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RESEARCH PAPER

Forecasting Bearing Capacity, Error Analyses and Parametric Analysis of Circular Footing Seating on the Limited Thick Sand-Layer with Eccentric-Inclined Load

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ABSTRACT: Bearing Capacity (BC) of the soil is one of the crucial parameters to construct any structure. A consistent soft computing models can reduce the cost and time by swiftly generate the required experimental data. This research presents, M5P model tree and feedforward backpropagation ANN model have been used to predict the BC of the circular footing resting on the limited thick sand-layer with eccentric-inclined load. To generate the proposed model, a set of 120 data are gathered from the literature. The results of M5P model tree achieved a coefficient of determination (R^2) of 0.96 for both training and testing phases. The Mean Absolute Percentage Error (MAPE) was 19.83% for training and 21.46% for testing. Whereas, for ANN model, R^2 is 0.98 and 0.97; MAPE is 18.20 and 16.29 for training and testing, respectively. The R^2 and MAPE results reveals that, the ANN model is better substitute method for predict the BC of the Circular Footing (CF) resting on the limited thick sand-layer with eccentric-inclined load than the M5P model. Further, model equations are developed to calculate the BC of the circular footing for the both the methods. Finally, sensitivity analysis concludes that the input parameter ratio of depth of the rigid rough base to width of footing (H/B) is the most influencing parameter to predict the desired output.

Keywords: Circular Footing, Sand, Bearing Capacity, Eccentric-Inclined Load, M5P Model Tree, ANN, Sensitivity Analysis.

1. Introduction

Shallow footings are most commonly used foundations in the construction of the buildings due to its low cost. The Bearing Capacity (BC) of uniform soil with normal/eccentric/eccentric-inclined load on all regular shaped footings (strip, square, circular and rectangular) was well established (Terzaghi, 1973; Vesic, 1973).

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However, a significant effect on the bearing capacity can be observed as the hard soil strata within a limited depth below the footing. In some real-time conditions, a limited thick soil stratum can be observed beneath the bedrock. In such situations, load carrying capacity of the shallow footing situated on the soil strata can be influenced by the boundary of the rigid bedrock.

A Finite Element Analysis (FEA) performed on the strip footing was positioned on sand under eccentric inclined load to determine the BC by Loukidis and Ygeionomaki (2017). Later, Sethy and Patra (2019) reported about the shallow circular footings resting on the limited thick sand-layer with eccentric-inclined load. Further, Sethy et al. (2020) conducted a simulation study on the same footing of Sethy and Patra (2019) by using FEA and Plaxis3D software. However, the process of physical and FEA needs enormous specialties and time. Now a days soft computing techniques playing a major role in reducing the time and cost.

From the past decade, machine learning techniques play a significant role in geotechnical engineering for forecasting, planning, and management of the non-linear data sets. As an example, Dutta et al. (2015) used an artificial neural network to forecast the deviator stress of sand blended with waste plastic strips (strip thickness: 0-5 mm thick). Initially, for forecasting the deviator stress eight input parameters were considered, after conducting the sensitivity analysis lest effected three input parameters were excluded and forecasted with five input parameters. The model having five input parameters forecast the deviator stress better than the previous eight input parameters model. The cohesion of limestone following Artificial Neural Networks (ANN) and ANN with Genetic Algorithm (GA) estimated by Ebid et al. (2021) and Khandelwal et al. (2018). The study concludes that, the GA-ANN model predicts the cohesion of the limestone more precisely than the ANN and multiple regression. ANN approach was used for the forecasting the resilient modulus of granular material (Saha et al., 2018). In this study seven physical properties of the granular material were used as an input parameter to predict the resilient modulus. The study concludes that; the developed ANN model is much superior than the regression model in predicting the resilient modulus.

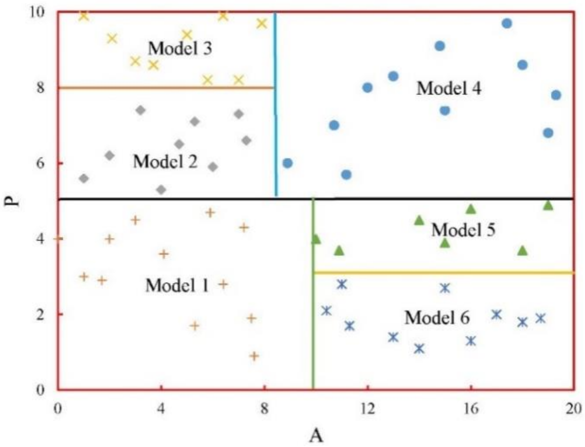
A research on the forecasting the BC of square footing resting on c-ϕ soil slope was done by Acharyya et al. (2020), where, two ANN model equations were proposed for predicting the BC and settlement of multi edges structural skirted footing positioned on the sand. Similarly, different studies performed on the forecasting the BC of rectangular footing on layered sand under inclined loading (Panwar, 2022); under-reamed piles compression capacity on sand and clay (Thottoth et al., 2024); cyclic load-induced settlement of strip footing on granular soil (Sasmal and Behera, 2021); BC for strip footing situated near sloping ground (Acharyya and Dey, 2019); unconfined compressive strength of fly ash stabilised organic clay (Gnananandarao et al., 2022); quarry dust modified unsaturated soil erodibility (Onyelowe et al., 2021) and erodibility of stabilized unsaturated lateritic soil (Onyelowe et al., 2022) using different soft computing techniques such as random forest regression, M5P model tree, SVM with different kernels and Artificial Neural Networks; the settlements of a raft reinforced with geogrid and geocell were predicted using ANN modelling with the help experimental data by Kumar et al. (2023).

The purpose of this article is to offer a strategy for forecasting the BC of circular footing resting on the limited thick sand-layer with eccentric-inclined load. The M5P model tree and ANN are utilized for present study. The data utilized in this study is collected from the published literature containing BC of circular footing and some other parameters. The total collected data sets are 120. This wide-ranging set of data can be sufficient for developing a model and predicting the desired output (Acharyya and Dey, 2019).

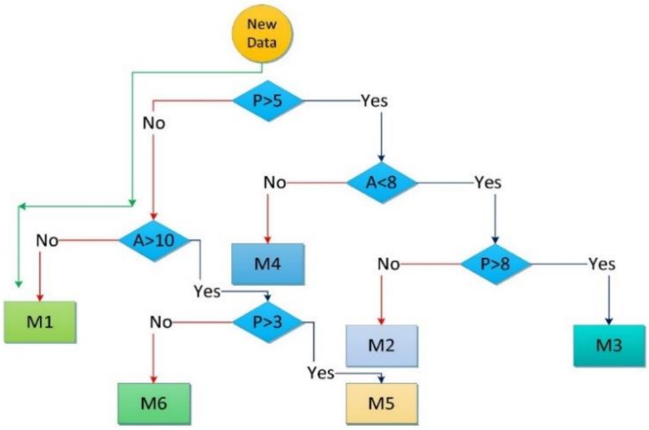
2. M5p Model Tree

The M5 tree algorithm was proposed by Quinlan (1992) and it had been refined and renamed as a M5P algorithm (Wang and Witten, 1997). The major advantage of the model trees is that, they are having capability to handle the large sets of data, each with a different set of variables and dimensions. They are renowned for their ability to deal with missing data as well.

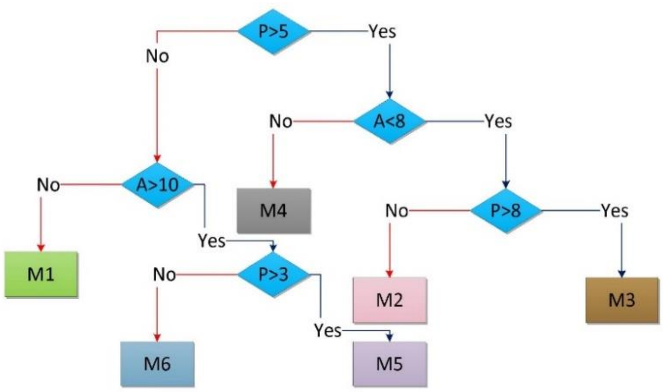
The M5P model architecture has depicted schematically in Figure 1. The M5P model begins with separating the input data into distinct sub sets, each sub-set has data records with a sharing feature (Figure 1a).



(a)



(b)



(c)

Fig. 1. The schematic view of the M5P model tree algorithm

To overcome the difference within the defined sub-set, Linear Regression Models (LRM) might be employed in this procedure. The information gathered in the preceding stage is then utilized to generate numerous nodes, each of which is segregated accord with the certain attribute (Figure 1b). This phase leads to construct a structure as like tree with the roots and leaves presented as top and bottom, respectively. New datasets are added to the tree. It progresses from the roots through the nodes until they reach to each leaf (Figure 1c). A math logical algorithm at every node helps the set of data to make its way to reach a leaf by validating a specified given set's value of data file containing the split value. This procedure enables the knowledge to be extracted from the tree model.

The M5P technique, which is an enhanced version of the M5 technique, has four basic phases. To form a tree, the input sets are dividing into multiple sub-sets in the first stage. The splitting criterion is used to reduce intra-subspace variation from the roots to the nodes. The variability is measured by the Standard Deviation (Stan. Dev.) values that reach the nodes. The Stan. Dev. reduction factor is used to develop the tree, which maximizes the expected reduction in error at each node, as shown in Eq. (1).

$$S.D.R = sd(S) - \sum_i \frac{S_i}{|S|} \times sd(S_i) \qquad (1)$$

where $S.D.R$: Stan. Dev. reduction; S: A group of data that is received by the node; S_i: Splitting the node based on an attribute generated set; and sd: is standard deviation.

The SDR equation was proposed by Wang and Witten (1997). However, having constructed the tree, when it comes to phase two, using the data related to each sub-space, in each sub-space, a LRM is constructed. At the time, to overcome the problem of overtraining, a pruning strategy is used. When the SDR for a LRM at the root of a subtree is lower than the predicted error for the subtree, an overtraining problem emerges. However, the pruning procedure can result in severe incompatibilities among neighboring LRM.

The smoothing technique is performed in the final phase to correct for this issue. To develop the final model for the leaf, the smoothing procedure integrates all of the models from the leaf to the root. The anticipated result of the leaf is processed as it returns to the root in this procedure. This value is combined with the LRM's anticipated value as follows for that node.

$$E' = \frac{ne + ka}{n + k} \qquad (2)$$

where E: is the estimated value, forwarded to the next higher node; e: is the estimated forwarding from below to the current node; a: is the model's estimated value for this node; n: is the number of training samples; and k: is a constant.

3. Artificial Neural Networks

An artificial neural network is comprised of multiple layers of neurons, where each layer is connected to the subsequent layer. The neurons in each layer receive inputs from the previous layer, and then calculate an output based on those inputs and their own weights and biases. The outcome from the resulted layer i.e., last layer is the final network output. In this research, the backpropagation algorithm is employed to train feedforward neural networks. This method involves iteratively fine-tuning the network's weights and biases to minimize the disparity between its output and the intended outcome. The process involves transmitting the error in reverse through the network, starting from the output layer and moving towards the input layer. The error at each neuron is then used to adjust the weights and biases of that neuron, in order to reduce the error in the next layer. The backpropagation algorithm is a powerful tool for training feedforward neural networks. It is comparably straightforward to put into practice, and it can be employed for training networks containing a substantial quantity of neurons. However, the algorithm can be slow to converge, especially for large networks. The flow of the ANN is presented in the Figure 2.

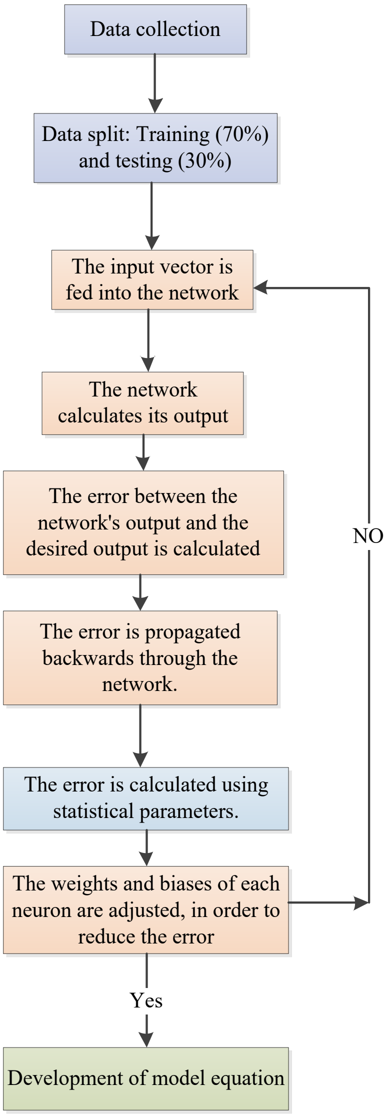


Fig. 2. Flowchart of the ANN model

4. Data Set

In the present research the total of 120 data points were collected from peer-reviewed literature (Sethy et al., 2020b). The data used in this study was generated through Plaxis3D software, employing both the Mohr-Coulomb (MC) model and the Hardening Soil (HS) model in the Finite Element Method (FEM). These FEM results were then compared to the laboratory experimental results conducted by Sethy et al. (2019). The outcomes from both the MC and HS models showed good agreement with the experimental results as reported by Sethy et al. (2020b). The differences between the results obtained in that study and those from previous research were found to be within a range of ± 20%. This compared comprehensive dataset was employed to predict the Bearing Capacity (BC) of a circular footing placed on a sandy layer with limited depth. The footing was subjected to eccentric-inclined loading during the analysis. The data consists of four attributes such as H/B, a/f, e/B and q_u where H, B, a, f, e and q_u: are depth of the rigid rough base, width of footing, load of inclination, angle of internal friction of soil, load eccentricity, and ultimate bearing capacity, respectively. The first three non-dimensional parameters (H/B, a/f and e/B) were used as an input parameter (q_u) in both the M5P model tree and ANN and the fourth parameter was used as an output parameter.

The range of data for the parameters for input and output are shown in Table 1. Further, typical input as well as output parameter ranges are depicted in the Figure 3 in terms of the histogram. Nevertheless, out of the entire dataset, 70% of the data was employed for training purposes, while the remaining 30% was allocated for testing.

Table 1. Data set for M5P and ANN modelling

Range of data	Input and output parameters			
	H/B	α/ϕ	e/B	q_u (kN/m^2)
Minimum	0.3	0	0	42.5
Maximum	5.5	0.489	0.15	880
Stan. Dev.	1.80	0.17	0.06	190.07

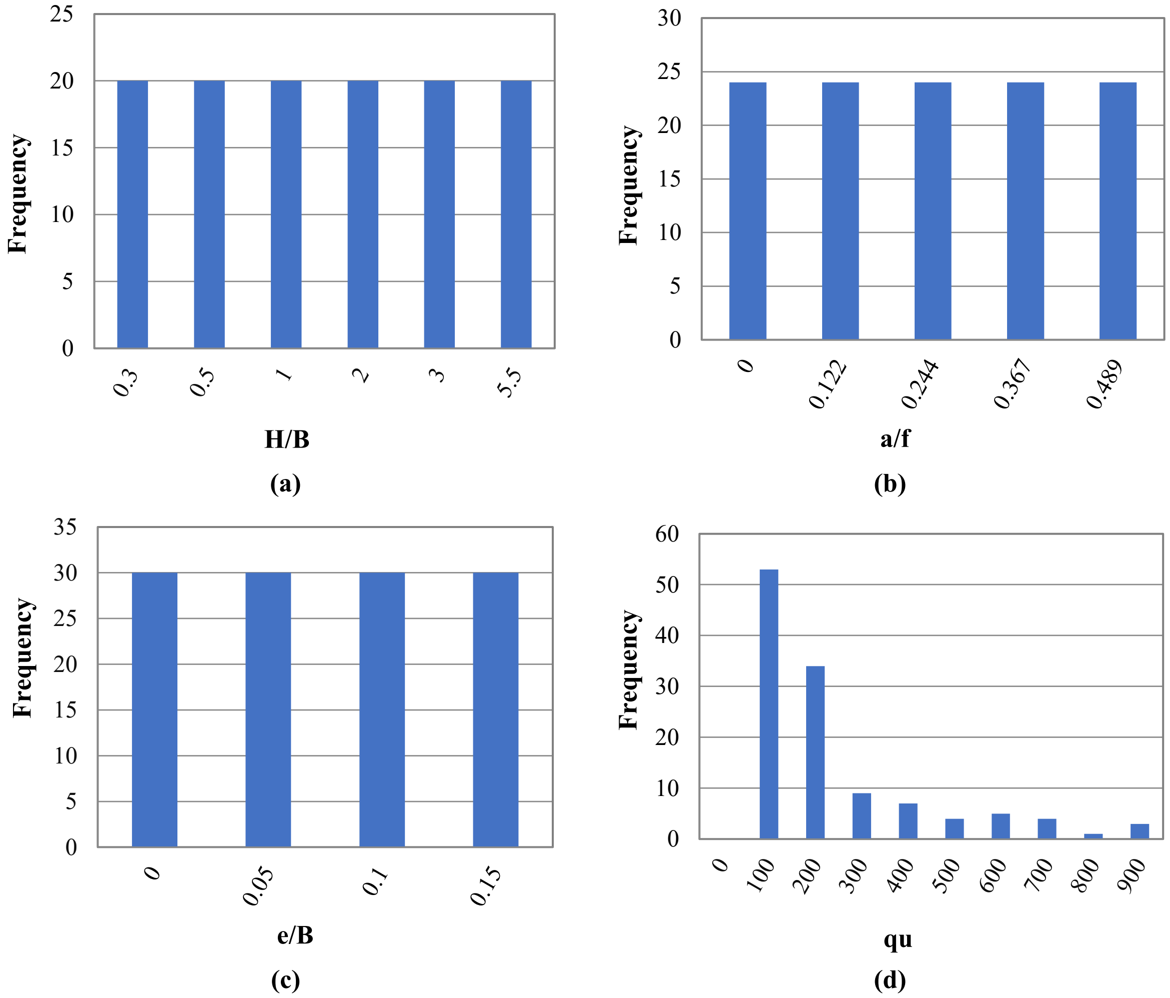


Fig. 3. Histograms of the data set: a) H/B: b) a/f; c) e/B; and d) q_u

5. Development of M5p Model Tree

To forecast the circular footing's BC on the limited thick sand-layer with eccentric-inclined load, many factors that may affect such as relative density, unit weight, soil friction angle, footing width, footing depth, sand layer limited thickness depth, type of footing material and so on. To attain the precise model, all these parameters need to be considered. However, it is exceedingly difficult to consider each specific attribute in the model in order to attain the desired output. Hence, majorly effecting parameters are converted in to the non-dimensional and used as an input parameter.

These non-dimensional parameters can also avoid the scaling impact of the circular footing. In the initial stage of this research, the function for estimating ultimate BC can be written as follows:

$$q_u = f\left(\frac{H}{B}, \frac{\alpha}{\phi}, \frac{e}{B}\right) \tag{3}$$

where, *H/B*, *a/f* and *e/B*: are no-dimensional parameters and the parameters already defined in the previous section.

6. Statistical Parameters of Error Analysis

The developed prediction model in the present study to forecast the BC of circular footing on the limited thick sand-layer with eccentric-inclined load developed by 120 data points gathered from the literature. There is no harmony among scholars when it comes to determining the desired level of prediction accuracy. Then, the quality prediction accuracy has become a major criterion. It was accomplished only after minimizing the error in the prediction (Ito, 1994; Sarle, 1995). In this paper, six statistical parameters were used such as: Mean Absolute Percentage Error (MAPE); Mean Square Error (MSE); Root Mean Square Error (RMSE); Mean Absolute Error (MAE); coefficient of correlation (r); and coefficient of determination (R^2).

MAPE can measure the relative performance among the various methods. Based on the calculated MAPE value the prediction accuracy is said to be good (MAPE between 10 to 20%), poor (MAPE between 20 to 50%) and inaccurate (MAPE over 50%) (Dutta et al., 2015). It is, not only provides the prediction quality, but also independent on the units of the measurements of the variable.

The calculated MSE gives accuracy measures as well as indications of the degree of dispersion; larger errors were given extra weight (Dutta et al., 2015). The squared difference among anticipated and actual observed data is taken into account by MSE in order to calculate the difference among predicted and observed data (Dutta et al., 2015). The root of MSE is the RMSE. It has the advantage of measuring within the same unit as the forecasted variables.

The MAE is one of the statistic parameters that determines overall accuracy and indicates the extent of the spread. All errors were also given the same importance when trying to calculate MAE. MAE is minimal (≈ 0) for perfect data fit and considerable for poor data fitting (Dutta et al., 2015). In comparison to MAE, MSE aggressively penalizes significant errors. MAE is determined by calculating from the same unit as MSE and RMSE, but large prediction errors are given less weight.

Further, the effectiveness of soft computing models is generally deciphered by coefficient of correlation ‘r’ and coefficient of determination ‘R^2’. The evaluation of this method is skewed. As a result, in addition to the unbiased statistical criteria, extra unbiased statistical criteria should be utilized. The predictive models with a 'r' and a 'R^2' closer to one stand for good predictions.

The intensity of the forecasting errors can be measured using the MAPE, MSE, RMSE, and MAE. RMSE and MSE have historically been popular due to their theoretical importance in statistical modelling. Nonetheless, certain researchers caution against utilizing them for assessing predictive precision due to their heightened susceptibility to deviations compared to MAE. Moreover, MAE and RMSE can also be utilized in tandem to discern the spread of errors within a prediction set. Always, the RMSE shall outweigh or be equivalent to the MAE.

As the gap between RMSE and MAE widens, the variance in individual errors within the dataset also increases. Moreover, when RMSE is equivalent to MAE (both ranges 0 to ∞), all errors share the same magnitude. RMSE and MAE can be called as negative score’s, with lower values being preferable. Besides, MAE can be regarded as a 'strong' measure for the output accuracy i.e. for prediction. The choice of an error measure has a significant impact on the results, which forecasting approaches are the generally accurate.

The mentioned statistical parameters and their mathematical is expressed as follows:

$$MAPE = \left[\frac{1}{n} \sum_{i=1}^{n} \left| \frac{q_{u_{ti}} - q_{u_{pi}}}{q_{u_{ti}}} \right| \right] \times 100 \tag{4}$$

$$MSE = \frac{1}{n} \sum_{i=1}^{n} \left(q_{u_{ti}} - q_{u_{pi}} \right) \tag{5}$$

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(q_{u_{ti}} - q_{u_{pi}} \right)} \tag{6}$$

$$MAE = \frac{1}{n} \sum_{i=1}^{n} \left| q_{u_{ti}} - q_{u_{pi}} \right| \tag{7}$$

$$r = \frac{q_{u_t} . q_{u_p} - n\overline{q_{u_t} . q_{u_p}}}{(n-1) \mathrm{S}_{\mathrm{q}_{\mathrm{u}_\mathrm{t}}} \mathrm{S}_{\mathrm{q}_{\mathrm{u}_\mathrm{p}}}} \tag{8}$$

$$R^2 = 1 - \frac{\sum_i \left(q_{u_{pi}} - \frac{1}{n} \sum_{i=1}^{n} q_{u_{ti}} \right)^2}{\sum_i \left(q_{u_{ti}} - \frac{1}{n} \sum_{i=1}^{n} q_{u_{ti}} \right)^2} \tag{9}$$

where $q_{u_{ti}}$: is target bearing capacity, $q_{u_{pi}}$: is predicted bearing capacity, q_{u_t}: is mean of targeted bearing capacity, q_{u_p}: is mean of predicted bearing capacity, $s_{q_{ut}}$: is standard deviation of the target bearing capacity, $s_{q_{u_p}}$: is standard deviation of the predicted bearing capacity, and n: is number of observations.

7. Developed M5P Model Equation

The training and testing technique is a most widely used technique to develop the learning algorithms to establish the model (Behnood et al., 2017). For training and testing the model, the train and test datasets are divided into two subsets using random partitioning. The model is trained using 70% of the data and then tested using the remaining 30% for evaluation of the desired model. Table 1 provides the minimum, maximum and Standard Deviation values for the total data for each of the individual parameters used to create the M5P model. The model tree that was created using the M5P technique is displayed in Eqs. (10-14). Theses equations can be used to determine the Bearing Capacity (BC) based on the conditions illustrated in Figure 4. Now, the statistical parameter results were used to see the prediction precision of the developed M5P model for forecasting the BC of CF is seating on the limited thick sand-layer with eccentric-inclined load. From the model M5P, the predicted results and the actual results were used to solve Eqs. (10-14). The calculated results are shown in the Table 2 for training as well as for testing.

$$BC = -510.84 \times \frac{H}{B} - 585.19 \times \frac{\alpha}{\phi} - 1274.64 \times \frac{e}{B} + 861.01 \quad (10)$$

$$BC = -545.73 \times \frac{H}{B} - 464.09 \times \frac{\alpha}{\phi} - 996.34 \times \frac{e}{B} + 732.08 \quad (11)$$

$$BC = -18.45 \times \frac{H}{B} - 161.30 \times \frac{\alpha}{\phi} - 404.40 \times \frac{e}{B} + 232.55 \quad (12)$$

$$BC = -17.26 \times \frac{H}{B} - 114.08 \times \frac{\alpha}{\phi} - 348.25 \times \frac{e}{B} + 207.89 \quad (13)$$

$$BC = -15.98 \times \frac{H}{B} - 170.40 \times \frac{\alpha}{\phi} - 321.93 \times \frac{e}{B} + 217.25 \quad (14)$$

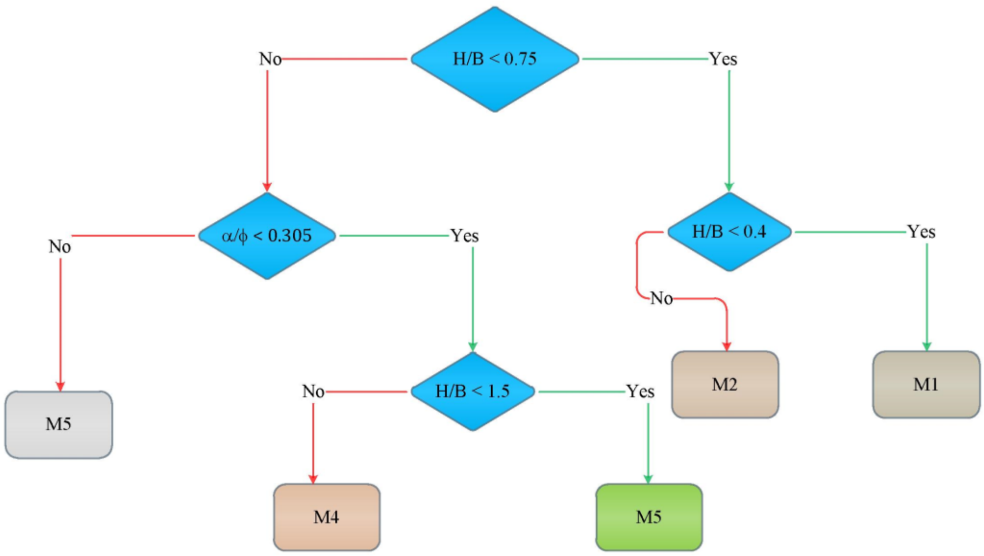


Fig. 4. Proposed M5P model tree to forecast the bearing capacity (q_u)

Table 2. Statistical parameters values for testing and training by M5P model

Statistical parameters	Training	Testing
R	0.99	0.99
R^2	0.96	0.96
MSE (kPa)	2273.72	1795.07
RMSE (kPa)	47.68	42.37
MAE (kPa)	32.13	28.57
MAPE (%)	19.83	21.46

Analyzing Table 2 reveals that, (keeping the above section in view-statistical parameters) the calculated MAPE value is lower than 20%, it reflects good accuracy of the present proposed prediction model (Behnood et al., 2017). Further, MSE, RMSE and MAE calculated results are small enough to represent the proposed model as good to predict the BC of the circular footing resting on the limited thick sand-layer with eccentric-inclined load.

Finally, r and R^2 calculated results (≈1) also represent the proposed model as good enough to predict the desired output (Behnood et al., 2017). The predicted versus (vs) actual values were compared in terms of the plot as demonstrated in Figures 5 and 6 for training and testing, respectively.

Close examination of Figures 5 and 6 unveils that, a comparison of predicted vs targeted values is within the 20% of the deviation line as reference to line of equality. The proposed soft computing model (M5P model tree) offers significant advantages. It excels at generating mathematical equations and enhances understanding of their formulation. Additionally, these models are easy to refine and apply. Besides, it is easier to improve and use these models. The model's limitation lies in its lack of high accuracy for higher values of the bearing capacity. Likewise, the modelling relies solely on experimental values for the data, and the inclusion of field data would significantly enhance its significance.

8. Preparation of ANN Model

The ANN technique is a dynamic method of information processing that develops connections between input variables (X_i) and output variables (Y_j) through interconnected neurons (represented by weight factor, w_j).

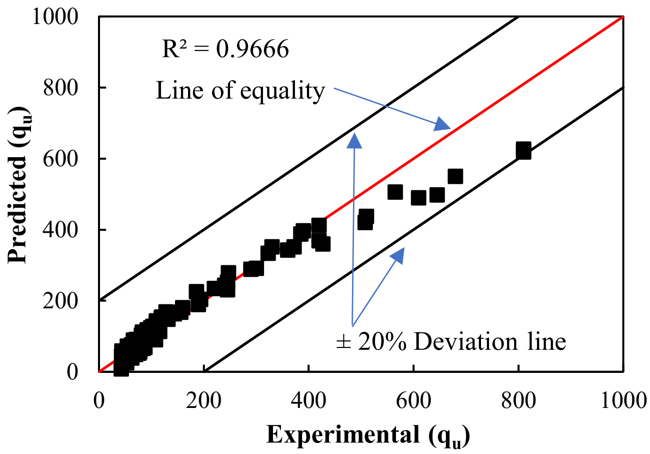


Fig. 5. M5P predicted vs experimental plot for training data

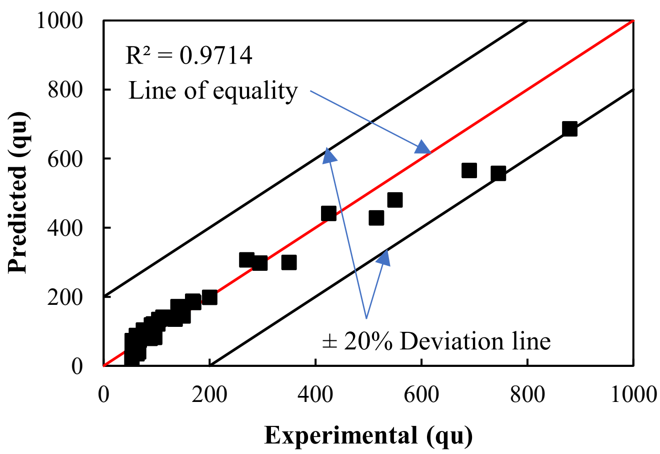


Fig. 6. M5P predicted vs experimental plot for testing data

It is crucial to emphasize that the input variables (X_i) and the output variables (Y_j) are typically normalized to x_i and y_j, respectively. The ANN models establish a relationship between the normalized input parameters x_i and the normalized output variables y_j as follows:

$$y_j = f\left(\sum_{i=1}^{n} w_{ji}x_i\right) + b_j \tag{15}$$

where *f*: is the transfer function employed which follows a form of sigmoidal function, w_{ji}: is the undetermined weight factors, b_j: is a bias.

A neural network model precisely adapts the weight factors (w_{ji}) and bias (b_j) in Eq. (15) by minimizing error function. Within the field of geotechnical engineering, the utilization of the ANN approach is extensive in the creation of prediction models. This involves harnessing comprehensive datasets derived from both experimental studies and numerical analyses. Overall, the creation of an ANN model encompasses a pivotal stage is the construction of the ANN architecture.

8.1. Selection of ANN Architecture

In this study, a three-layered ANN model was constructed, as depicted in Figure 7. The ANN model comprises of three layers: the input layer, the hidden layer, and the output layer. Determining the suitable quantity of hidden layers and neurons for each of these hidden layers constitutes an intricate undertaking within the design of ANNs. As outlined by Boger and Guterman (1997), a proposition has been made that the quantity of neurons in the hidden layer could be set at 70% of the data of the input layer. Besides, they indicated that if the number of hidden layer neurons is insufficient, supplementary neurons can be subsequently incorporated into the output layer. Conversely, it is advised by Linoff and Berry (1997) that the ideal count of neurons within the hidden layer should be kept below twice the quantity of neurons in the input layer.

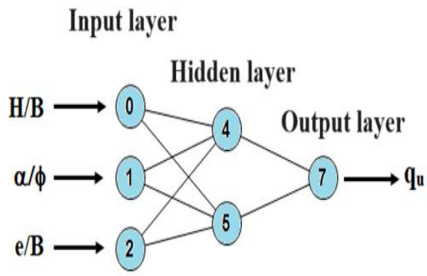


Fig. 7. Architecture of ANN model

Meanwhile, Blum (1992) put forth the notion of an intermediate size for the hidden layer neurons, lying between the size of the input and output layers. Given these observations, it was resulted at the recommended method for ascertaining the quantity of neurons in the hidden layer (70% of input layer), which is based on simple guidelines presented by Boger and Guterman (1997). This approach is also endorsed by Ito (1994) and Kůrková (1992) who have adopted a similar strategy.

For this study, single hidden layers were utilized to reduce the complexity of the ANN model, which was a crucial factor for this research to develop the model equations. A hidden layer in a neural network serves a crucial role in enabling the network to learn and represent complex patterns and relationships within the input data. The success of the neural network model depends significantly on knowing when to stop the training process. Overtraining the neural network can lead to noisy results, while insufficient training can result in poor predictions and a lack of generalization for new data.

Hence, a trial-and-error technique was employed to adjust the number of iterations for both training and testing datasets. In the present study the mean square error was calculated between the real and forecasted values across various iterations, ultimately pinpointing the iteration that yielded the lowest mean square error. This served as the optimal reference for identifying the neural network structure, as illustrated in Figure 8. The training process was halted upon reaching a low value for the average error function to avoid potential overfitting effects caused by further iterations.

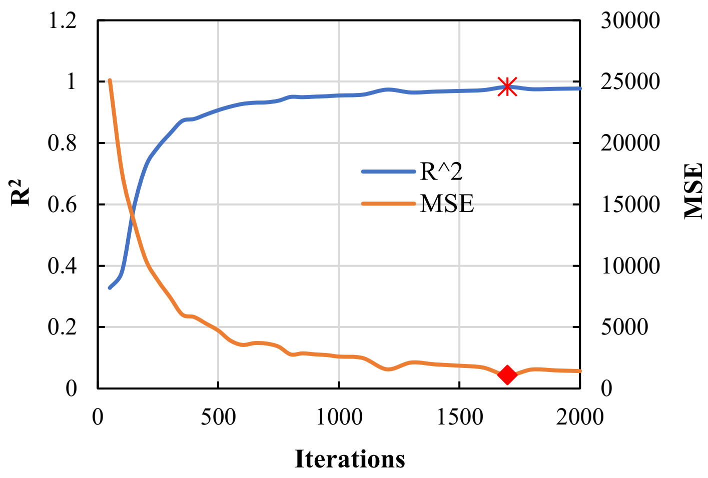


Fig. 8. Optimal hidden layer neurons

After careful consideration, the number of iterations was set to 1700 for this experiment. As a result, the neural network model selected for this experiment followed a configuration of 3-2-1, which denotes the count of neurons in the input, hidden, and output layers, respectively, for the purpose of constitutive modelling.

8.2. Prediction of Baring Capacity Using ANN

After successful construction of architecture for ANN model, it was fed with desired input parameters in order to calculate the output. In this process weights and bias was generated as presented in Table 3. The predicted output is compared with the original data for training (Figure 9) and testing (Figure 10).

Further, to understand the prediction accuracy of the proposed ANN model, performance measures were used (as discussed in section "statistical parameters"). The calculated values of performance measures are tabulated in Table 4.

Table 3. ANN model connection weights and biases

Hidden neurons	Weights (w_{jk})				Biases	
	H/B	α/ϕ	*e/B*	q_u	b_{hk}	b_0
1	12.14	1.10	1.03	-5.04	0.28	9.47
2	15.98	0.54	0.23	-7.74	0.45	-

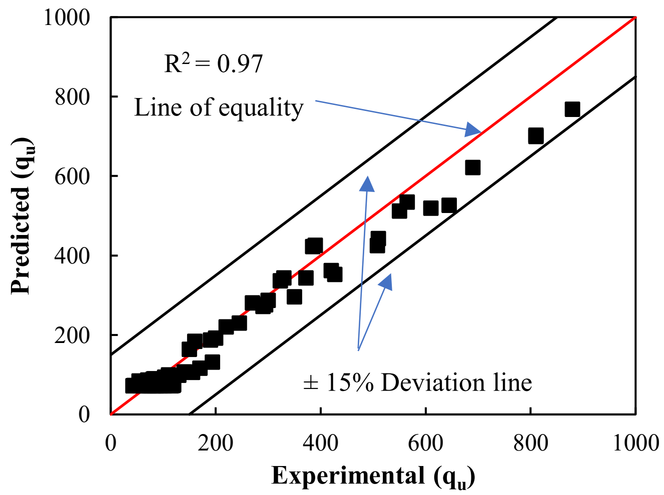


Fig. 9. Comparing actual q_u with ANN predicted q_u on training data

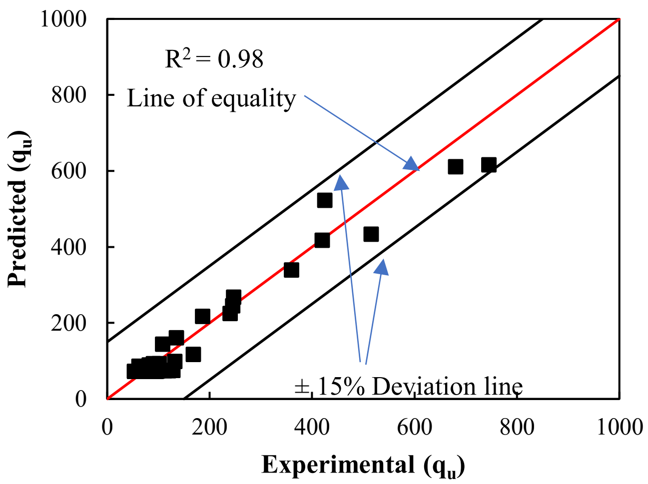


Fig. 10. Comparing actual q_u with ANN predicted q_u on testing data

Table 4. The statistical metrics for both the training and testing datasets by ANN model

Statistical parameters	**Training**	**Testing**
R	0.99	0.98
R^2	0.98	0.97
MSE	1567.96	1394.23
RMSE	39.60	37.34
MAE	28.98	24.88
MAPE (%)	18.20	16.29

9. The Equation of the ANN Model for 'Q_u' Based On the Trained Neural Network

The fundamental mathematical equation of the ANN connecting the input variables and the output can be expressed as follows:

$$q_u = f\left\{b_0 + \sum_{k=1}^{h}\left(w_k \times f\left[b_{hk} + \sum_{j=1}^{m}\left(w_{jk} \times X_j\right)\right]\right)\right\} \quad (16)$$

As a result, the model equation for the output may be formed using the ANN model's training weights. In this work, a model equation for circular footing bearing capacity was created utilising the values of the weights and biases presented in Table 2 as per the equations below.

$$A = 0.28 + 12.13\frac{H}{B} + 1.1\frac{\alpha}{\phi} + 1.03\frac{e}{B} \quad (17)$$

$$B = 0.45 + 15.98\frac{H}{B} + 0.54\frac{\alpha}{\phi} + 0.23\frac{e}{B} \quad (18)$$

$$E = 9.46 - \frac{5.04}{(1+e^{-A})} - \frac{7.74}{(1+e^{-B})} \quad (19)$$

$$q_u = \frac{1}{(1+e^{-E})} \quad (20)$$

The bearing capacity value produced from Eq. (9) is in the range of [-1, 1] and must be denormalized as Eq. (21):

$$q_{u_{actual}} = 0.5(q_u + 1)(q_u \max - q_u \min) + q_u \min \quad (21)$$

where q_u *max* and q_u *min*: are the maximum and minimum values of bearing capacity, respectively, as used in the data set.

10. Sensitivity Analysis for M5P Model

Sensitivity analysis was used to determine which input parameter had the most influence for predicting the q_u of circular footing resting on the limited thick sand-layer with eccentric-inclined load by M5P model. With the use of statistical measures (r, R^2, MSE, RMSE, MAE and MAPE), one input parameter was removed for each case and its influence on the bearing capacity of the soil was measured as shown in Table 5.

Table 5. M5P model's parametric analysis on training data

Input combinations	Input parameters removed	r	R^2	MSE	RMSE	MAE	MAPE (%)
H/B, α/ϕ, e/B	--	0.99	0.97	2117.38	46.02	29.30	18.93
α/ϕ, e/B	H/B	0.76	0.23	30083.37	173.45	134.61	91.28
H/B, e/B	α/ϕ	0.97	0.88	6508.77	80.68	51.93	31.08
H/B, α/ϕ	e/B	0.99	0.92	4091.38	63.96	39.36	25.39

Study of the Table 5 demonstrates that *H/B* is the most effective input parameter on forecasting the bearing capacity. This is because variations in the thickness of the rigid soil affect pressure distribution, which can lead to either an increase or decrease in the footing's bearing capacity. Second one is the ratio of the load of eccentricity to the angle of internal friction (*α/ϕ*) followed by the ratio of load of inclination to width of the footing (*e/B*). Finally, from the Table 5, it can be concluded that removing the other parameter has no significant impact on forecasting the BC of the layered soil with limited thickness, in contrast to *H/B*. It should be mentioned that the ratio H/B had a significant part in predicting the q_u of the layered soil with limited thickness.

10.1. Sensitivity Analysis for ANN Model

In this section, an examination is conducted to analyze the influence of individual variables on the bearing capacity (output) via parametric analysis. To conduct the sensitivity analysis, two methods were employed based on the weight configuration, as outlined by David (1991) and Olden and Jackson (2002).

In the initial method (David, 1991; Kůrková, 1992), the connection weights of each neuron in the hidden layer were divided into several (based on the type of modeling) components, a concept later verified by Onyelowe et al. (2021, 2022). These components were subsequently connected to each corresponding input neuron. On the other hand, the second method, as proposed by Olden and Jackson (2002), entailed the computation of the cumulative product of the ultimate weights of connections (spanning from input neurons to hidden neurons and from hidden neurons to output) across all input neurons (Onyelowe et al., 2022). To determine the contribution of individual variables corresponding to a given input, the procedures presented by David (1991) and Olden and Jackson (2002) was followed. The outcomes derived from these procedures are shown in Figure 11.

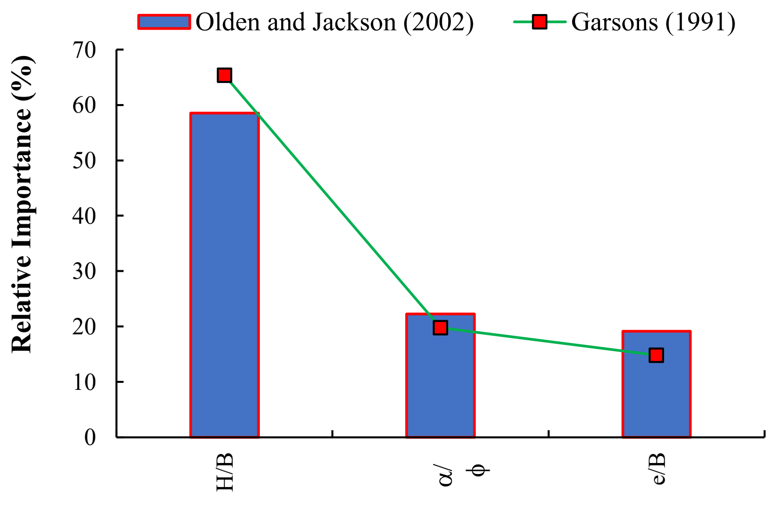


Fig. 11. Parametric analysis for ANN model

Figure 11 clearly indicates that the dominant influential factor (*H/B*) is followed by (*a/f*), and (*e/B*) based on the both methods of David (1991) and; Olden and Jackson (2002). Finally, it can be concluded that, both methods yielded identical results from the parametric analysis.

11. Comparison of M5P Tree and ANN Models

After successfully completing the M5P model tree and ANN modelling, a comparison is made among these models to assess their competency, along with an examination of the parametric analysis results. The outcome of the proposed M5P model tree and ANN model is visualized on scatter plots, depicting the measured versus predicted values for both training (Figures 5 and 6) and testing (Figures 9 and 10) datasets. Representing the data points of predicted versus measured outputs along with a line of equality and a ± 15% error line serves as an appropriate method for illustrating the model's predictive capacity.

The results of the statistical parameter from the M5P model tree (Table 2) and ANN model (Table 4) reveals that, both modes are good in prediction the desired q_u as r, R^2 are close to 1 and remaining MSE, RMSE, MAE and MAPE are small values (as discussed in Section 6).

But, the M5P model tree is inferior than the ANN model as evidence from the comparison of Tables 2 and 4. A comparison was made among the models using line diagram as data number vs q_u as depicted in Figure 12. This figure shows that both M5P and ANN models representing lines are overlapped with the experimental data. Hence, both models can be useful to predict the desired q_u. In conclusion, the parametric analysis conducted to assess the influence of individual variables on the output q_u highlights that (*H/B*) emerges as the primary contributing independent factor, followed by (α/ϕ) and (*e/B*). The influence of independent parameter on predicting the q_u was found same in both methods (M5P and ANN) as per the Table 5 and Figure 11.

12. Conclusions

This paper deals with the problem of prediction of bearing capacity of a circular footing subjected to an eccentric-inclined load placed on sand with a limited thick.

The objectives of this paper were:

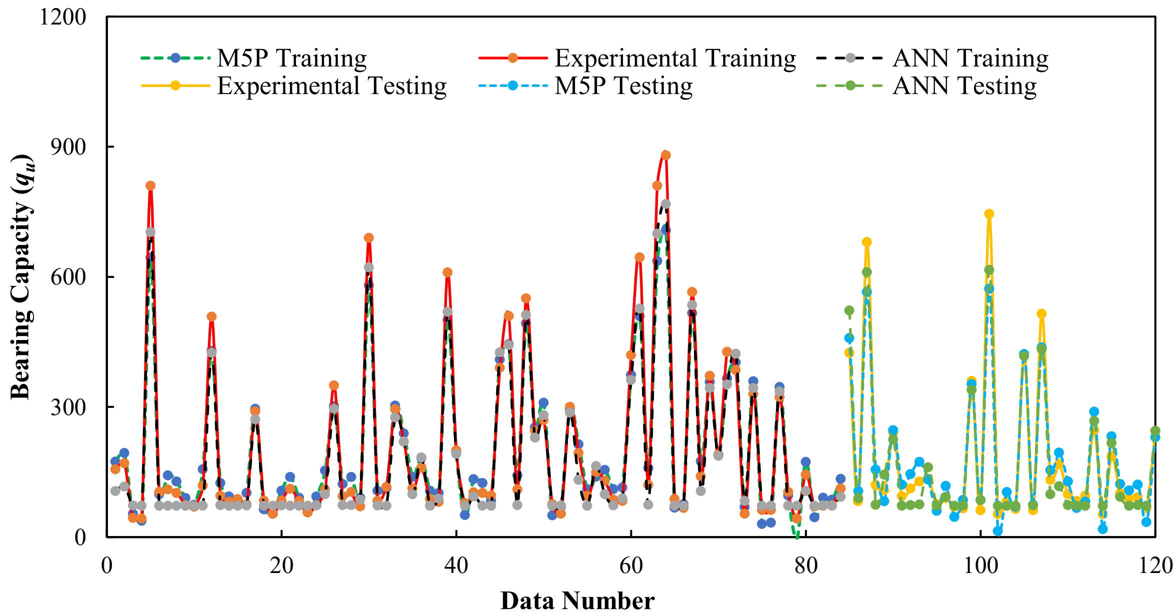


Fig. 12. Comparison of experimental vs M5P tree and ANN models training and testing data

- To evaluate how well the M5P tree model and ANN model can predict the load-bearing capacity of a circular footing on sand under specific conditions, like an eccentric-inclined load with limited thickness of sand.
- To conduct a comparison between the developed M5P model tree and ANNs.
- To develop the model equations based on both M5P tree and ANN models.
- To perform sensitivity analysis to see the influence of each input parameter on the output prediction.

The results show that the ANNs model is effective in predicting the bearing capacity of a circular footing under an inclined load on sand with limited thickness. It performs much better than the M5P model. With the ANN model, data can be inputed after training to predict outcomes. On the other hand, the M5P model requires the entire process each time a prediction is to be made. To evaluate the performance of both models, measures like MSE, RMSE, MAE, MAPE, R^2 and r used. In both models, MSE, RMSE, MAE and MAPE had low values, while r and R^2 were close to 1 for both models. This suggests that both models can predict the bearing capacity of a circular footing under an inclined load on sandy ground with limited thickness, but the ANN model is considered the best.

A sensitivity analysis was performed to understand the importance of each input parameter on bearing capacity (q_u). The results indicated that *H/B* is the most significant input parameter, followed closely by α/ϕ and *e/B*. The benefit of utilizing M5P and ANNs techniques is their ease of updating when new data becomes available.

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