performed the SS and MCS technique using Excel spreadsheet environment to estimate the reliability of a soil slope. In the current study, the effect of cross-correlation of cohesive soil parameters on P_f of a braced excavation has

been examined. MCS and SS techniques have been utilized to estimate the P_f of a BES for the construction of a box drain in the city of Cuttack, India. SS is an advanced version of MCS developed by Au and Wang (2014) that requires a relatively smaller number of samples compared to MCS for predicting P_f with desired accuracy. The risk analyses have been carried out for developing random realizations of the soil parameters (i.e., cohesion and unit weight) along the depth of the braced cut. The effects of correlation between the soil parameters on the probability of failure are also investigated.

This study is helpful for estimating the risk of failure for the safety of the construction.

2. Research Significance

The probabilistic failure response of a large and important civil engineering project should ideally be carried out to determine the confidence level in the designed system. FORM, second order reliability method, first order second moment method, MCS, and SS are some of the techniques which have been used successfully in the past by the investigators (Wong, 1985; Lee and Kwak, 1987; Juang et al., 2019; Yang and Ching, 2019; Low, 2021). However, the study of the past literature shows that SS method has rarely been used for PA of BES. To obtain reliable results using MCS, it is necessary to consider a large number of samples, i.e., in the order of 10000 or more (Au and Wang, 2014). SS is an advanced version of MCS in which the PA is carried out in a few levels.

The samples in each level are generated in a way such that they are shifted towards the failure region in subsequent levels, and thus, the failure response of the system can be quickly simulated in SS analysis utilizing fewer samples compared to MCS.

In the present paper, the results of PA of a BES using SS of a real-life project are reported. A 9.0 m deep cut is supported with a braced excavation technique for the

construction of a box drain in the state of Cuttack, Odisha, India. The results have been further compared with the results obtained using MCS to establish the relative superiority of SS over MCS. The spatially variable and cross-correlated random fields of cohesion and unit weight are developed for representing the uncertainties arising due to changes in soil properties within the domain. The failure probabilities of the system are presented for both correlated as well as uncorrelated. The authors believe that the present work can shed important light on the probabilistic failure response of deep excavation projects.

3. Study Area

The study area is the city of Cuttack in the state of Odisha situated at latitude $20^{\circ} 31' 23''$ and $20^{\circ} 52' 30''$ north and longitude $85^{\circ} 47' 17''$ and $85^{\circ} 78' 80''$ east. The state government has undertaken a project to construct a box drain for a 3.0 km long stretch inside the city. The construction of the box drain requires excavation up to 8.0-9.0 m depth from the present ground level.

Figure 1a presents the 3.0 km long alignment of the box drain to be constructed in the city of Cuttack. Figure 1b shows the details of the box drain. It should be noted that the soil below the box drain has very low shear strength, necessitating that the low shear strength soil below the box drain level should be replaced by coarse sand of 3.0 m depth. Therefore, it is necessary to excavate up to 9.0 m depth from the ground level.

An extensive site investigation was carried out to determine the necessary geotechnical parameters of the soil in the construction site. Soil samples were collected from a total of 31 boreholes dug at the site along the 3.0 km long stretch, each extending at a distance of 100.0 m of the proposed box drain. All necessary geotechnical parameters, such as grain size distribution, Atterberg limits, cohesion, unit weight, moisture content, and compression index, were determined in the laboratory.

However, only cohesion and the unit weight values along the depth of different boreholes have been gathered at various chainage locations, as shown in Figures 2a and 2b, because these two parameters are

primarily required for braced excavation analysis. Note that in this current study, analysis results have been done at chainage distance 3+310 only.

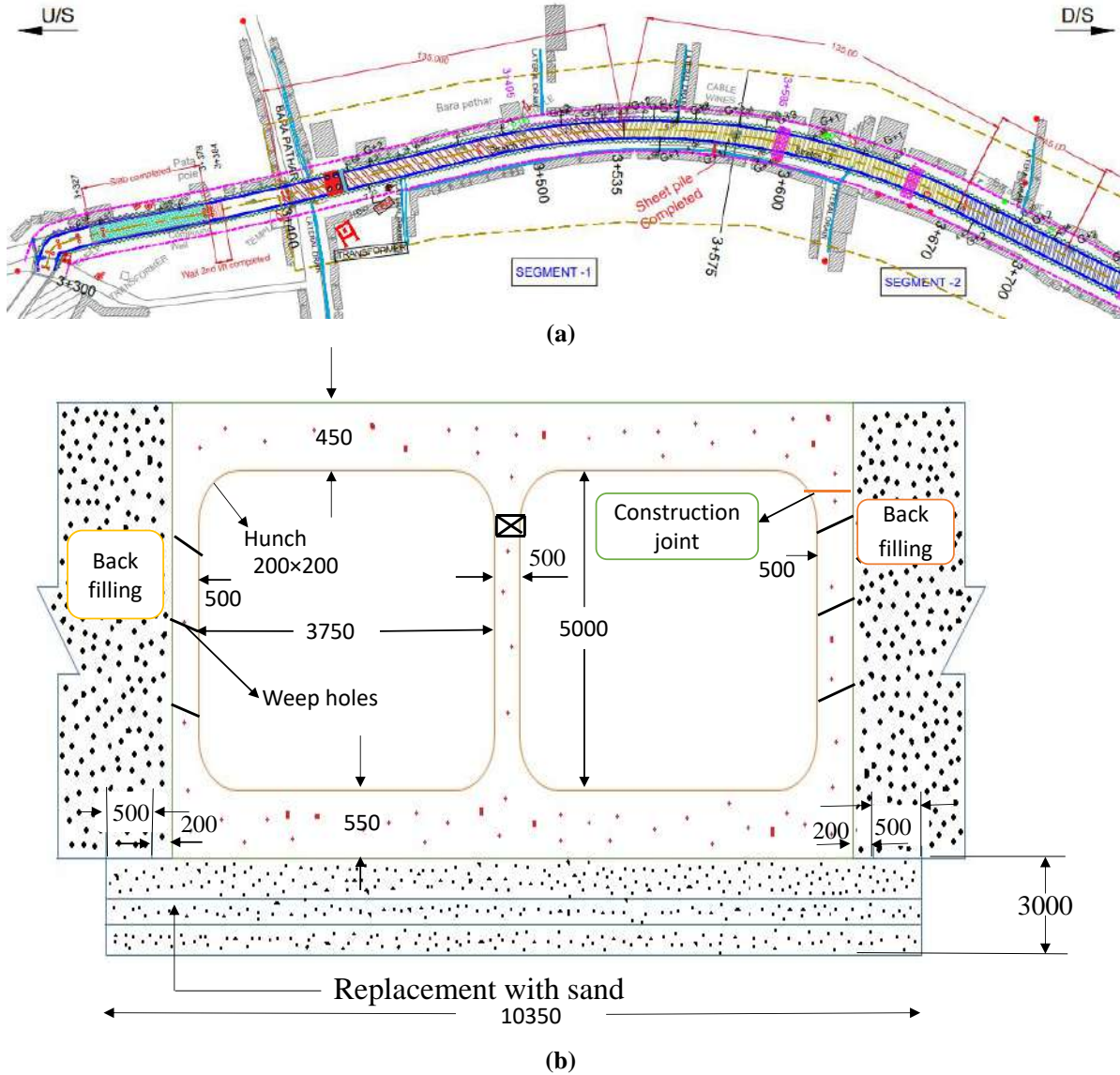
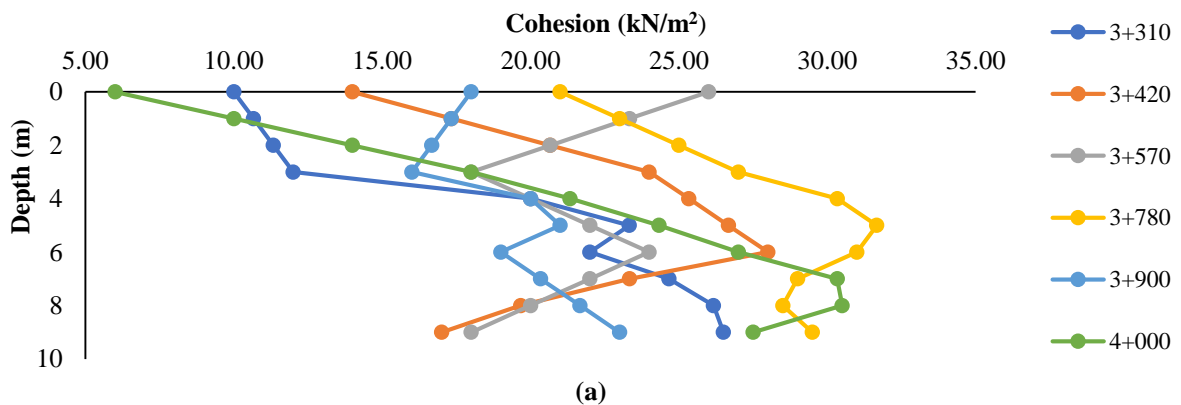


Fig. 1. a) 3.0 km long alignment of the box drain in Cuttack, Odisha; and b) Proposed box drain profile (all dimensions are in mm)



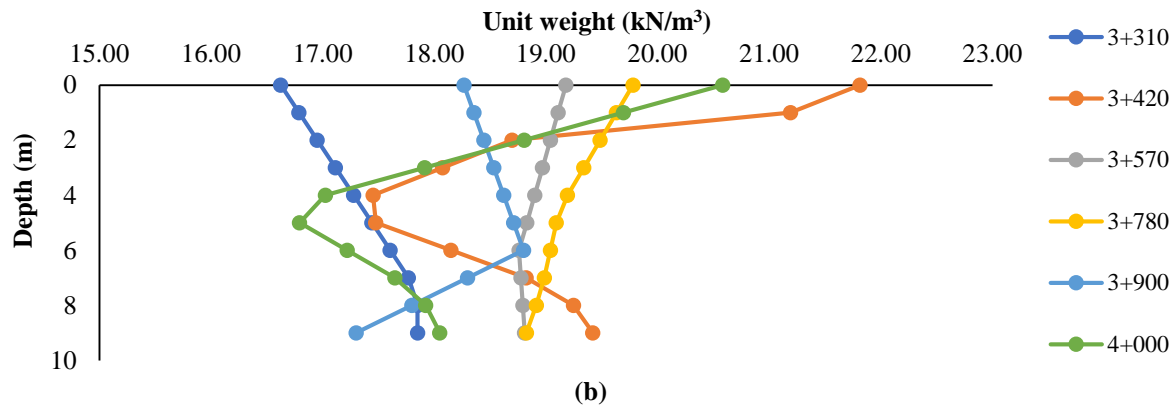


Fig. 2. a) Mean value of unit weight along the depth for a borehole at various chainage distances; and b) Mean value of cohesion along the depth for a borehole at various chainage distances

4. Methodology

The Lateral Earth Pressure (LEP) in a braced cut depends on soil type, construction method, and equipment. The LEP varies because of spatial changes in soil parameters both horizontally and vertically. For the braced cuts: 1) design of cuts in sand; and 2) design of cuts in clay, Peck (1969) and Sivakugan and Das (2009) proposed an APD as shown in Figure 3. The APD of cuts in sand, soft and medium clay, and stiff clay are depicted in Figures 3a to 3c, respectively. The magnitude of lateral APD (σ_a) is shown in Eqs. (1-2). Peck (1969) classified clays based on the non-dimensional stability number (N_s) as expressed in Eq. (3). The value of $N_s > 4$ indicates the soft to medium clay, and $N_s < 4$ stands for stiff clay. Eqs. (4) and (5) present the value of σ_a for the soft to medium clay and stiff clay, respectively.

$$\sigma_a = 0.65\gamma HK_a \quad (1)$$

$$K_a = \tan^2 \left(45^\circ - \frac{\phi'}{2} \right) \quad (2)$$

$$N_s = \frac{\gamma H}{c} \quad (3)$$

$$\sigma_a = \gamma H \left[1 - \left(\frac{4c}{\gamma H} \right) \right] \text{ and } \sigma_a = 0.3\gamma H \quad (4)$$

(whichever is more)

$$\sigma_a = 0.2\gamma H \text{ to } 0.4\gamma H \text{ (with an average of } 0.3\gamma H) \quad (5)$$

where c : is the undrained cohesion, ϕ' : is the effective angle of internal friction, γ : is the unit weight of soil, K_a : is Rankine active pressure coefficient. The soil sample in the study area is mostly cohesive soil. It has been observed that the soil parameters (i.e.,

c' and γ) vary depending on the excavation depth. Since different clay layers have been encountered in the cut, the average value of c' and γ can be calculated using Eq. (6) and Eq. (7) as follows.

$$c_{av} = \frac{1}{H} (c_1 H_1 + c_2 H_2 + \dots + c_n H_n) \quad (6)$$

$$\gamma_{av} = \frac{1}{H} (\gamma_1 H_1 + \gamma_2 H_2 + \dots + \gamma_n H_n) \quad (7)$$

where, H_1, H_2, \dots, H_n : are the thicknesses of soil layers, c_1, c_2, \dots, c_n : are the cohesions of each layer.

4.1. Load on Strut

Bracing strut and excavation bracing frame support braced excavation by propping the vertical face (Bahrami, 2019). This research employs four designated struts (A, B, C, D) shown in Figure 4. Additionally, sheet piles are assumed to have hinges at struts B and C. Including hinges at struts B and C increases shear forces and bending moments, ensuring a conservative design. The forces F_A, F_B, F_C , and F_D are the reaction forces (per unit length) at the strut levels. The reaction forces were determined through equilibrium calculations. The load on the strut can be accomplished by utilizing the formula $P_i = F_i s_i$ ($i = A, B, C$, and D).

Notably, subscripts P, F , and s denote the load, force, and spacing between strut levels, respectively; while superscript i indicates the strut level. Interested readers can refer to the works of Sivakugan and Das (2009) for further details on braced excavation procedures.

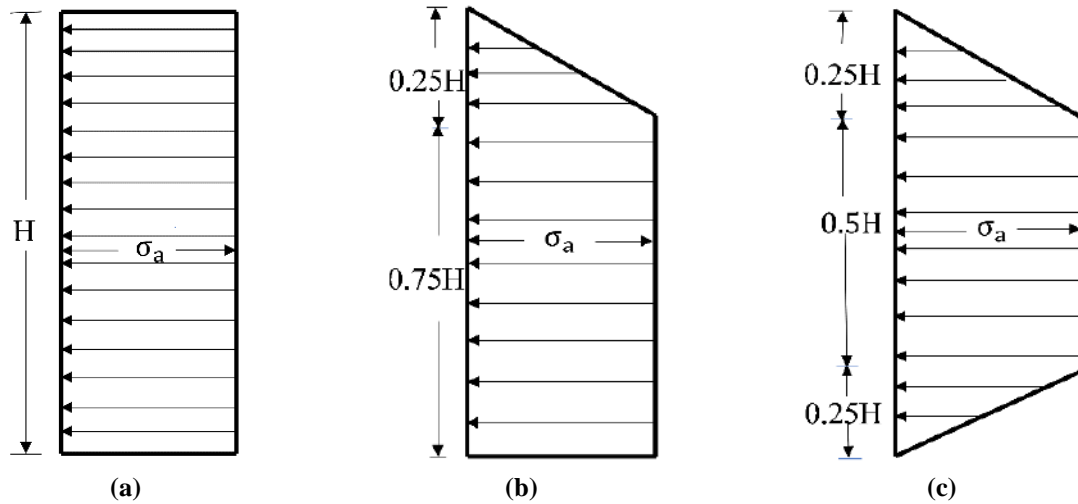


Fig. 3. APD envelope of: a) Sand; b) Soft to medium clay; and c) Stiff clay (Peck, 1969)

4.1.1. Sheet Piles

A steel sheet pile has been used to support the braced cut. Calculations are performed to determine the maximum bending moment (M_{\max}) for each of the sections depicted in Figure 4b. The section modulus (S) of the sheet pile can be calculated using Eq. (8).

$$S = \frac{M_{\max}}{\sigma_{all}} \quad (8)$$

where σ_{all} : is the allowable flexural yield stress of steel.

4.1.2. Wales

Steel structural members that transfer load from the diaphragm wall to the strut are called wales. Wales is fastened to the sheet pile at points meeting lateral support requirements. Figure 4a depicts the basic installation structure of the braced cut. The maximum moment for the pinned connection of the wales at any level can be determined using Eq. (9). The Factor of Safety (FOS) is defined using Eq. (10) based on the maximum 'S' value available at the site. This study reports the maximum value of $S = 2000 \text{ cm}^3$ at the site.

$$M_{\max} = \frac{F_i(s_i)^2}{8} \quad (9)$$

$$FOS = \frac{\text{maximum section modulus of wales}}{\text{available section modulus}} \quad (10)$$

4.2. Monte Carlo Simulation (MCS)

The MCS method is a mathematical procedure for continuously evaluating an empirical operator having a random variable with a known probability distribution (Liu et al., 2020). For obtaining the desired accuracy level of P_f , the number of samples to be generated by MCS should be at least equal to $10/P_f$ (Kim, 2000; Kar and Roy, 2022). For illustration, to obtain $P_f = 0.001$ accuracy, the total number of samples to be generated by MCS should be at least equal to 10,000. The P_f of the braced excavation is calculated as the ratio of the number of samples with $FOS < 1.0$ to the total number of generated samples. The P_f using MCS can be determined using Eq. (11).

$$P_f = \frac{\text{Number of failed sample}}{\text{Total number of generated sample}} \quad (11)$$

4.3. Subset Simulation (SS)

The SS approach is an adaptive stochastic simulation process created for effectively estimating low levels of P_f . The fundamental idea behind SS is to represent low failure probabilities as products of smaller conditional failure probabilities, making it easier to estimate these smaller conditional probabilities with less computational effort (Au and Wang, 2014; Gao et al., 2019; Tian et al., 2021).

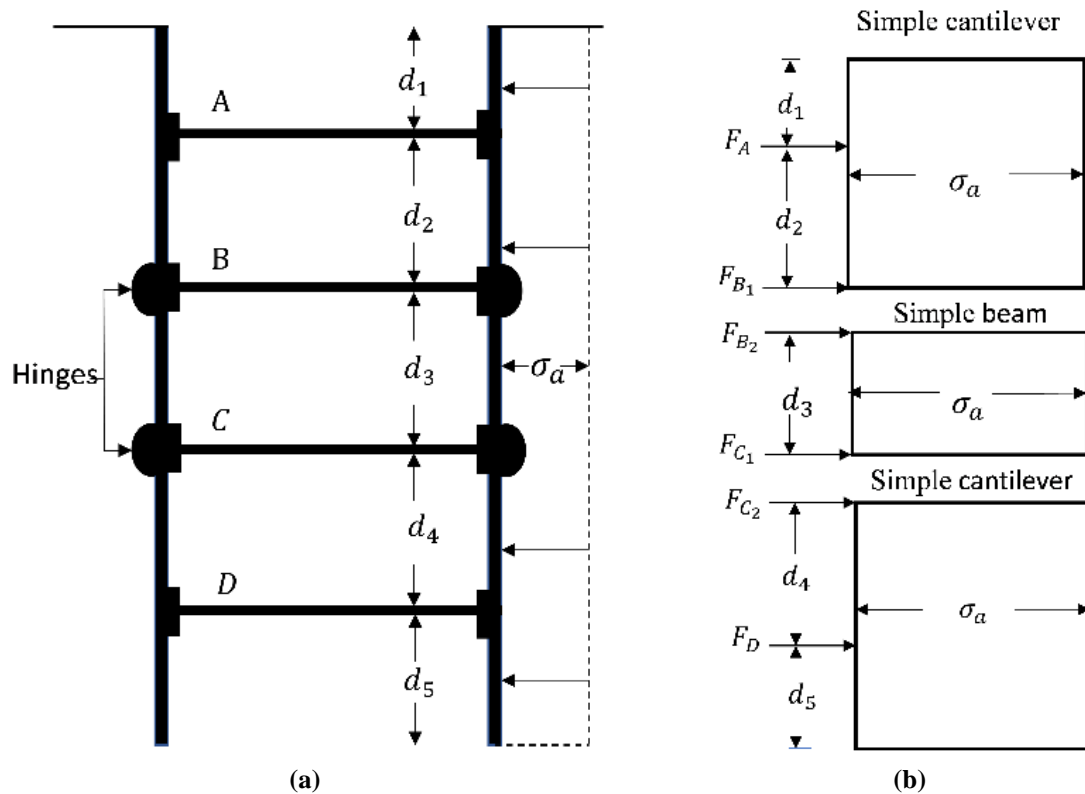


Fig. 4. Braced excavation system with four struts: a) Excavation support configuration; and b) Idealized apparent pressure diagrams for strut loading analysis

Detailed information regarding Markov chain of Monte Carlo simulations method is referring to Au and Wang (2014). For any engineering system, P_f can be defined as the probability of FOS lower than f_s , i.e., $P_f = P(\text{FOS} < f_s)$. In accordance with SS, the P_f can be represented as shown in Eq. (12) (Au et al., 2010; Wang et al., 2011).

$$P_f = P(F_m) = P(F_1) \prod_{j=2}^m P(F_j | F_{j-1}) \quad (12)$$

where $F_j = \{\text{FOS} < fos_j, j = 1, 2, \dots, m\}$: are a set of failure events that occur in the intermediate stages, and they are specified by a succession of lowering the intermediate threshold values $f_{os_1} > f_{os_2} > \dots > f_{os_m} = f_{os}$; $P(F_1) = P(\text{FOS} < f_{os_1})$ and $P(F_j | F_{j-1}) = P(\text{FOS} < f_{os_j} | f_{os_{j-1}})$, $j = 2, 3, \dots, m$. During SS, the intermediate threshold value $f_{os_1} > f_{os_2} > \dots > f_{os_{m-1}}$ are determined adaptively so that $P(F_1)$ and $P(F_j | F_{j-1})$, $j = 2, 3, \dots, m - 1$ always correspond to the specified conditional probability p_o .

5. Probabilistic Analysis of Braced Excavation

To perform the PA, a package of worksheets and function Add-Ins in Excel called UPSS 3.0, MS-Excel based platform integrating both MCS and SS based probabilistic simulation codes written in Visual Basic Programming language has been used. This platform was created by Au and Wang (2014).

The PA procedure using UPSS 3.0 comprises a few steps, namely: a) the development of a Deterministic Modelling (DM) worksheet for deterministic analysis of the problem under consideration; b) a Uncertainty Modelling (UM) worksheet that is used to create the random fields of the variables involved; c) a Probabilistic Modelling (PM) worksheet that is generated by linking the probabilistic parameters defined in the UM sheet with the DM sheet; and d) finally, a run sheet where the MCS and SS simulation are carried out. At the end of the above procedure, output is generated in the form of Complimentary

Cumulative Distribution Function (CCDF) plots, which are very useful in identifying the failure state of the system.

5.1. Random Field Generation Procedure

In order to account for the possibility of random heterogeneity in soil shear strength parameters, it was hypothesised that the soil parameters (i.e., c' , ϕ , and γ) would follow a lognormal distribution (Javankhoshdel and Bathurst, 2016; Touma, 2018) with mean (μ) and variance (σ^2) of soil parameter. In this investigation, c' and γ are considered to be lognormally distributed along the depth of the vertical cut. Thus, $\ln c'$ and $\ln \gamma$ are lognormally distributed with mean $\mu_{\ln c'}$, variance $\sigma_{\ln c'}^2$ and $\mu_{\ln \gamma}$, variance $\sigma_{\ln \gamma}^2$, respectively. Eq. (13) shows the lognormal distribution of any random variable x , with a μ_x and σ_x in a lognormal field.

$$x = \exp(\mu \bar{I} + \sigma \bar{L} \bar{\epsilon}) \quad (13)$$

where μ : is the mean; σ : is the standard deviation of x ; \bar{I} : represents a dimensional unit vector; \bar{L} : depicts the dimension lower triangular matrix and $\bar{\epsilon}$: shows the dimensional standard Gaussian vector.

A correlation matrix \bar{R} is developed in such a manner that satisfies $\bar{R} = \bar{L} \bar{L}^T$. The correlation between $\ln[x_{z_i}]$ and $\ln[x_{z_j}]$ at a depth z_i and z_j represent as $R_{ij} = e\left(\frac{-2|z_i - z_j|}{\lambda}\right)$ where λ : is the correlation length, and \bar{L} : represents the lower triangular matrix.

5.2. Material Cross Correlation Between Soil Parameters

The objective of the study is to analyse the relationship between cross-correlated random soil parameters (i.e., c' and γ) and the P_f during the braced excavation procedure. To calculate the cross-correlation coefficient (ρ) between two correlated random variables X_i and X_j , expressed as $\rho_{X_i X_j}$ can be calculated using Eq. (14).

$$\rho = \rho_{X_i X_j} = \frac{\text{cvar}_{X_i X_j}}{\sigma_{X_i} \sigma_{X_j}} \quad (14)$$

where σ_{X_i} and σ_{X_j} : are the standard deviation of the random variables X_i and X_j , respectively. $\text{cvar}_{X_i X_j}$: stands for the covariance of random variables X_i and X_j .

For more details on the implementation of cross-correlation, one can refer to the literature (Javankhoshdel and Bathurst, 2016; Li et al., 2023). In this current work, the random variables c' and γ are expressed by Eqs. (15) and (16). In particular, the covariance matrix is given in Eq. (17).

$$c = \sigma_c Z + \mu_c \quad (15)$$

$$\gamma = \rho \sigma_\gamma Z + \sigma_\gamma \sqrt{1 - \rho^2} Z + \mu_\sigma \quad (16)$$

$$\Lambda = \begin{bmatrix} \sigma_c^2 & \rho \sigma_c \sigma_\gamma \\ \rho \sigma_\gamma \sigma_c & \sigma_\gamma^2 \end{bmatrix} \quad (17)$$

6. Results and Discussion

In this study, PA for a BES at a depth of cut 9.0 m was performed using MCS and SS with UPSS add-ins 3.0 in MS Excel. For this, both deterministic and PA were conducted to compute the FOS against braced cut failure. The correlated spatially distributed random fields of c' and γ of soil following log normal distribution along the depth using the concepts of Pearson Correlation matrix and Markov Correlation function. According to Table 1, the soil parameter c' was varied to have values of 10%, 30%, and 50% for both Case 1 and Case 2, while the value of γ was varied to have values of 5% and 7%, respectively (Duncan, 2000; Shahin and Cheung, 2011).

The "DM" worksheet (see Appendix Figure 1A) is used to conduct a deterministic analysis of the BES. The worksheet presents the analysis that has been performed for a braced cut at a depth of 9.0 m using four struts. The worksheet includes the calculation of strut force, design of sheet piles, and design of Wales. Notably, the resulting FOS calculation is based on the design of Wales. The U.M worksheet was created using MS Excel by generating a \bar{R} and \bar{L} for soil parameters

(i.e., c' and γ) at different depths.

The U.M worksheet includes Probability Density Functions (PDFs) and random values of cohesion and unit weights. Notably, this worksheet has been created separately for both with and without cross-correlation, that has been presented in Appendix Figures 2A and 3A, respectively. Note that, cross-correlation in between c' and γ was considered positive i.e., $\rho_{c',\gamma} = 0.1$. Also, it is noted that this study was analysed with correlation length of $\lambda = 1.0$ m and $\lambda = 2.0$ m. The U.M sheet consists of uniform i.i.ds generated by the RAND () function and standard normal variables generated using the NORMINV () function to convert i.i.ds to standard Gaussian variables. The random variables that are generated at various depths in the U.M. sheet are then averaged prior to being linked to the deterministic sheet, and the resulting worksheet is called the P.M sheet.

6.1. Selection of Correlation Coefficient

It has already been discussed that for

cohesive soil, the soil parameters are positively correlated. In this study, distinct correlated coefficients (0.1, 0.3, 0.5, and 0.7) were investigated, and PA was performed using MCS. Figure 5 deduced that the effect correlated coefficient on P_f . This plot has been plotted considering specific FOS and their corresponding P_f .

As the value of ρ decreases, P_f increases for a constant value of FOS. However, increasing the value of ρ has a very low effect on P_f . The correlation coefficient $\rho_{c',\gamma} = 0.10$ has been chosen because the maximum P_f of the BES has been reported in this paper.

6.2. MCS Analysis Results

Table 2 displays the results of a 10,000-sample MCS analysis for the BES. The simulations were conducted for correlating the length of $\lambda = 1.0$ m and $\lambda = 2.0$ m, at depths of cut 9.0 m, respectively. The data shows that P_f of the BES increases when COV of c' and γ are increased.

Table 1. Summary of the COV of details soil parameters

		COV (%)			Range of COV	References
		10	30	50		
Case 1	c' (kN/m ²)	10	30	50	10% - 70%	Shahin and Cheung (2011)
	γ (kN/m ³)	5	5	5	3% - 7%	Duncan (2000)
Case 2	c' (kN/m ²)	10	30	50	10% - 70%	Shahin and Cheung (2011)
	γ (kN/m ³)	7	7	7	3% - 7%	Duncan (2000)

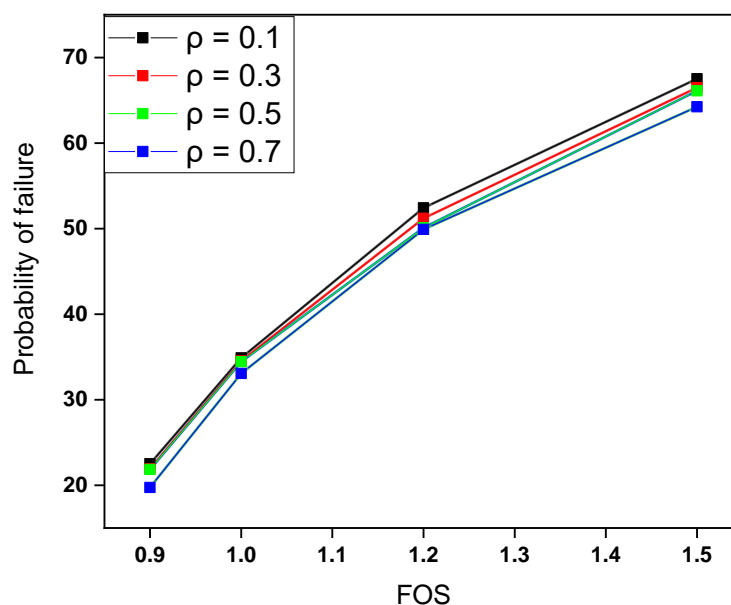


Fig. 5. Plot in between P_f and FOS at different values of the correlation coefficient

Furthermore, it is seen that P_f value increases when λ increases. Also, it is noticed that if the cross-correlation between c' and γ is considered, there is a further increase in P_f of the system. Therefore, considering the correlated structure of related variables is crucial to ensure conservative results. For clarity, CCDF plot in Figure 6 (uncorrelated) and Figure 7 (correlated) has been presented. It can be seen from the CCDF, as the COV value increases, the curve of the CCDF plot shift to the right, indicating more failures.

Figures 8 and 9 show histograms for the worst-case scenario $v_{c'} = 50\%$ and $v_\gamma = 5\%$ using MCS, with and without correlation, at a depth of cut 9.0 m for $\lambda = 2.0$ m. Herein, notation $v_{c'}$ indicates the COV of cohesion and v_γ indicates the COV of unit weight. In Figure 8, among 10,000 samples, 3487 had FOS values below one (FOS < 1). Therefore, the P_f was calculated as $\left(\frac{3487}{10000}\right) \times 100 = 34.87\%$, and the resolution of P_f was determined to be $\left(\frac{1}{10000}\right) \times 100 = 0.01\%$.

Table 2. Results obtained from MCS at a depth of cut 9.0 m

Correlation length, λ	COV, c (%)	COV, γ (%)	Number of samples generated	Number of failed samples having FOS < 1 when $\rho = 0.0$	Number of failed samples FOS < 1 when $\rho = 0.1$	P_f (%) for $\rho = 0.0$	P_f (%) for $\rho = 0.1$
$\lambda = 1$	10	5	10000	0	294	0	2.94
	30	5	10000	78	2456	0.78	24.56
	50	5	10000	994	3452	9.94	34.52
	10	7	10000	0	385	0	3.85
	30	7	10000	13	2390	0.13	23.90
	50	7	10000	807	3439	8.07	34.39
$\lambda = 2$	10	5	10000	0	314	0	3.14
	30	5	10000	202	2523	2.02	25.23
	50	5	10000	1551	3487	15.51	34.87
	10	7	10000	0	394	0	3.94
	30	7	10000	90	2530	0.9	25.30
	50	7	10000	1335	3480	13.35	34.80

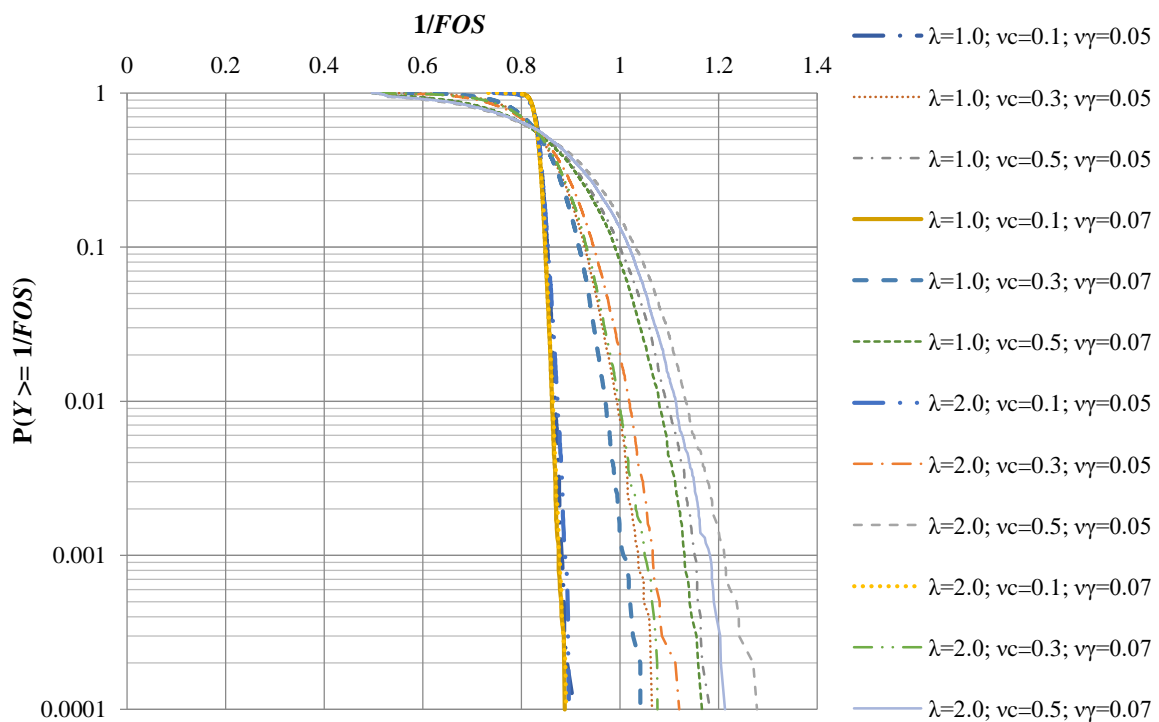


Fig. 6. CCDF plot between P_f and $1/FOS$ using MCS at a depth of cut 9.0 m (without correlation)

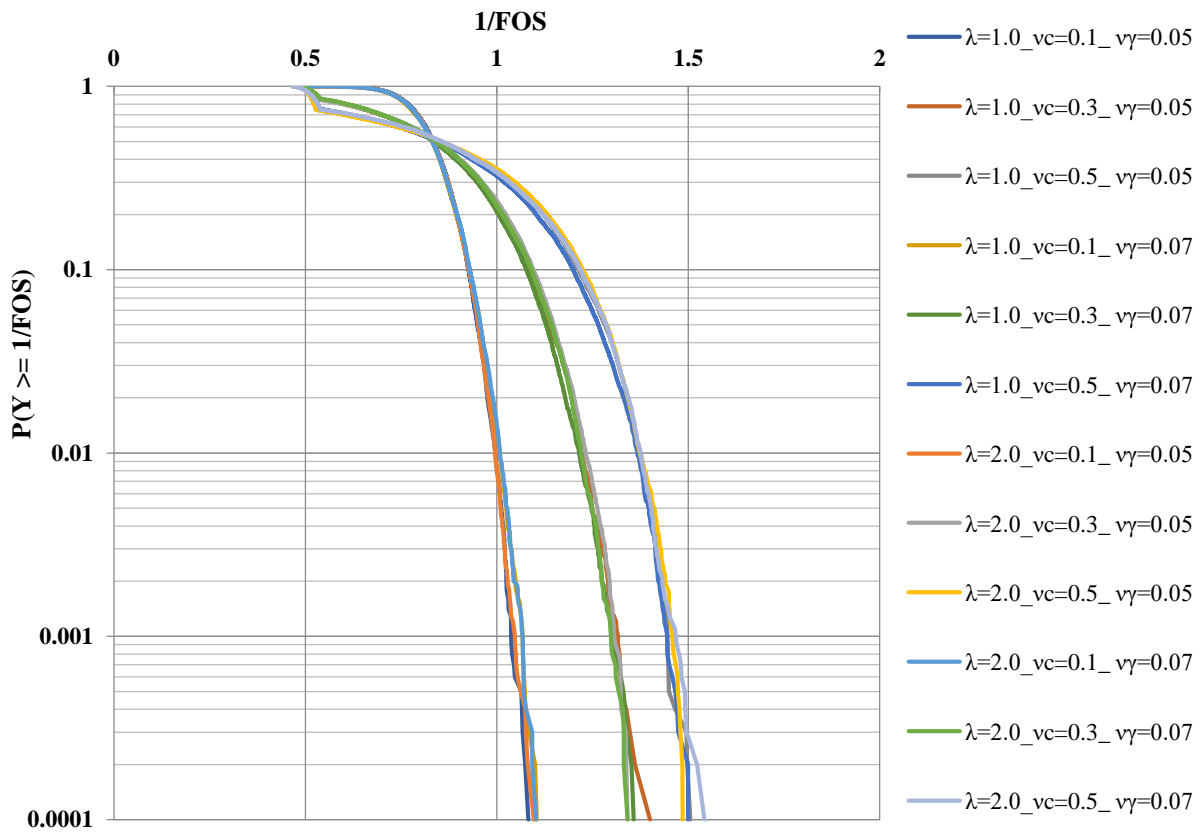


Fig. 7. CCDF plot between P_f and $1/FOS$ using MCS at a depth of cut 9m (with correlation)

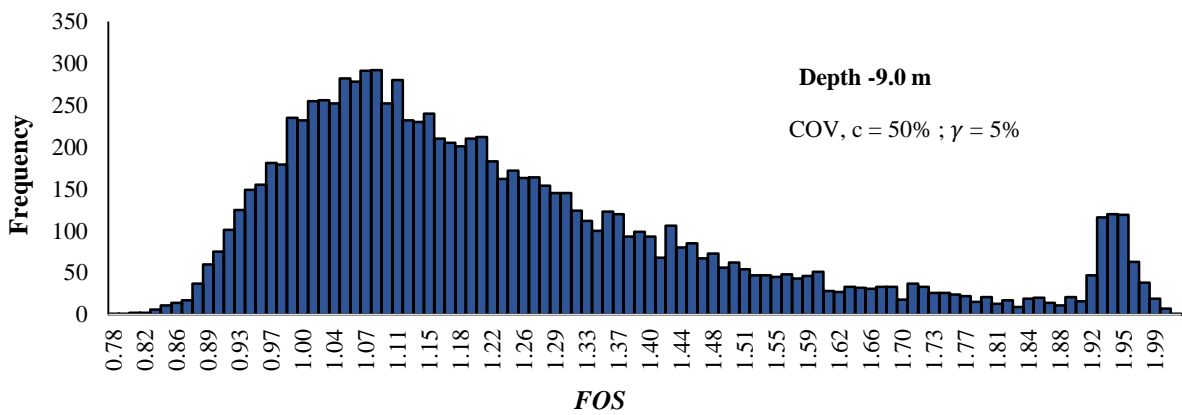


Fig. 8. Histogram at a depth of cut 9.0 m at $\lambda = 2.0$ m (without correlation)

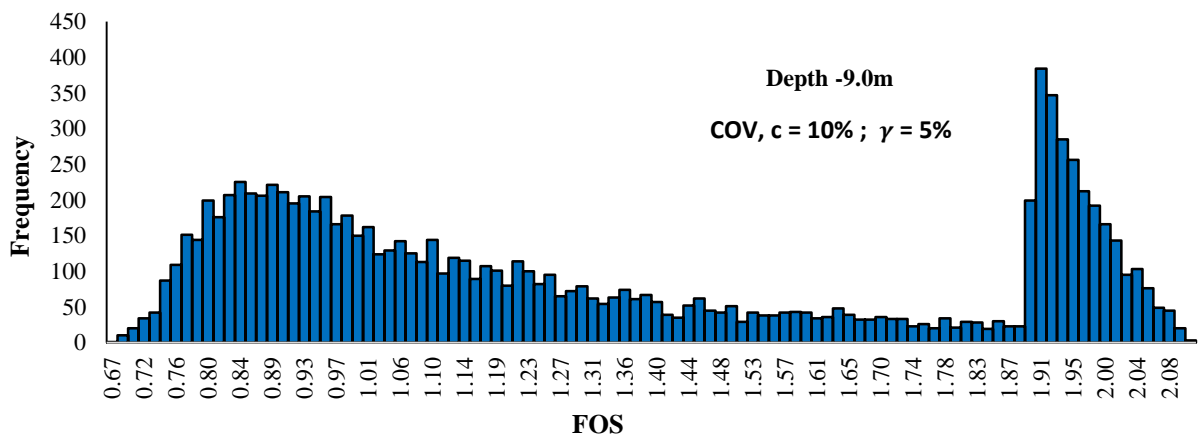


Fig. 9. Histogram at a depth of cut 9 m at $\lambda = 2.0$ m (with correlation)

6.3. Comparative Study for with and without Cross-Correlation in MCS

Comparing the results of the simulations with and without cross-correlation using MCS, it was found that the P_f increases significantly when cross-correlation ($\rho_{c',\gamma} = 0.1$) was considered. It can be deduced from Table 2, there have not been obtained any failed sample in a system when $\nu_{c'} = 10\%$ and $\nu_\gamma = 5\%$, and $\nu_{c'} = 10\%$ and $\nu_\gamma = 7\%$ with a correlation length of $\lambda = 1.0$ m and $\lambda = 2.0$ m are considered. However, when cross-correlation was considered, a significant number of failed

samples were observed at $\lambda = 1.0$ m and $\lambda = 2.0$ m.

6.4. SS Analysis Results

In this study, SS was used with 1400 random samples distributed across three simulation stages (Levels 1, 2, and 3) with 1 run, $N = 500$, and $p_0 = 0.1$. The total number of samples is calculated as $N + m \times (1 - p_0)$ i.e., $500 + 3 \times (1 - 0.1) = 1400$. As an example, when $\lambda = 1.0$ m and $\rho = 0.0$, in the third row of Table 3, there were 5, 450, and 500 failures in Levels 1, 2, and 3, respectively.

Table 3. Results of SS at depths of cut of 9.0 m (with cross correlation)

Correlation, λ	$\nu_{c'}$ %	ν_γ %	Total samples produced	Number of failed samples having FOS < 1 when $\rho = 0.0$			Number of failed samples having FOS < 1 when $\rho = 0.1$			P_f (%) for $\rho = 0$	P_f (%) for $\rho = 0.1$
				Level 1	Level 2	Level 3	Level 1	Level 2	Level 3		
$\lambda = 1$	10	5	1400	0	0	0	0	110	500	0	3.2
	30	5	1400	0	7	500	75	450	500	1.14	25
	50	5	1400	5	450	500	123	450	500	11	34.6
	10	7	1400	0	0	0	0	155	500	0	4.32
	30	7	1400	0	0	85	79	450	500	0.17	25.8
	50	7	1400	0	386	500	125	450	500	8.72	35
$\lambda = 2$	10	5	1400	0	0	0	0	113	500	0	3.26
	30	5	1400	0	96	500	82	450	500	2.92	26.4
	50	5	1400	29	450	500	126	450	500	15.8	35.2
	10	7	1400	0	0	0	0	180	500	0	4.6
	30	7	1400	0	0	477	84	450	500	0.95	26.8
	50	7	1400	24	450	500	130	450	500	14.8	36

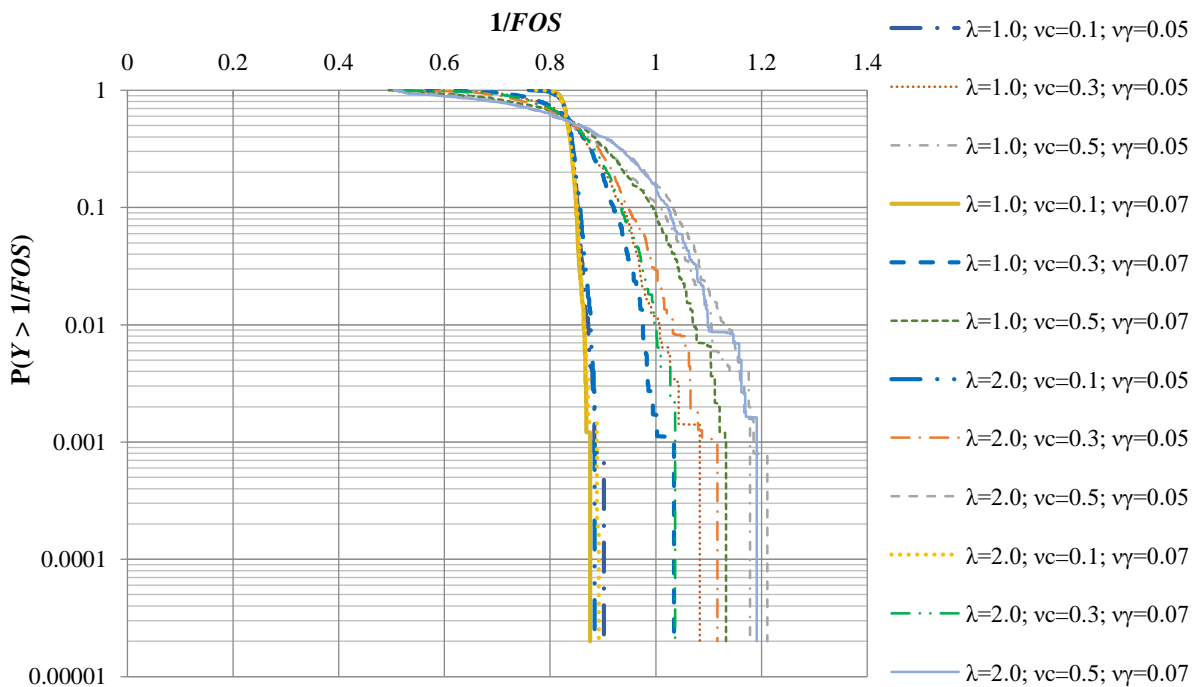


Fig. 10. CCDF plot between P_f and $1/FOS$ using SS at a depth of cut 9.0 m (without cross-correlation)

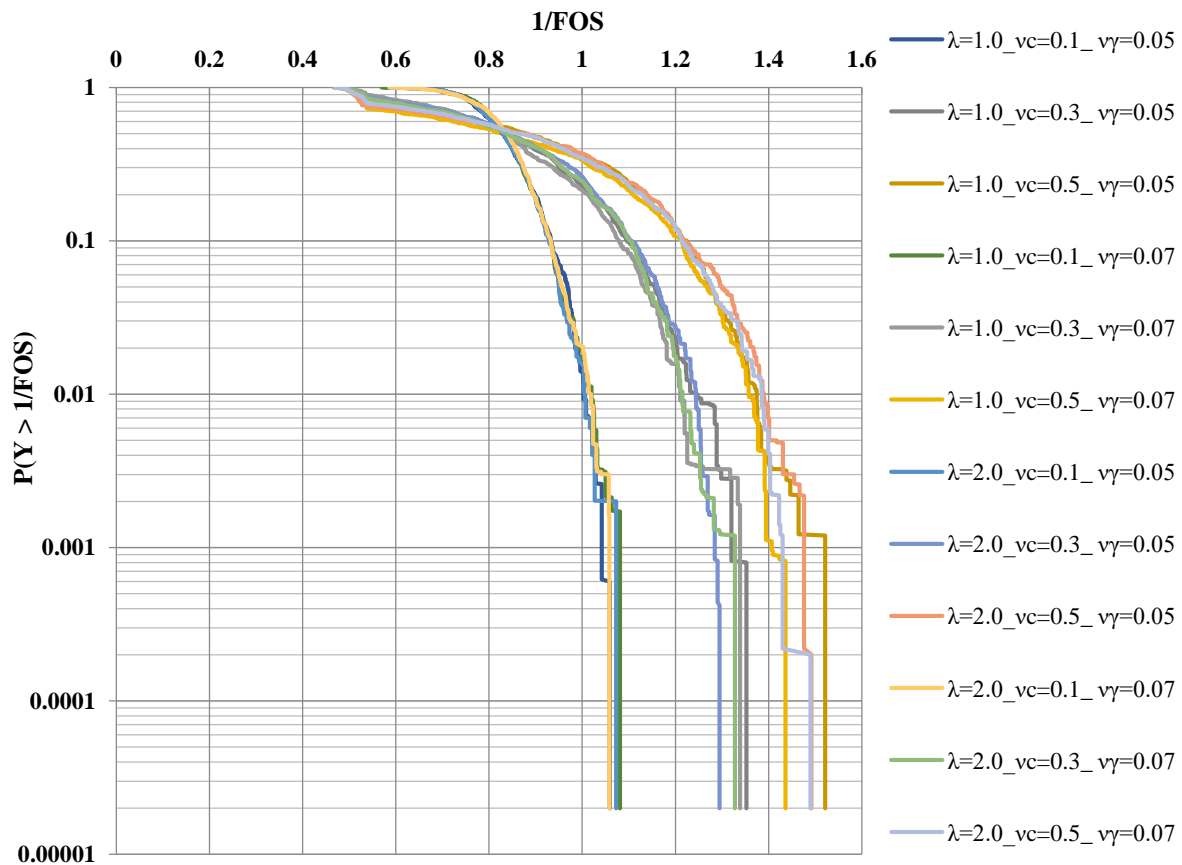
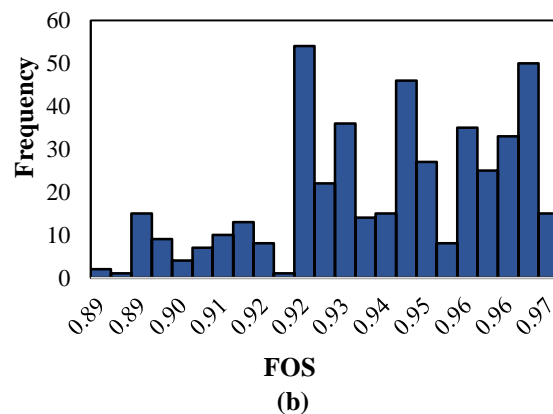
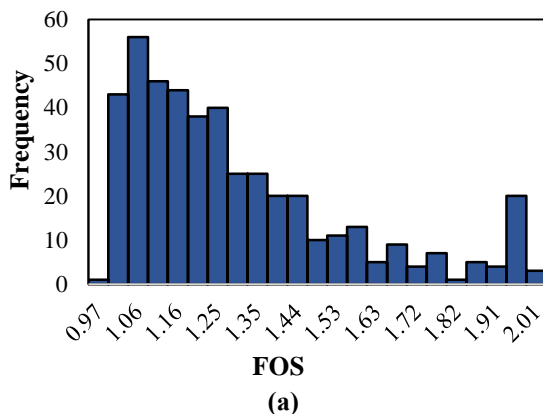


Fig. 11. CCDF plot between P_f and $1/FOS$ using SS at a depth of cut 9m (with cross-correlation)

The probability of occurrence at Levels 1, 2, and 3 is 0.9, 0.09, and 0.01, respectively. Therefore, P_f can be found using Eq. (12) as $\left(0.9 \times \frac{5}{450} + 0.09 \times \frac{450}{450} + 0.01 \times \frac{500}{500}\right) \times 100 = 11\%$. Table 3 displays the results of an SS analysis for the BES using 1400 samples. This simulation shows that P_f of the BES increases when $\nu_{c'}$ and ν_γ are increased. Furthermore, it is seen that P_f value increases when λ increases.

Additionally, it is noted that the system's P_f continues to rise when the cross-correlation between c' and γ is taken into account. The results show that SS provides similar outcomes to MCS with fewer random samples needed. CCDF plots in Figures 10 and 11 depict SS results at various COV levels with $\rho = 0$ and $\rho = 0.1$ for a 9.0 m cut depth. Histogram plots (Figures 12 and 13) for the worst-case P_f scenario at a 9.0 m cut depth are presented using SS.



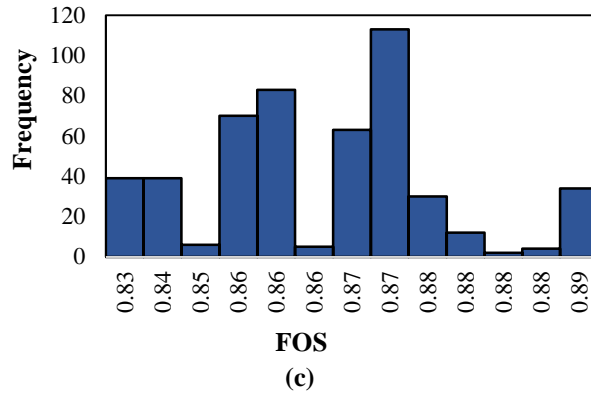


Fig. 12. Histogram plot in SS for the worst P_f condition (without correlation): a) Out of 450 samples, 29 samples fail at level -1; b) Out of 450 samples, 450 samples fail at level -2; and c) Out of 500 samples, 500 samples fail at level -3

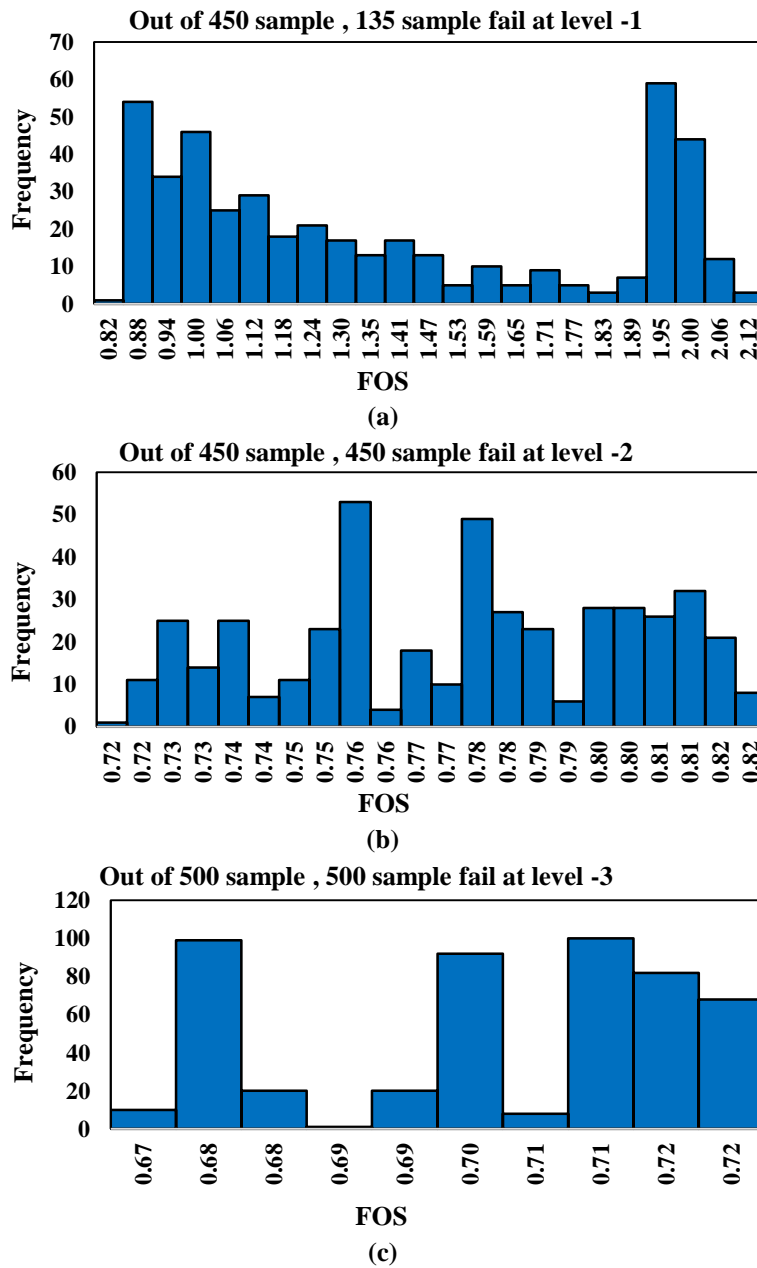


Fig. 13. Histogram plot using SS for worst P_f condition (with cross-correlation): a) Out of 450 samples, 29 samples fail at level -1; b) Out of 450 samples, 450 samples fail at level -2; and c) Out of 500 samples, 500 samples fail at level -3

The presented histograms clearly show failures at each simulation level. In Figure 12, out of 1400 samples, 29 failed at level - 1, and all samples failed at Level 2 and 3.

6.5. Comparative Study with and without Cross-Correlation in SS

Table 3 shows no failures at any level when $v_{c'}$ was 10% and v_{γ} was 5% and 7% for both $\lambda = 1.0$ m and $\lambda = 2.0$ m. In Table 3 (Row 1), considering the same COVs for soil parameters but accounting for cross-correlation between c' and γ , 110 samples failed at Level 2, and 500 samples failed at Level 3 for $\lambda = 1.0$ m. This suggests that incorporating cross-correlation between c' and γ leads to an increase in the P_f .

6.6. Computational Cost of SS over MCS

This section discusses the comparative analysis between the employed MCS and SS simulations in terms of cost. In this study, the UPSS 3.0 Spreadsheet environment was utilized, which is open source. For the simulation analysis, the software was operated on a 64-bit operating system with an x64-based processor, CPU @ 2.40GHz, and 8.00 GB Ram. To get the same level of probability, SS demonstrates advantages in efficiency over direct MCS for achieving comparable levels of probability. Specifically, SS requires a smaller sample size, resulting computation time of 16.37 seconds across three levels of simulation. In contrast, MCS required 1.20 minutes for the same analysis. On the other hand, MCS requires a high computational cost and time, if a greater number of samples are used for simulation.

7. Summary and Conclusions

The current study primarily emphasises conducting a PA of a braced excavation by utilising a spreadsheet version of Microsoft Excel to perform MCS and SS simulations. In this work, PA for braced cuts was carried out at a depth of cut 9.0 m utilizing four struts. For this, the soil parameter c' and γ were considered as uncertain parameters and modelled with a lognormal random

field, and Cholesky lower triangular matrix were created. In addition, four different correlation coefficient values between soil parameters (i.e., c' and γ) on the response P_f were investigated. Based on the results obtained on P_f , the best correlation coefficient (i.e., $\rho = 0.1$) were considered for this investigation.

Furthermore, the comparative analysis was conducted to investigate the effects of braced excavation at various depths, utilizing techniques such as MCS and SS to evaluate the c . Additionally, the study examined various levels of COV and varied soil parameters c' to 10%, 30%, and 50%, while also altering the γ to 5% and 7%. Based on the obtained results in this investigation, the following conclusions can be drawn:

- i. Four different correlation coefficients (0.1, 0.3, 0.5, and 0.7) were investigated, and probabilistic analysis was conducted by both MCS and SS methods. It is seen that $\rho = 0.10$ yields maximum P_f for the box drain system.
- ii. It is seen that the consideration of correlation between the soil parameters (i.e., c' and γ) yields higher P_f value compared to the situation when the soil parameters are deemed uncorrelated.
- iii. When the correlation length is increased in both MCS and SS, the P_f increases dramatically.
- iv. When MCS and SS are conducted for probabilistic analysis, it is found that SS is superior at a low level of failure. In addition, the obtained results by SS were approximately similar to those obtained by MCS, which also shows that both methods are applicable for probabilistic analysis. SS requires a smaller number of samples to report P_f with the same level of accuracy as MCS. This fact indicates that SS has better efficiency than MCS in terms of both memory storage space and computational time.

8. Data Availability

The dataset used in this study is available upon request. Please contact to

corresponding author for access to the data.

9. Declaration

Artificial intelligence-based tools were utilized for limited assistance in language editing, and improving the clarity of the manuscript.

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UNCERTAINTY MODELLING SHEET										
Depth (z in m)	0	1	2	3	4	5	6	7	8	9
Correlation length	1	1	1	1	1	1	1	1	1	1
Parameter 1 - Cohesion										
Mean of Cohesion (KN/m ²)	11.00	11.00	11.33	12.00	20.00	23.33	22.00	24.67	26.17	26.50
Std dev of Cohesion	1.10	1.10	1.13	1.20	2.00	2.33	2.20	2.47	2.62	2.65
C.O.V	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Mean of ln(C)	2.393	2.393	2.423	2.480	2.991	3.145	3.086	3.200	3.260	3.272
Std dev of ln(C)	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100
Parameter 2 - Bulk Density										
Yb (KN/m ²)	16.625	16.788	16.952	17.115	17.605	17.442	17.605	17.768	17.850	17.850
Std dev of Yb	0.831	0.839	0.848	0.856	0.880	0.872	0.880	0.888	0.893	0.893
C.O.V	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
ln(Yb)	2.810	2.819	2.829	2.839	2.867	2.858	2.867	2.876	2.881	2.881
Std dev of ln(Yb)	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050
Random Sample generation										
Depth	0.000	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000
Uniform I.I.D	0.727	0.213	0.360	0.091	0.798	0.219	0.533	0.504	0.729	0.472
Std normal I.I.D (z)	0.605	-0.796	-0.359	-1.332	0.833	-0.777	0.084	0.010	0.610	-0.071
Pdf value p(z)	0.332	0.291	0.374	0.164	0.282	0.295	0.398	0.399	0.331	0.357
L*Z	0.487	-0.859	-0.521	-1.222	0.723	-0.757	0.095	0.090	0.595	-0.070
Cholesky Transformation Matrix Written and Computed by Matlab										
	0	1	2	3	4	5	6	7	8	9
0	1									
1	0.135335	1								
2	0.018316	0.13409	1							
3	0.002479	0.018147	0.13409	1						
4	0.000335	0.002456	0.018147	0.13409	1					
5	4.54E-05	0.000332	0.002456	0.018147	0.13409	1				
6	6.14E-06	4.50E-05	0.000332	0.002456	0.018147	0.13409	1			
7	8.32E-07	6.09E-06	4.50E-05	0.000332	0.002456	0.018147	0.13409	1		
8	1.13E-07	8.24E-07	6.09E-06	4.50E-05	0.000332	0.002456	0.018147	0.13409	1	
9	1.52E-08	1.11E-07	8.24E-07	6.09E-06	4.50E-05	0.000332	0.002456	0.018147	0.13409	1
Generation of lognormal field for cohesion and unit weight by cholesky transformoion										
LN(cohesion)	2.442	2.307	2.371	2.358	3.063	3.069	3.096	3.209	3.319	3.265
Cohesion (KN/m ²)	11.491	10.047	10.706	10.570	21.389	21.530	22.100	24.765	27.629	26.185
Cavg (KN/m ²)	19.43563									
LN(Yb)	2.8340	2.7765	2.8031	2.7776	2.9031	2.8198	2.8717	2.8807	2.9105	2.8773
Yb (KN/m ²)	17.0136	16.0631	16.4953	16.0811	18.2297	16.7736	17.6671	17.8261	18.3658	17.7654
Ybavg (KN/m ²)	17.2717									

Fig. 3A. Uncertainty model worksheet without cross-correlation



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