






Hybrid Neuro-Fuzzy ML and MC Simulation-Based Reliability Analysis of Simply Supported Beam

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ABSTRACT: This paper introduces an innovative approach utilizing hybrid Adaptive Neuro Fuzzy Inference System (ANFIS) models for the reliability-based design of structural beams. While structural reliability analysis with hybrid ANFIS models remains largely unexplored, existing studies primarily rely on rudimentary simulation models. To address this gap, this study employs Particle Swarm Optimization (PSO) and Genetic Algorithms (GA) to enhance the performance of the ANFIS model. The Machine Learning (ML) models are validated on three Monte-Carlo datasets of size 1000, 2500, and 5000. The findings demonstrate satisfactory performance across all ML models, with the hybrid ANFIS models exhibiting superior predictive capabilities compared to traditional methods. Among the hybrid ANFIS models, ANFIS-PSO emerges as the most robust. The reliability indices and Probability of Failure (POF) values are calculated for the predicted values and compared with actual values. It is concluded that the ANFIS-PSO-based methodology is the most robust model and outperforms the other models. It is noteworthy that while the ANFIS-PSO model demonstrates exceptional performance, all models presented in this study serve as valuable tools for reliability-based structural design, offering robust alternatives to conventional methodologies.

Keywords: Reliability Analysis, Machine Learning, Hybrid ANFIS, Simply Supported Beam, Monte-Carlo Simulation.

1. Introduction

Reliability analysis is a crucial aspect of structural engineering that ensures the safety, durability, and cost-effectiveness of engineered structures and contributes to the sustainable development and prosperity of societies. It helps in identifying potential failures, optimizing design, managing risks, ensuring compliance with standards and

regulations, optimizing life cycle costs, and maintaining public confidence and trust (Haldar and Mahadevan, 2000; Harr, 1987).

Reliability analysis helps engineers identify potential failure modes and take preventive measures to mitigate risks. Traditional deterministic methods in structural engineering, such as deterministic structural analysis, limit state design, elastic analysis, a factor of safety, and code-based

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design methods, rely on fixed parameters and do not account for uncertainties in material properties or loading conditions (Naess et al., 2012; Strauss et al., 2009; Toratti et al., 2007). Deterministic structural analysis solves structural mechanics equations using fixed input parameters, while limit state design ensures structures remain within predefined safety limits under all loading conditions (Acito et al., 2011; Singh et al., 2020).

Elastic analysis assumes structural materials behave linearly and elastically, but does not account for nonlinearities or uncertainties in material behavior. The factor of safety divides the maximum expected load or stress a structure can withstand by a predetermined factor of safety, based on engineering judgment, historical data, and conservative assumptions about uncertainties in material properties and loading conditions (Ching and Phoon, 2011; Haldar and Mahadevan, 2000). Code-based design methods provide guidelines for calculating loads, selecting materials, and sizing structural members to ensure compliance with safety and performance requirements. However, traditional deterministic methods may not fully capture the complexities and uncertainties inherent in real-world applications, leading to a growing recognition of the importance of probabilistic approaches like Monte Carlo Simulation (MCS) and reliability analysis for comprehensive assessments of structural safety and performance.

Reliability analysis methods, like MCS, overcome the limitations of traditional deterministic methods in structural engineering (Cardoso et al., 2008; Li et al., 2013; Naess et al., 2012; Papadrakakis et al., 1996). They consider uncertainties, incorporate conservative design assumptions, and offer a comprehensive risk management framework. They also offer greater flexibility and adaptability, allowing engineers to modify input parameters and incorporate new data to refine design decisions. Additionally, it aids

in design optimization by facilitating an agreement between performance, affordability, and safety. Conventional wisdom holds that the factor of safety method is the most prudent; however, this overly conservative approach is unavoidably economically unsound. The probability of failure too remains high (Sivakumar Babu and Srivastava, 2007).

Fore, the odds of failing are considerable (Sivakumar Babu and Srivastava, 2007). These methods produce conservative, inefficient designs by exaggerating the impact of uncertainty. The optimization process in Reliability-Based Design (RBD) takes the structural failure probability with uncertainties into consideration as a probability constraint. RBD offers a system with profound trust and cost-effective structural design outcomes. The First Order Reliability Method (FORM) is a highly efficient method for finite element model-based reliability analysis, which has been extensively studied in different engineering problems (Ayyub and Haldar, 1984; Kar and Roy, 2022; Lee and Mosalam, 2005; Samui et al., 2016). FORM approximates the limit state function near its most probable point of failure. However, it has limitations such as assuming linearity, limited accuracy for nonlinear problems, and inability to capture higher-order effects (Hao et al., 2019). Liu et al. (2022) proposed multi-objective reliability-based design optimization, while Hao et al. (2021) proposed the Nested Stochastic Kriging (NSK) model. Machine Learning (ML) based reliability analysis is an emerging approach that leverages the capabilities of ML algorithms to analyze the reliability of structural systems (Zargari et al., 2024). ML has been successfully used in many engineering applications (Kumar et al., 2024a,b,c,d,e; Kumar and Kumari, 2024; Kumar and TN, 2023; Kumar et al., 2023g). ML-based reliability analysis overcomes these limitations by nonlinear handling, flexibility, adaptability, and improved prediction accuracy (Cardoso et al., 2008;

Kumar et al., 2024d; Saraygord Afshari et al., 2022a).

However, most of the studies are focused on basic ML models, e.g., Artificial Neural Networks (ANNs), Support Vector Machines (SVMs), and kriging methods (Saraygord Afshari et al., 2022b). ANN-based reliability analysis faces challenges in reliability analysis due to its inherent complexity. These include capturing complex performance functions, calculating partial derivatives of implicit functions, and slow convergence rates.

ANNs' non-linear transformations make derivative computation less straightforward than traditional methods. Slow convergence can hinder the efficiency of reliability analysis. Overfitting occurs when a model learns noise or random fluctuations in training data, leading to poor performance on unseen data (Kumar et al., 2023f).

Burges (1998) highlighted that SVMs are powerful classification and regression tools due to their generalization capabilities. However, their application was limited due to high algorithmic complexity, significant memory demands, and challenges in Kernel selection. These factors make training computationally expensive and time-consuming, especially for large datasets or high-dimensional spaces. Choosing the right Kernel function and parameters could significantly affect the model's performance. While ANN and Fuzzy Logic (FL) offer various advantages, they also possess certain limitations. The advantages of both ANN and FL were realized through neuro-fuzzy models.

The Adaptive Neuro Fuzzy Inference System (ANFIS) is a system that integrates Neural Network (NN) and AI-based fuzzy logic. It functions like a Neural Network during the learning phase and like fuzzy logic during the execution phase. ANFIS has been demonstrated to outperform ANN and FL when used individually (Atsalakis et al., 2018; Godil et al., 2011; Pezeshki and Mazinani, 2019; Pradeep et al., 2021).

Mohammadzadeh et al. (2023) determined that ANFIS demonstrated

higher reliability compared to ANN, Gene Expression Programming (GEP), and other traditional models. Their study presented a hybrid neuro-fuzzy ML models-based reliability analysis of structural beams using MCS. Şener et al. (2024) applied hybrid ANFIS models for the characteristics of air bearings, concluded to outperform traditional ANN and ANFIS models.

Particle Swarm Optimization (PSO)-ANFIS and Genetic Algorithm (GA)-ANFIS models have successful applications in many fields of structural engineering (Achouri et al., 2023; Karimi Sharafshadeh et al., 2023; Ly et al., 2021; Mishra et al., 2021), however, hybrid ANFIS models-based reliability analysis of structural beams has not been performed till now. This paper addresses the literature gap, the authors propose a state-of-the-art reliability-based design of structural beams using MCS. Computers have revolutionized various sectors, with ML accelerating research and improving system efficiency.

Traditionally, experiments were crucial for solving engineering problems, but high resource and equipment costs led to a surge in ML models for estimating the deflection of beams. Thus, the research presented in the study presents a good novelty for future research in the field.

1.1. Disadvantages of Existing Methods

The observational method does not permit design modifications during construction based on observed behavior. Traditionally, the Factor of Safety (FS) approach has been used to manage uncertainties and variability in design. However, this method relies on experience and often leads to overly conservative analyses and a significant probability of failure, lacking a rational problem-solving basis. The First Order Second Moment (FOSM) method is effective for reliability analysis of pile bearing capacity but is time-consuming and challenging for multivariable relationships, often yielding flawed results for non-normal distributions. Methods like the multi-tangent-plane

surface method, Response Surface Method (RSM), and multi-plane surfaces method improve accuracy but are limited to nonlinear convex or concave limit state surfaces.

ANNs are successful but struggle with explicit performance functions and complex partial derivatives, leading to slow convergence and overfitting. SVMs offer good generalization but suffer from high algorithmic complexity and memory requirements, along with Kernel selection challenges. Fuzzy-based reliability analysis is effective but demands extensive data, expertise, and the tedious development of rules and membership functions.

1.2. Motivation of the Work

ML-based FORM provides a novel and robust alternative to traditional methodologies for the reliable design of structures, however, most of the studies in the literature are simulated on scarce datasets or utilise primitive soft-computing models. Hybrid neuro-fuzzy simulation models have got very limited applications in structural engineering and have not been utilised so far for the reliability analysis of structural beams. A unique hybrid learning algorithm that optimizes the ANFIS parameters more efficiently than traditional methods. Moreover, combining FORM with MCS provides a way to verify and correct the reliability estimates. By performing a large number of random simulations, the true failure probability can be estimated, and the error in the FORM approximation can be assessed and adjusted accordingly. By harnessing the capabilities of PSO-ANFIS and GA-ANFIS, the study aims to improve the accuracy and efficiency of predicting deflection of structural beams and utilize it to propose a state-of-the-art new methodology for the reliability-based design of structures.

2. Research Methodology

The following subsections describe the theoretical background of the ML models,

dataset preparation, and reliability analysis.

2.1. ML Models Used in the Study

The ANFIS is a hybrid computational model that merges the adaptive learning features of neural networks with the intuitive decision-making processes of fuzzy logic. It was proposed by Jang (1993) as a method for approximating or modeling systems that are difficult to describe mathematically.

ANFIS models consist of four main components: fuzzy logic, Fuzzy Inference Systems (FIS), neural networks, and hybrid systems. Fuzzy logic is a mathematical framework for dealing with uncertainty and imprecision, while FIS models model relationships between inputs and outputs based on fuzzy logic principles. Neural networks are computational models inspired by the structure and function of the human brain, consisting of interconnected nodes organized into layers.

ANFIS models have a structure resembling a neural network, with layers for input, fuzzy inference, and output. The parameters of the ANFIS model are learned from data using techniques such as gradient descent or least squares estimation. During training, the model adjusts its parameters to minimize the difference between actual and predicted outputs. ANFIS models have been applied to various applications, including system identification, prediction, control, and pattern recognition, due to their ability to handle complex, nonlinear relationships and uncertainty in data. The hybridization of ANFIS and Optimization Algorithms (OAs) combines the learning and adaptive capabilities of ANFIS with the global search and optimization abilities of OAs.

The goal is to improve the performance of ANFIS by optimizing its parameters using OAs. The utilization of optimization techniques in regression ML algorithms is witnessing an increasing inclination and interest. PSO is a population-based optimization algorithm that iteratively improves candidate solutions by simulating

the movement of particles in a search space.

It was first proposed by Kennedy and Eberhart (1995) and is based on the principles of swarm intelligence and optimization algorithms. Swarm intelligence refers to the collective behavior of decentralized, self-organized systems, where groups of simple agents can exhibit complex, intelligent behavior when interacting with each other and their environment. PSO is based on the metaphor of a swarm of particles moving through a multidimensional search space, with each particle representing a potential solution.

Optimization algorithms are methods for finding the best solution among feasible solutions within a defined search space. These solutions are evaluated based on an objective function that quantifies their quality or fitness. Key components of PSO include the particle, position, velocity, fitness, personal best, global best, inertia weight, and acceleration coefficients. The algorithm steps include initializing the swarm with random positions and velocities, evaluating the fitness of each particle, updating their positions and velocity, and repeating the process until a termination condition is met. Termination criteria for PSO iterations can be met based on various criteria, such as reaching a maximum number of iterations, achieving a target fitness value, or stagnation of the swarm. Overall, PSO is widely used for solving optimization problems in various domains due to its simplicity, efficiency, and ability to handle nonlinear and multimodal optimization problems.

GAs are evolutionary algorithms derived from natural selection and genetics, used for optimization and search problems. They are inspired by the process of natural selection, where organisms with favourable traits are more likely to survive and reproduce, leading to the evolution of populations better adapted to their environments. GAs represent potential solutions to optimization problems as chromosomes, with each chromosome representing a candidate solution. The

algorithm starts with an initial population of chromosomes, which is influenced by the population size. Each chromosome is evaluated based on its fitness, indicating the quality of each solution. The selection process determines which chromosomes will reproduce and form the next generation, with higher fitness values being more likely to be selected. The process of recombination, also referred to as crossover, involves the exchange of genetic material between selected sets of parent chromosomes, resulting in the formation of novel sets of chromosomes. The process of mutation involves the introduction of random alterations to the genetic material of offspring chromosomes. This mechanism serves to maintain genetic diversity and prolong the time it takes for the population to reach suboptimal solutions. By implementing a replacement strategy, it can be ensured that the population will exhibit improvement with the progression of each subsequent generation. GAs undergo iterative processes until they reach a specific threshold, such as the maximum number of generations, or until they discover a solution that satisfies their criteria. The effectiveness of genetic algorithms depends on appropriate parameter settings, representation, and problem characteristics.

2.2. Hybridization of the ANFIS Model

Optimizing ANFIS parameters using OAs like GAs or PSO can significantly improve the performance of ANFIS models. The model has been shown to improve accuracy and convergence speed, handle high-dimensional and complex problems more efficiently, and reduce search space. However, the hybrid ANFIS models may suffer from overfitting if the training data size is small and the number of parameters to be optimized is large.

However, hyper parameter tuning for the best-performing model using the hit-and-trial method and regularization techniques like early stopping are utilized to avoid overfitting. The hybridization of

ANFIS and PSO combines the learning and adaptive capabilities of ANFIS with the global search and optimization abilities of PSO. The model uses PSO to optimize fuzzy sets and weights in ANFIS, searching the parameter space to find the optimal combination that minimizes the cost function. The process involves initializing the population of particles in PSO, evaluating their fitness using ANFIS, and updating their personal best, global best, and ANFIS parameters. The performance and robustness of ANFIS models by optimizing their parameters using the evolutionary principles of GAs. The process involves encoding, initialization, evaluation, selection, crossover, mutation, replacement, and termination. The encoding process involves a chromosome representation of the parameters of the ANFIS model. The encoding scheme represents the fuzzy sets, membership function parameters, and neural network weights. After the initialization of random individuals, the next phase is training and validating the ANFIS model using the parameters encoded in each chromosome and measuring its performance on a dataset. The selection, mutation, and crossover are repeated till the termination criteria are met.

The best individual (chromosome) represents the optimized parameters for the ANFIS model. The optimum hyper parameter of hybrid model of ANFIS-GA and ANFIS-PSO model are mentioned as follows: Number of fuzzy rules is fixed 8 for all models, number of epochs are 500 for ANFIS, ANFIS-GA and ANFIS-PSO model, initial step size is 0.02, step size increase and decrease rate are 1.0 and 0.5, respectively, cognitive acceleration and social acceleration both are taken as 2 for all the proposed models.

2.3. Monte-Carlo Simulation

MCS is a valuable tool for analyzing and understanding complex systems and processes in the presence of uncertainty. It is a computational technique that approximates the behavior of complex systems or processes by performing

repeated random sampling. It originated during the Manhattan Project, where scientists faced mathematical problems related to neutron diffusion in nuclear materials. The name "Monte Carlo" refers to the famous Monte Carlo casino in Monaco, known for its games of chance, reflecting the randomness inherent in the method.

MCS involves several key steps, including problem formulation, probability distributions, random sampling, and simulation runs. The basic idea is to generate random variables for uncertain parameters in the model and use these to compute the outcomes. The simulation is typically repeated thousands or even millions of times to create a distribution of possible outcomes. The probability density functions of the samples are the same as those of the population. The mechanism involves defining the problem, developing a mathematical model, specifying probability distributions for the uncertain variables, generating random samples using a random number generator, running simulations for each set of random inputs, and analyzing the results to understand the range, mean, variance, and other statistical properties of the outcomes. The fundamental concept underlying a MCS is to assign a range of values to an uncertain variable in order to observe the resulting outcomes, and subsequently calculate an average of these outcomes to obtain an approximate estimate. The primary focus of the method is the iterative selection of random samples.

In the case that the variable is uncertain, the procedure will assign it a random value. Subsequently, the model is executed, and the resulting outcomes are recorded. Multiple iterations or simulations are conducted to generate paths, and the result is derived through accurate mathematical operations.

2.4. Dataset Preparation

For the generation of the dataset, a simply supported beam of span (L) 5000 mm is taken, and applied to a uniformly distributed load of ' w ' N/mm (Figure 1).

The beam span, the distance between two supports of a beam, is a crucial geometrical parameter that influences its structural behavior and deflection. The input parameters are the load applied and the modulus of elasticity (E) of the beam. The modulus of elasticity, also known as Young's modulus, measures a material's stiffness by comparing stress to strain in the linear elastic region of its stress-strain curve. The uncertainty in E can arise from several factors, including material inhomogeneity, variations in manufacturing processes, and measurement errors during testing. The output parameter, deflection of the beam (δ), is calculated from Eq. (1).

$$\delta = \frac{L}{325} - \frac{5WL^4}{384EI} \quad (1)$$

The dataset for the study is generated in MATLAB environment using the 'randn' command while keeping L and moment of inertia (I) constant. According to the Bureau of Indian Standards (1984), the prescribed maximum deflection of the beam (δ_{\max}) is set at $L/325$, which translates to 15.385 mm for a span of 5000 mm. First, a dataset of size 800 is generated, which is randomly divided into training and testing datasets after normalization of the datasets, which corresponds to 70% and 30% of the total datasets, respectively. Keeping the same input variables and their probability distribution, random datasets of sizes 1000, 2500, and 5000 are generated in a MATLAB environment. Once the model is trained and tested on the training and testing datasets, respectively, the simulated model

is tested on the MC datasets, keeping the same training dataset. Descriptive statistics provide a summary of the main aspects and characteristics of the input and output parameters of the used dataset. Descriptive statistics are fundamental for gaining insights into the distribution and characteristics of a dataset, aiding in the initial exploration and understanding of the data. In this study, the minimum, maximum, mean standard error, median, standard deviation, kurtosis, and skewness of the input and output variables are calculated and summarized in Table 1.

From the presented result, it can be observed that the kurtosis for all variables is negative, and the skewness is almost negligible for all parameters used in this study. The minimum value of modulus of elasticity is 25000 N/mm², the applied load varies from 10 N/mm to 255 N/mm, and the corresponding deflection varies from 0.477 mm to 15.37 mm.

2.5. Performance Evaluation

Assessing the performance of ML models is crucial for understanding how well they predict the deflection of a simply supported beam. The choice of performance metrics depends on the type of problem being addressed (regression, classification) and the nature of the data. Interpretability and relevance of metrics are important in engineering problems like beam deflection, guiding better decision-making in structural design. Comparative analysis helps select the most appropriate model for the specific task.

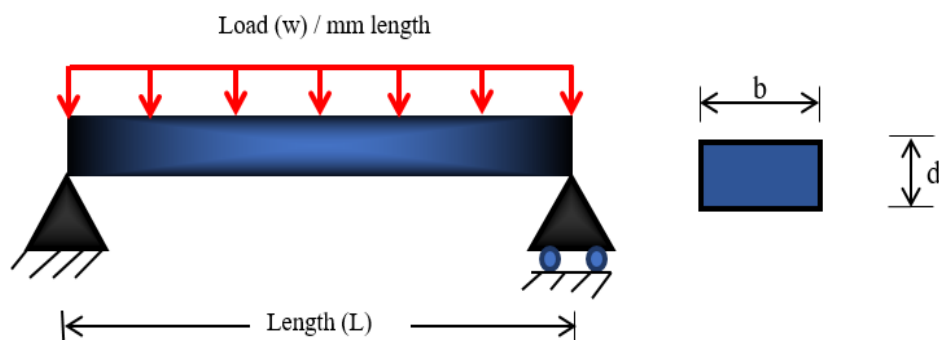


Fig. 1. The simply supported beam taken for the study

Table 1. Descriptive statistics of the input and output datasets

Descriptive statistic	Modulus of elasticity (E)	Applied load (W)	Deflection of beam (d)
Minimum	25000	10	0.477
Maximum	31622.7	255	15.37
Mean	28397.33	132.50	7.09
Standard error	87.32	2.55	0.14
Median	28483.26	132.50	7.03
Standard deviation	2469.72	72.20	3.93
Kurtosis	-1.35	-1.20	-1.08
Skewness	-0.08	0.00	0.08

Table 2. Mathematical expression and the ideal value of performance metrics

Performance metrics	Mathematical expression	Ideal value
Root Mean Square Error (RMSE)	$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (d_i - y_i)^2}$	0
Mean Absolute Error (MAE)	$MAE = \frac{1}{n} \sum_{i=1}^n y_i - d_i $	0
Weighted Mean Absolute Percentage Error (WMAPE)	$WMAPE = \frac{\sum_{i=1}^n \left \frac{d_i - y_i}{d_i} \right \times d_i}{\sum_{i=1}^n d_i}$	0
Coefficient of determination (R^2)	$R^2 = \frac{\sum_{i=1}^n (d_i - d_{avg})^2 - \sum_{i=1}^n (d_i - y_i)^2}{\sum_{i=1}^n (d_i - d_{avg})^2}$	1
Willmott's index of agreement	$WI = 1 - \left[\frac{\sum_{i=1}^n (d_i - y_i)^2}{\sum_{i=1}^n \{ y_i - d_{avg} + d_i - d_{avg} \}^2} \right]$	1
Nash-Sutcliffe Efficiency (NS)	$NS = 1 - \frac{\sum_{i=1}^n (d_i - y_i)^2}{\sum_{i=1}^n (d_i - d_{avg})^2}$	1
Variance Account Factor (VAF)	$VAF = \left(1 - \frac{\text{var}(d_i - y_i)}{\text{var}(d_i)} \right) \times 100$	100
Performance Index (PI)	$PI = \text{adj. } R^2 + (0.01 \times VAF) - RMSE$	2
Expanded uncertainty (U_{95})	$U_{95} = 1.96(SD^2 + RMSE^2)^{\frac{1}{2}}$	0
Global Performance Indicator (GPI)	$GPI = MBE \times RMSE \times U_{95} \times t_{stat} \times (1 - R^2)$	0

Overfitting concerns can be addressed by evaluating model performance on both training and testing datasets. In this study, some common performance metrics used to assess the performance of constructed models are presented in Table 2. Where d_i and y_i denote the actual and model-predicted i^{th} value of deflection of the simply supported beam, respectively, d_{avg} denotes the average of the actual deflection of a simply supported beam, and n represents the total number of samples. For an ideal model, the value of these performance metrics should be equal to their ideal value as presented in Table 2. The overall methodology flowchart of the present study is presented in Figure 2.

2.6. Reliability Analysis

The First Order Reliability Method (FORM) is a prevalent technique in reliability analysis across engineering and other disciplines. It operates on the principle of identifying the most probable point of failure within the domain of the limit state function, which delineates the boundary between failure and non-failure regions in the space of random variables. In FORM, reliability is measured and expressed through the Reliability Index (β). Consider that the demand, denoted as E' , represents the expected deflections of an engineering system, while the capacity, denoted as L_m , represents the limiting deflection. Both E' and L_m are typically

uncertain variables.

The system's margin of safety is captured by the limit state function, also known as the performance function, which is a mathematical expression of the failure criterion. This function defines the relationship between random variables and the occurrence of failure. The Performance function (p) is given by Eq. (2).

$$p = g(E', L_m) = E - L_m \begin{cases} > 0. \text{ Safe} \\ = 0. \text{ Limit state} \\ < 0. \text{ Failure} \end{cases} \quad (2)$$

The procedure for calculating the Reliability Index (Sivakumar Babu et al., 2006) is as follow:

1) Express the basic variables (R, D) in the standard non-dimensional form (Eq. (3)):

$$p_E = \frac{E - \mu_E}{\sigma_E}, \text{ and } Z_{L_m} = \frac{Q - \mu_{L_m}}{\sigma_{L_m}} \quad (3)$$

Transform the limit state function into reduced variables, resulting in a representation as a straight line (Eq. (4)):

$$g(E', L_m) = E' - L_m \quad (4)$$

2) The FORM algorithm operates on the principle of finding the Most Probable Failure Point (MPFP), which is the point on the limit state surface where the failure probability is maximized. The reliability index (β) (Eq. (5)) is determined by the shortest distance from the origin to the function $f(E', L_m)$ as depicted in Figure 1. The comprehensive methodology for the reliability analysis is detailed in Figure 2.

$$\beta = \frac{\mu_E - \mu_{L_m}}{\sqrt{(\sigma_E^2 + \sigma_{L_m}^2)}} \quad (5)$$

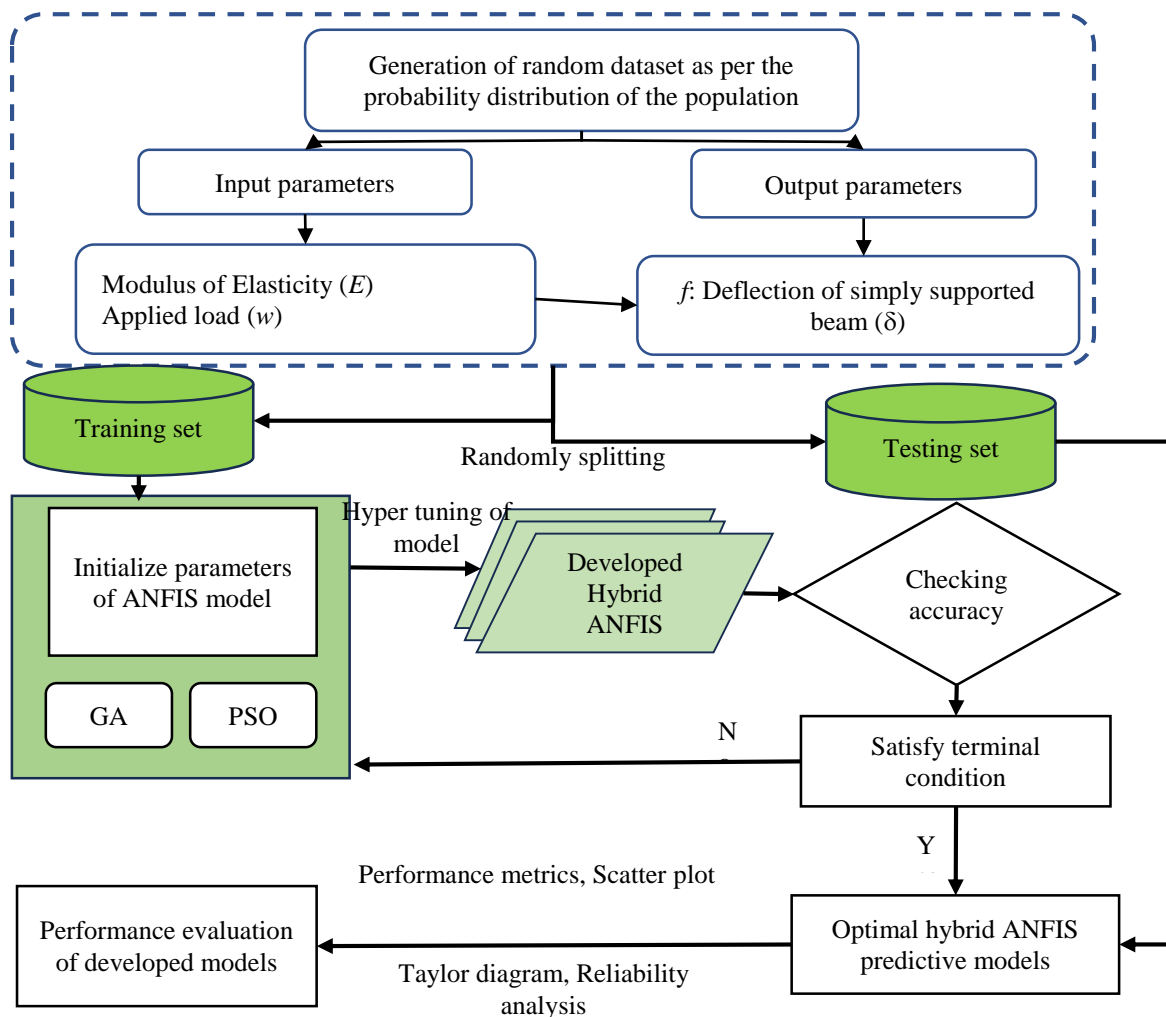


Fig. 2. Methodology flowchart of the present study

3. Results and Discussion

3.1. Performance Evaluation of Developed Models

The performance comparison of developed ML models depends on various factors, including the quality and quantity of data, feature engineering, hyper parameter tuning, and the specific characteristics of the algorithms. In this study, ten most widely used performance metrics were evaluated and analyzed to compare the performance of developed ML models. Table 3 shows the comparison of

developed models in quantitative in both the training and testing phases. From the presented result, it can be observed that the developed ANFIS-PSO model obtained the R^2 value of 1.00 for both training and testing followed by ANFIS-GA ($R^2 = 0.996$ for both training and testing) and ANFIS ($R^2 = 0.979$ for training and 0.978 for testing). Thus, the performance of the proposed ANFIS-PSO model is superior to that of the other proposed models. Tables 4-6 show the comparative study of performance parameters of the Monte Carlo simulation of the proposed models.

Table 3. Performance comparison of ANFIS, ANFIS-GA, and ANFIS-PSO

	ANFIS		ANFIS-GA		ANFIS-PSO	
	TR	TS	TR	TS	TR	TS
R^2	0.979	0.978	0.996	0.996	1.000	1.000
WMAPE	0.071	0.073	0.029	0.031	0.006	0.007
NS	0.976	0.974	0.996	0.995	1.000	1.000
RMSE	0.041	0.040	0.018	0.018	0.005	0.005
VAF	97.672	97.531	99.570	99.513	99.968	99.962
PI	1.915	1.913	1.974	1.973	1.994	1.994
WI	0.994	0.993	0.999	0.999	1.000	1.000
MAE	0.032	0.032	0.013	0.013	0.003	0.003
U_{95}	0.255	0.240	0.274	0.257	0.268	0.250
GPI	0.000	0.000	0.000	0.000	0.000	0.000

Table 4. Performance comparison of MC simulation of ANFIS model

Parameters	Number of MC simulations		
	1000	2500	5000
R^2	0.983134	0.988298	0.988653
WMAPE	0.111658	0.08258	0.14495
NS	0.936297	0.954645	0.907558
RMSE	0.039626	0.032952	0.061134
VAF	97.65969	97.60643	97.17235
PI	1.920099	1.931401	1.899219
WI	0.983324	0.98731	0.97408
MAE	0.033551	0.024708	0.054569
U_{95}	0.146565	0.140616	0.184207
GPI	-0.00029	-5.8E-05	-0.00031

Table 5. Performance comparison of MC simulation of ANFIS-GA model

Parameters	Number of MC simulations		
	1000	2500	5000
R^2	0.989759	0.990091	0.99086
WMAPE	0.070654	0.078838	0.107482
NS	0.962309	0.959592	0.934483
RMSE	0.030481	0.031103	0.051467
VAF	97.75297	97.91371	97.34815
PI	1.936804	1.938117	1.912856
WI	0.989679	0.988769	0.981447
MAE	0.02123	0.023588	0.040463
U_{95}	0.140366	0.141227	0.181112
GPI	-5.2E-05	-4.6E-05	-0.00013

Table 6. Performance comparison of MC simulation of ANFIS-PSO model

Parameters	Number of MC simulations		
	1000	2500	5000
R ²	0.987791	0.989815	0.991043
WMAPE	0.093538	0.094255	0.110237
NS	0.950969	0.953495	0.934309
RMSE	0.034765	0.033367	0.051535
VAF	97.15216	97.71313	97.62601
PI	1.924543	1.933571	1.91575
WI	0.986397	0.986986	0.981623
MAE	0.028107	0.028201	0.0415
U95	0.138663	0.140525	0.183124
GPI	-8.4E-05	-5.8E-05	-0.00015

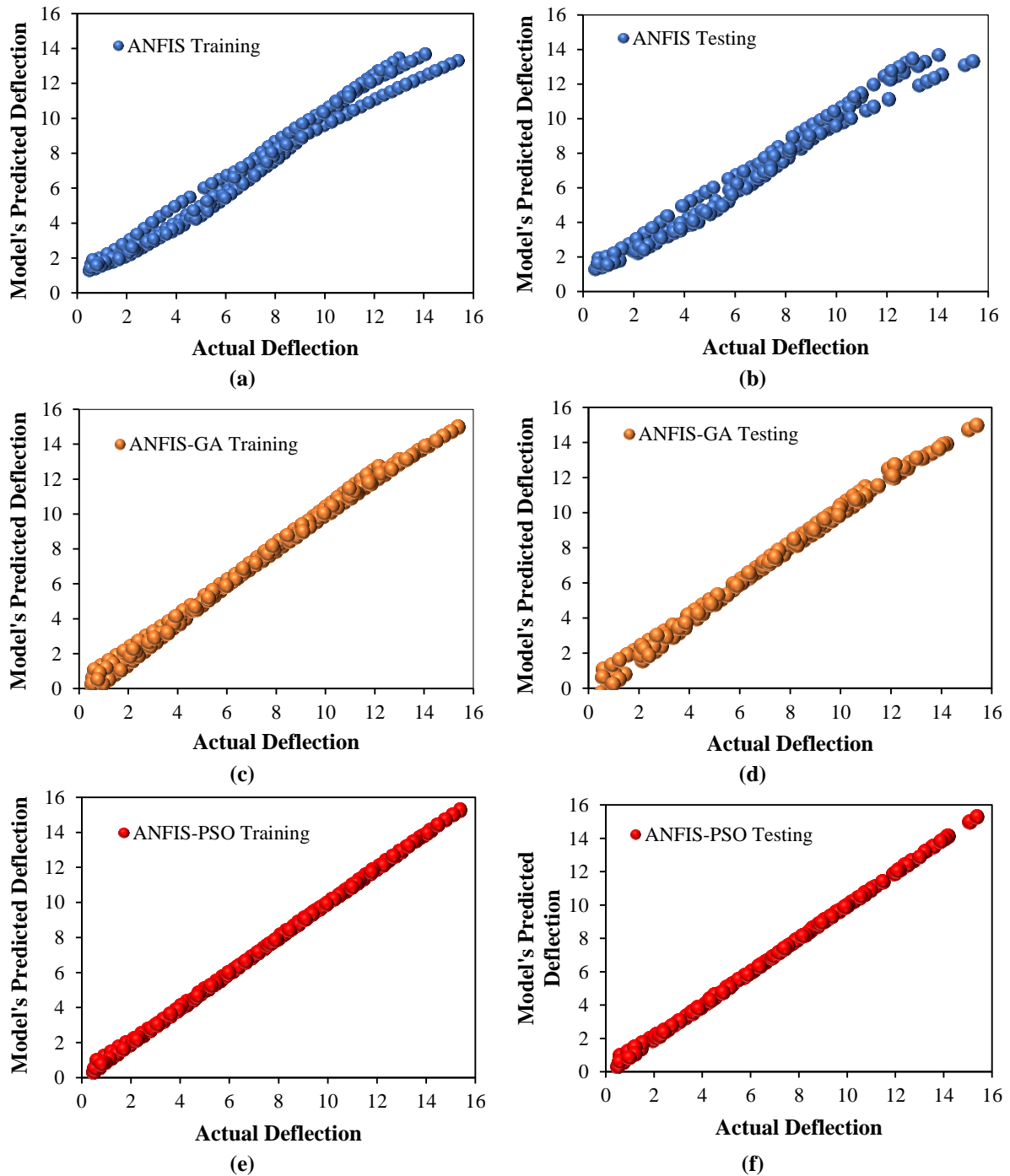


Fig. 3. Scatter plot for developed models of training and testing phase: a) ANFIS training; b) ANFIS testing; c) ANFIS-GA training; d) ANFIS-GA testing; e) ANFIS-PSO training; and f) ANFIS-PSO testing

Higher values of accuracy parameters such as R^2 , NS, VAF, PI, and WI confirm the excellent performance of the developed models, similarly, lower values of error parameters such as RMSE, WMAPE, MAE, and GPI exhibit the excellent model's performance in predicting the deflection of SS beam. From the results presented in Tables 4-6, it can be observed that the obtained value

of performance metrics is considerable and excellent for all the proposed models.

However, the performance metrics for 5000 simulation datasets are more excellent for all proposed models. A scatter plot is a graphical representation of data points in a two-dimensional space, where each point represents the values of SS beam deflection.

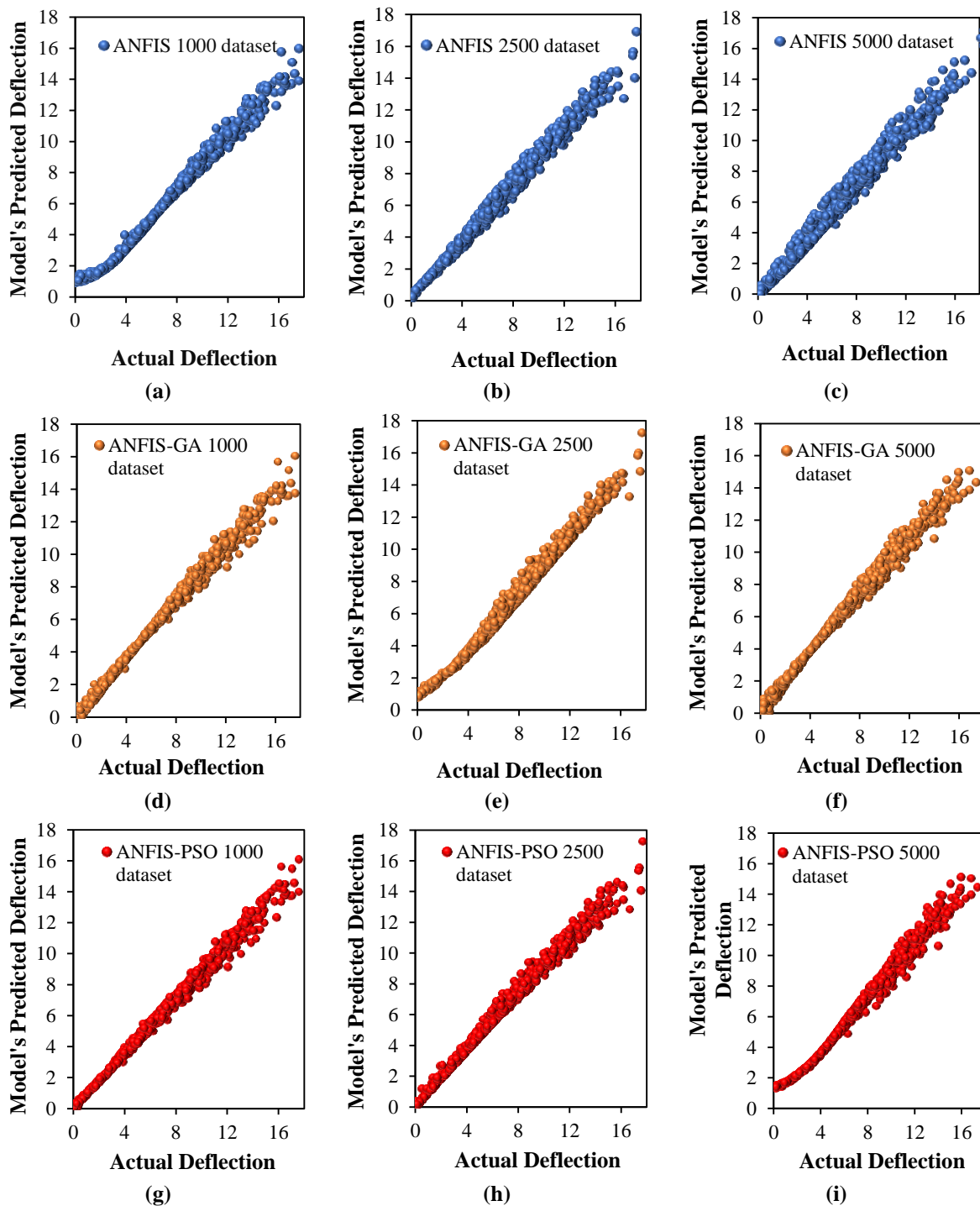


Fig. 4. Scatter plot for all models for different MC simulations: a) ANFIS, 1000 dataset; b) ANFIS, 2500 dataset; c) ANFIS, 5000 dataset; d) ANFIS-GA, 1000 dataset; e) ANFIS-GA, 2500 dataset; f) ANFIS-GA, 5000 dataset; g) ANFIS-PSO, 1000 dataset; h) ANFIS-PSO, 2500 dataset; and i) ANFIS-PSO, 5000 dataset

In the context of ML models, scatter plots are often used to visualize the relationship between the predicted and actual values of deflection of the SS beam. Ideally, the ML models is performing well, the points on the scatter plot should align along a diagonal line (45-degree angle) from the bottom-left to the top-right, indicating that the predicted values are close to the actual values. Deviations from this diagonal line suggest discrepancies between the actual and predicted values.

The scatter plot for the developed models is presented in Figure 3 for the training and testing phase, and Figure 4 shows the scatter plot of MC simulation plots for 1000, 2500, and 5000 MC simulations, respectively. From the presented result, it can be observed that the developed ANFIS-PSO model obtained a denser pattern along the diagonal line in both the training and testing phases.

3.2. Accuracy Matrix

The accuracy matrix is a heat map matrix of the performance metrics value, which is used to assess the accuracy level of proposed models. Without having to look at the values of the performance metrics, researchers are able to quickly evaluate the amount of accuracy obtained by the proposed models against the corresponding performance metrics. It was stated earlier that various performance metrics need to be determined in order to assess the performance of developed models from several standpoints. As a consequence, a heat map matrix has been shown to assess the quick review of the developed model’s accuracy. The accuracy matrix of the constructed ANFIS and hybrid model of ANFIS are shown in Figure 5, and the accuracy matrix for the different Monte Carlo simulations for all constructed models is presented in Figure 6.

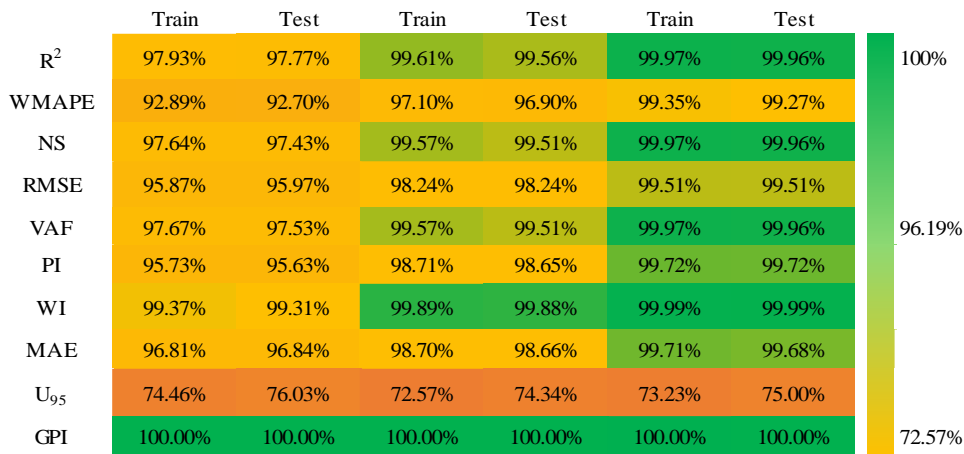


Fig. 5. Accuracy matrix for the training and testing phases of all the models

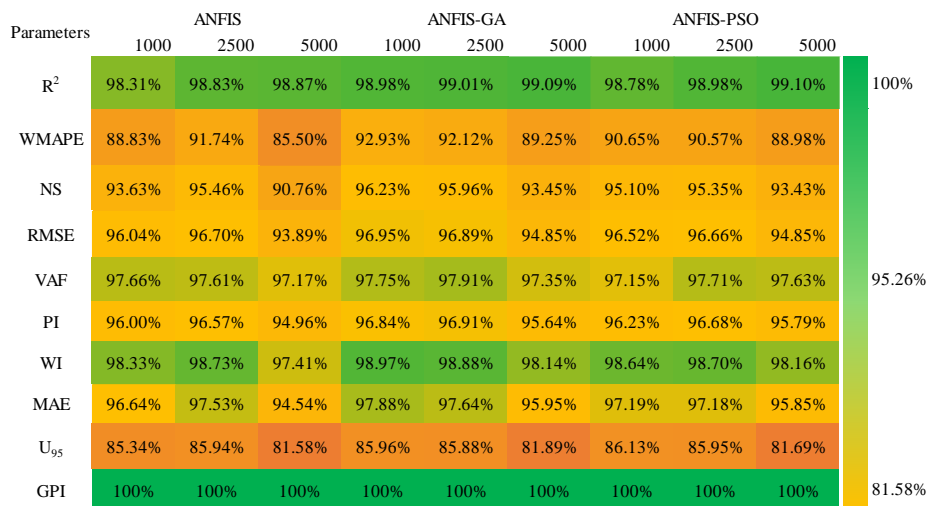


Fig. 6. Accuracy matrix for Monte Carlo simulations of all models

Herein, the overall performance of constructed models can be observed from the presented figure, the ANFIS-PSO is the best-performing model in predicting the deflection of simply supported beam, followed by ANFIS-GA and ANFIS model in the training and testing sets. As per the result of the accuracy matrix, the constructed ANFIS-PSO model attained the best accuracy corresponding to each performance metric.

3.3. Taylor Diagrams

A Taylor diagram is a graphical representation used to assess the skill of proposed models or forecasts compared to observations. It was introduced by Taylor (2001) as a tool for visually summarizing multiple performance metrics in a single plot. The Taylor diagram provides insights into how well a model or simulation reproduces key statistical characteristics of observed data. The main components of the Taylor diagram are:

i) Correlation coefficient axis (x-axis): the correlation coefficient between the model and observed data is typically represented on the x-axis. This axis shows how well the model reproduces the variability of the observations. A perfect model would lie on the rightmost edge with a correlation coefficient of 1.

ii) Standard deviation ratio axis (y-axis): the ratio of the standard deviation of the model to the standard deviation of the observations is represented on the y-axis. This axis accounts for the spread of data points around the mean. Ideally, a model

that perfectly replicates the observed variability would lie on the top of the plot with a standard deviation ratio of 1.

iii) Reference point (origin): the origin of the Taylor diagram (0, 1) represents a model that perfectly reproduces the standard deviation of the observations but has no correlation with them. Points on the diagram are plotted relative to this reference point.

iv) Radial lines: radial lines extending from the origin represent constant values of the correlation coefficient. Each concentric circle corresponds to a specific correlation coefficient, with the outermost circle typically representing perfect correlation (1.0).

v) Distance from origin: the distance of a point from the origin indicates how well the model reproduces both the correlation coefficient and standard deviation ratio. Points closer to the reference point have lower skill, while points farther away indicate better performance.

The Taylor diagram for all proposed models for the training and testing phase is presented in Figures 7a and 7b, respectively. The Taylor diagram for MCS is presented in Figures 8a to 8c. From the presented figures, it can be observed that the ANFIS-PSO model achieved the highest accuracy and correlation coefficient in both the training and testing phases. However, for 1000 and 2500 MCS, all models achieved almost similar accuracy, and for 5000 simulations ANFIS-PSO model attained the best accuracy among the other proposed models.

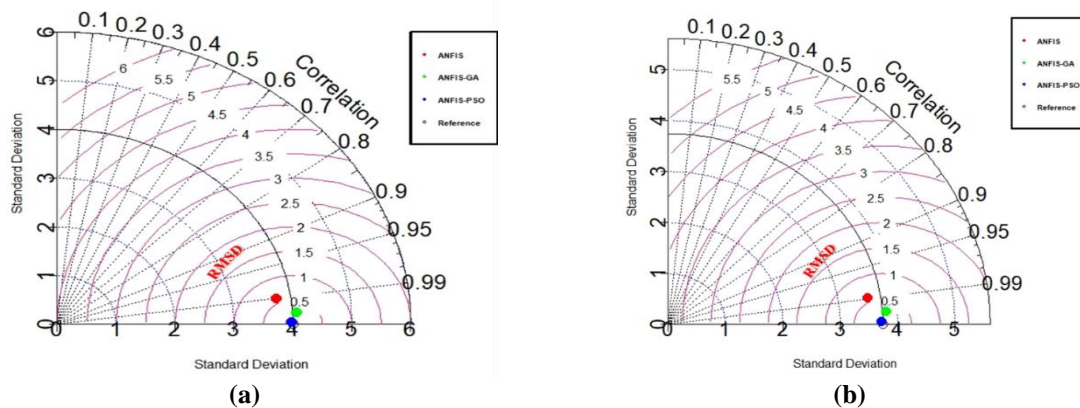


Fig. 7. Taylor diagram for: a) Training phase; and b) Testing phase

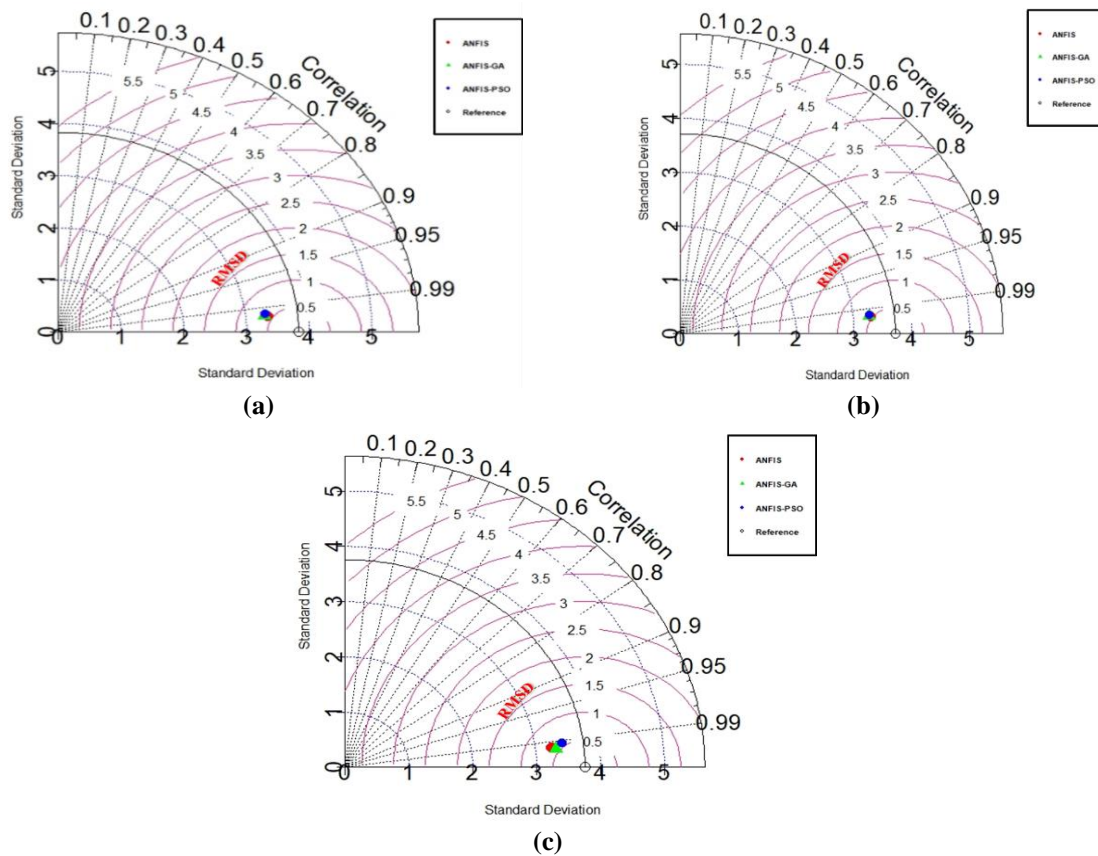


Fig. 8. Taylor diagram for: a) 1000 MC simulation; b) 2500 MC simulation; and c) 5000 MC simulation

3.4. Reliability Indices

The reliability indices of the performances of the ML models are calculated from the predicted values and compared with the reliability indices calculated from the actual values and termed as the predicted reliability index and the actual reliability index, respectively. The corresponding Probability of Failure

(POF) is also calculated and compared with the actual POF values. The capacity of the system or limiting deflection (L) is taken as $L/325$, i.e., 15.385 mm for the span of 5000 mm, as per Bureau of Indian Standards (1984). The reliability indices and POF are reported in Figures 9 and 10, respectively, calculated as per the methodology given in Section 2.5.

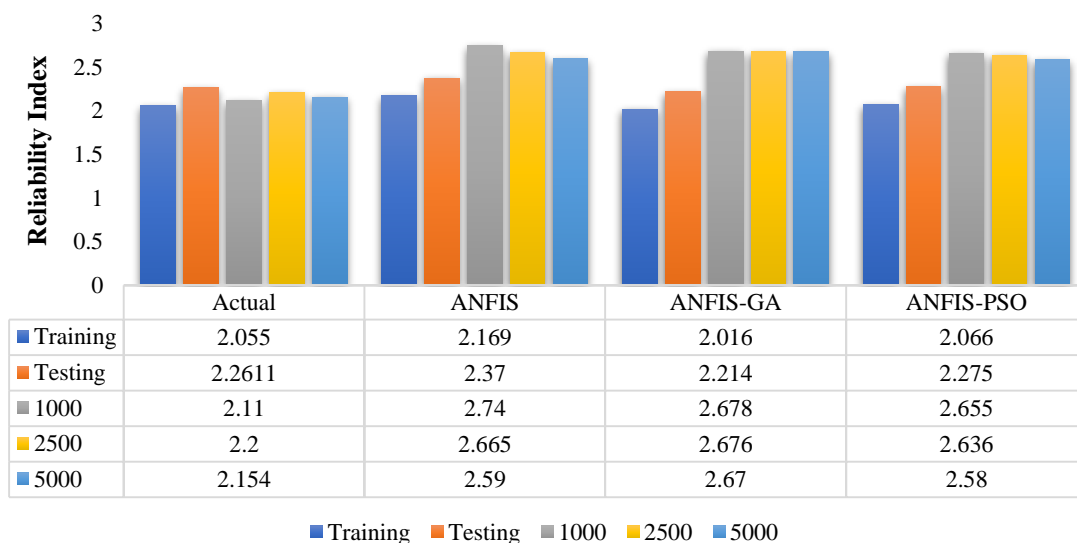


Fig. 9. Comparison of reliability indices of the proposed models

The β values for the ANFIS-PSO (2.066 and 2.275) (Figure 9) model are close to the actual values (2.055 and 2.2611) in both the training and testing phases. The reliability indices predicted by the ANFIS and ANFIS-GA models also predict the reliability index satisfactorily; however, while ANFIS (2.169 and 2.37) over-predicted the values, ANFIS-GA (2.016 and 2.214) under-predicts it. The performances of the ML models reduce slightly in the MC simulation phase, however, ANFIS-PSO is the best-performing model. Corresponding to the actual reliability indices of 2.11, 2.2, and 2.154 in the MC simulation of 1000, 2500, and 5000 datasets, respectively, the values calculated from the ANFIS-PSO model predictions are 2.655, 2.636, and 2.658, respectively, compared to 2.74, 2.665 and 2.59 of ANFIS and 2.678, 2.676 and 2.58 of 1000, 2500 and 5000 MC datasets, respectively. The POF values (Figure 10) of the ANFIS-PSO model are close to the actual values, compared to the other two models. Thus, ANFIS-PSO is the most reliable model; however, the performances of the other two models are also satisfactory.

4. Conclusions

This study proposed hybrid neuro-fuzzy ML models based on the reliability-based design of structural beams using Monte Carlo simulation. For this purpose, 800 random datasets were generated, with

modulus of elasticity and applied load as input parameters and beam deflection as output parameters. The models were simulated and tested on the dataset, and the corresponding reliability indices were calculated. To check the reliability of the models, MC simulation was performed, and corresponding reliability indices were calculated and compared to the actual reliability index. The important conclusions of the study are:

1. ANFIS-PSO ($R^2 = 1$, RMSE = 0.007) is the most robust model in the training and testing phase, followed by ANFIS-GA ($R^2 = 0.996$, RMSE = 0.018) and ANFIS ($R^2 = 0.978$, RMSE = 0.04). The hybridization improves the performance of the ANFIS model.

2. The performance of the models is robust in the MC simulation datasets; however, the performance of the hybrid ANFIS models reduces slightly, which highlights the importance of checking the model performance in the MC simulation. Another significant takeaway is the enhancement in the performance of the hybrid ANFIS models as the number of datasets increases, which confirms the robust simulation of the models.

3. ANFIS-PSO ($\beta = 2.275$, POF = 0.0115) was concluded to be the most robust model, having the reliability index and POF close to the actual value ($\beta = 2.261$, POF = 0.0118). The other two models also gave satisfactory performance and were concluded to be robust as well.

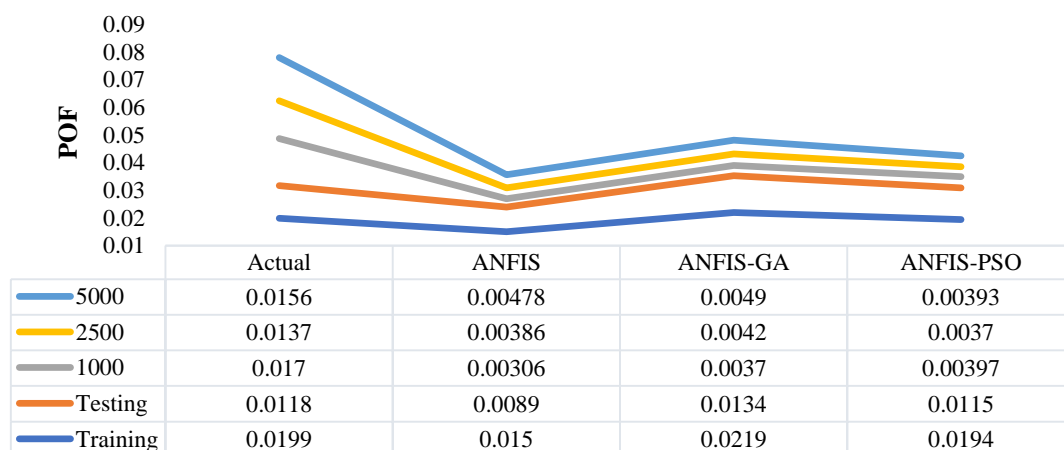


Fig. 10. POF values of the ML models

To enhance future investigations, it is crucial to broaden the scope by simulating more hybrid ANFIS models for comprehensive reliability analyses and comparing them with the ANFIS-PSO framework. Subjecting ML models to rigorous training and evaluation across varied and diverse datasets is essential to validate their effectiveness as robust methodologies for practical designs related to simply supported beams and similar structures. Moreover, it is imperative to integrate additional reliability-based analyses for conducting thorough comparisons between the outcomes generated by the FORM and the models formulated in this research. Furthermore, the ML-driven framework suggested in this study offers possibilities for expanding reliability analyses to cover a wider range of civil engineering structures.

Expanding these investigations will enhance this paper's comprehension and utilization of ML techniques in structural reliability assessments, thus progressing the field of civil engineering. It is crucial to acknowledge that machine learning, although a potent tool, does not offer a universal solution. The successful application of machine learning in the laboratory setting necessitates meticulous examination of the particular problem, the accessibility and quality of data, the suitable selection of algorithms, and frequent validation and enhancement of models to guarantee accurate and reliable outcomes.

5. Declarations

The authors declare that no generative AI or AI-assisted technologies were used during the writing, editing, or data visualization phases of this manuscript.

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