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RESEARCH PAPER

Reduction Factors for Laterally Loaded Pile Groups Accounting for Pile Cross Sections and Soil Properties

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ABSTRACT: P-y analysis approach is among the most widely used methods implemented for simulating the pile behavior under lateral loading. To obtain the p-y curve of a pile group, that of a single pile should be modified by a group reduction factor which is conventionally determined by the pile spacing to diameter ratio and the pile row number in the group. The pile section types and soil characteristics i.e. internal friction angle are among other important factors affecting the group reduction factor. However, the influence of these parameters has not been investigated in previous researches, thoroughly. In this study, continuum models of eight pile groups with different conditions regarding pile section types and soil properties were built and analyzed. It was found that the group reduction factor of the pile group with square tube section is less than that of the group of piles with pipe sections, due to group effects. On the other hand, for the pile groups of both section types, the greater internal friction angle, increases the shadowing and edge effects which results in lower group reduction factor. Moreover, the group reduction factors calculated based on numerical modeling results were compared with the recommendations of AASHTO and FEMA guidelines. It was also shown that the continuum model results conform with the results of experimental studies.

Keywords: Group Reduction Factor, Internal Friction Angle, Pile Group, P-y Curve, Section Properties, Soil-Pile Interaction.

1. Introduction

In addition to axial force, piles should be designed to withstand lateral load. The lateral forces may be applied to the piles used in the structures such as walls retaining the soil pressure, port structures influenced by the mooring forces, offshore structures that are under the influence of waves and marine currents, high-rise structures which are affected by the wind and different types of structures founded on piles in earthquake-prone zones. The piles experience lateral deformation under lateral loading and interact with the surrounding soil. Therefore, the analysis of piles under lateral load is complicated and the lateral load effects on the pile behavior should be analyzed by appropriate methods (Asaadi et al., 2017; Hajitaheriha et al., 2021).

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In general, piles are rarely used individually, so the study of pile group behavior is of utmost importance in geotechnical engineering. On the other hand, the lateral bearing capacity of a single pile differs from that of the same pile in a pile group. Each pile under lateral force, creates a shear zone in the front soil, and by overlapping these areas created by the nearby piles, the shear area is enlarged. If the spacing between the adjacent piles in a row or in different rows decreases, there will be more interference. The pile-soil-pile interaction is mainly due to the effect of adjacent piles in a row of piles on each other (edge effect) as well as the mutual effects of piles in two adjacent rows (shadowing effect). These two group effects are shown in Figure 1.

To consider the non-linear behavior of the soil-pile interaction, p-y curves are typically utilized, where, p is the soil resistance and, y is the horizontal displacement. It should be noted that in this method, the pile group effects are taken into account by applying a correction coefficient called P-multiplier (P_m) for each row of the pile group (Figure 2). Because of the decrease in the pile bearing capacity due to group effects, this corrective factor is less than or equal to one.

The p-y curves are empirical curves that model the non-linear behavior of the soil. These curves are derived from the piles behavior under horizontal load in real conditions. Using these curves is very common for simulating the soil behavior in professional practice. Depending on the type of soil at each depth, the corresponding p-y curve is determined. Then, these curves describe the behavior of non-linear springs to be used along the piles for analysis. There are programs specifically developed for analyzing piles under horizontal loads using p-y curves (Das, 2015).

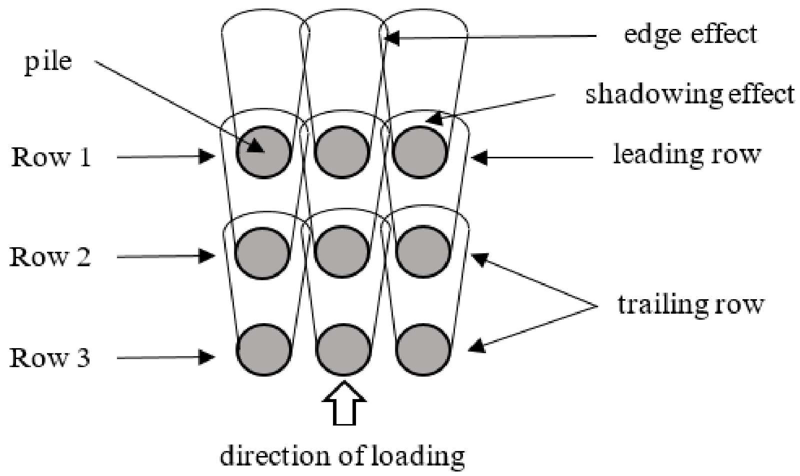

Fig. 1. Rows of pile group and overlapping zones (Fayyazi et al., 2012b)

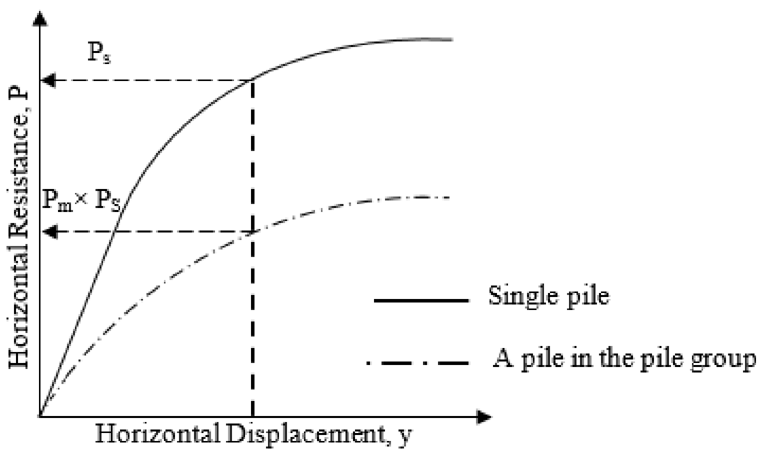

Fig. 2. P-multiplier definition (Fayyazi et al., 2014)

The maximum value of corrective coefficient i.e. P_m in a pile group, is for the first row which is called the leading row (Figure 1), and the lower values of P_m are corresponding to the next rows called trailing rows. Hence, as pile rows get closer to the lateral load, the P-multiplier becomes smaller (Brown et al., 1988).

P-multipliers can be obtained by performing a full-scale test. In the same way, many studies have been conducted in clay and sand (Ruesta and Townsend, 1997; Rollins et al., 1998; Brown et al., 2001; Rollins and Sparks, 2002; Rollins et al., 2003; Snyder, 2004; Rollins et al., 2005; Walsh, 2005; Rollins et al., 2006). But, physical modelling with full scale is difficult and costly. Hence, centrifuge experiments have also been used to calculate P_m as conducted by McVay et al. (1995, 1998) and Taghavi and Muraleetharan (2016). Another method for calculation of P_m, is to model the pile groups on the basis of continuum mechanics (Law and Lam, 2001; Fayyazi et al., 2012a,b, 2014; Tehrani et al., 2016; Asaadi et al., 2017). In this way, it is possible to solve a variety of problems involving different types of piles and soils. The artificial intelligence-based methods have been also utilized to solve geotechnical engineering problems (Saeedi Azizkandi and Fakher, 2014; Kohestani et al., 2017; Avval and Derakhshani, 2019; Khorrami and Derakhshani, 2019; Jafariavval and Derakhshani, 2020; Khorrami et al., 2020) amongst which the P-multipliers were estimated by the Genetic Programming and Model Tree approaches (Khoshroo and Derakhshani, 2020; Talebi and Derakhshani, 2019).

As per cyclic nature of seismic loads, the leading and trailing rows are interchanged by changing direction of lateral load. In order to account for this load reversal, one can consider the average P_m value of different rows. This average corrective coefficient is referred to as the “group reduction factor”. Due to the above-mentioned problems associated with experimental studies, available laboratory data are limited, so that it is difficult to study about group reduction factors. To solve this problem, data from numerical modelling under various conditions can be used to compute the P-multipliers and consequently the group reduction factors.

In this study, the group reduction factor is calculated for groups of piles with different sections in various soils using a three-dimensional continuum modeling approach. The ability of the Finite Element Method (FEM) for numerical modeling of the pile group behavior was first investigated by modelling a full-scale experiment and comparing the predictions and observations. Then, the model is used to evaluate various groups of piles with either pipe or square tube sections in the soils of different internal friction angle values. Next, the effects of variations of these key parameters on the group reduction factor are assessed. Finally, the group reduction factors computed based on numerical results of this study, are compared with those recommended in codes of practice and experimental data.

2. Validation of the Continuum Model

To verify the performance of the continuum model, an existing well-documented laboratory research was considered. Christensen (2006) tested a full-scale 3×3 pile group under lateral load in a multilayered soil. In this experiment, the piles were 324 mm in diameter with a thickness of 9.5 mm and a length of 12.8 m. The position of piles in pile group and the loading direction are depicted in Figure 3. Properties of the soil profile in the test site are given in Table 1.

Christensen (2006) calculated the row reduction factors of the pile group. By simulating the behavior of the tested pile group using p-y curves, he computed the P_m values of 1, 0.7 and 0.65, for the leading row as well as the second and third (trailing) rows. The group reduction factor which is equal to the average reduction factors of the rows, was 0.783 for this experiment.

Christensen test was used to validate the continuum model built using FEM in the Abaqus3D (Hibbitt et al., 2016; Rezaei et al., 2016). In Figure 4, the model geometry is presented in which the pile positions in the soil as well as the FE mesh are visible. Solid elements were used to model piles and soil in the continuum model. The Mohr-Coulomb constitutive model was used in the continuum model. Additionally, according to the sensitivity analysis, for the soil model dimension being sufficiently large and boundary does not affect quality of modeling results, the 5D and 10D intervals are respectively considered from outer piles in the perpendicular direction to the lateral load and in the lateral load direction (Fayyazi et al., 2014).

To verify the accuracy of the numerical modeling, the results of the Christensen test were compared with the corresponding results from the continuum model. To this end, two important curves of total group load-deflection curve and bending moment distribution along the pile are exhibited in Figures 5 and 6, respectively. As can be seen, the values obtained from numerical modeling and those measured in the test are in good agreement.

Table 1. Characteristics of different soil layers in the study of Christensen (2006)

Depth (m)	Unit weight (kN/m^3)	Shear modulus (kN/m^2)	Poisson's ratio	Friction angle (degrees)	Dilation angle (degrees)	Undrained shear strength (kN/m^2)
0-2.1	16.7	29600	0.3	40	0	0
2.1-2.4	16.8	15800	0.3	40	0	0
2.4-2.7	19.1	7000	0.3	0	0	41
2.7-3.7	19.1	7000	0.3	0	0	50
3.7-4.6	19.1	8100	0.3	0	0	40
4.6-6.3	18.1	17800	0.3	38	0	0
6.3-8	19.1	6500	0.3	0	0	57
8-12.8	16.7	22600	0.3	33	0	0

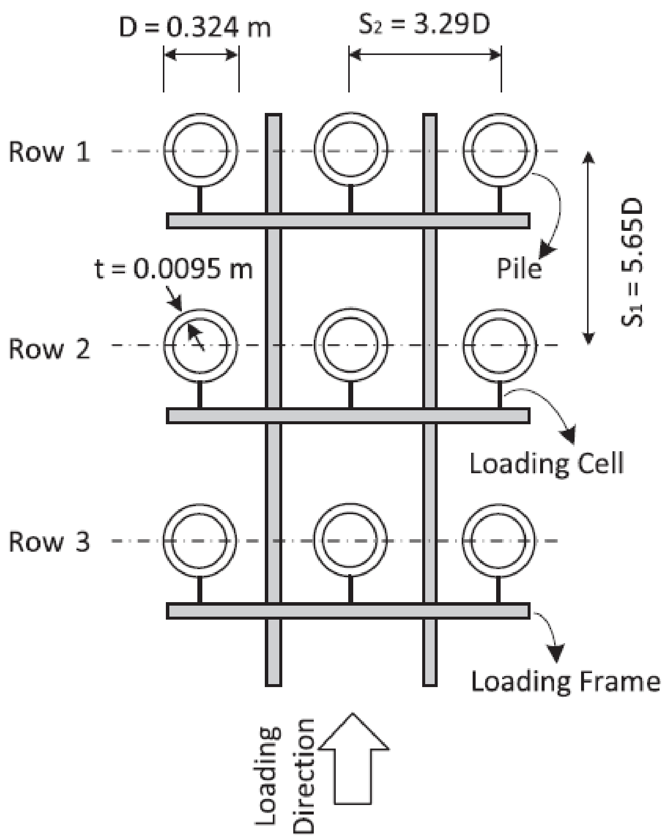

Fig. 3. Plan of the pile group test conducted by Christensen (2006)

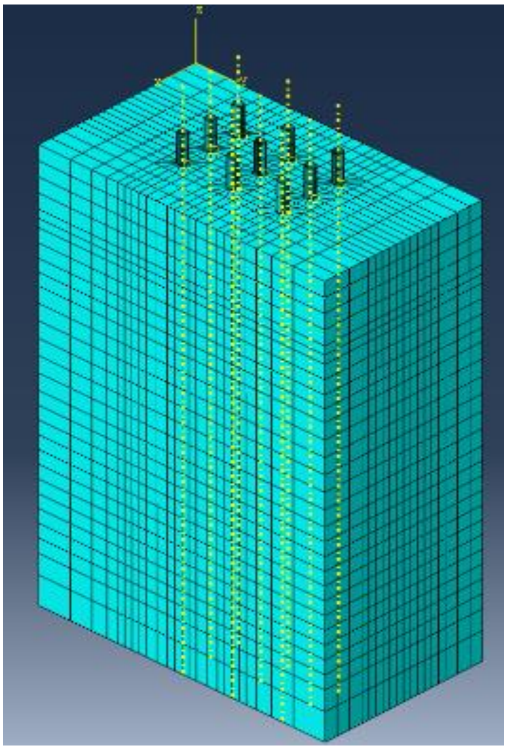

Fig. 4. Model geometry

In summary, comparison of the pile group response from continuum model with Christensen test, indicate that the 3D numerical modeling can efficiently simulate the pile group behavior under lateral loading.

3. Numerical Modeling

To study pile section effects on group reduction factor, two sections were considered. The first one is a circular hollow section with the diameter of 30 cm, which is briefly referred to as “Pipe”. The second section called “Square Tube”, is a hollow square with its EI (flexural rigidity) equal to that of the pipe section and a height of 25.14 cm. These sections are demonstrated in Figure 7. In both cases, the thickness of section is 1 cm and the pile length is 10 meters. The pile groups were modeled in dry and cohesionless soils of different internal friction angles using the Mohr-Coulomb constitutive law. Table 2 lists different values of parameters used for building different models to study the effects of pile sections on the group reduction factor in granular soils with various properties.

To evaluate the influence of granular soil internal friction angle on the group reduction factor, when utilizing different sections, the friction angle values were chosen to be 25, 30, 35 and 40 degrees.

As shown in Figure 8, the dimensions of the model were selected in accordance with the sensitivity analysis described in the verification section. As can be seen, a 3×3 pile group was modeled with the ratio of pile center to center spacing to the pile diameter of 3 (i.e. S/D = 3). Piles were made of St37 steel and the soil behavior was analyzed with the Mohr-Coulomb constitutive model.

3.1. Preliminary Results of Pile Group Analysis

In this section, a brief review is given about the group pile lateral load and bending moment via presentation of the analysis results of a sample group pile. Figure 9 presents the p-y curves of the single pile and the corresponding pile the middle of the group. The curves are the plotted for the pipe and the square tube pile sections while the maximum lateral deflection was considered to be 5 cm.

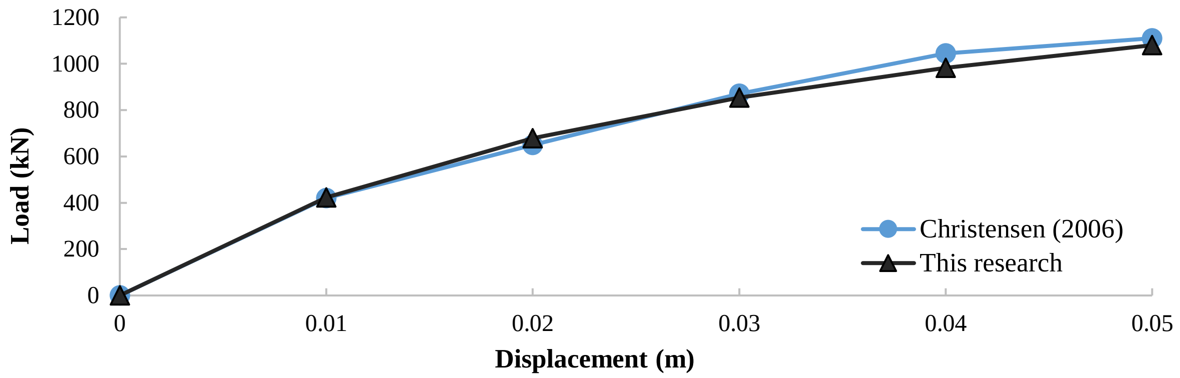

Fig. 5. Computed versus observed total group load-displacement curve

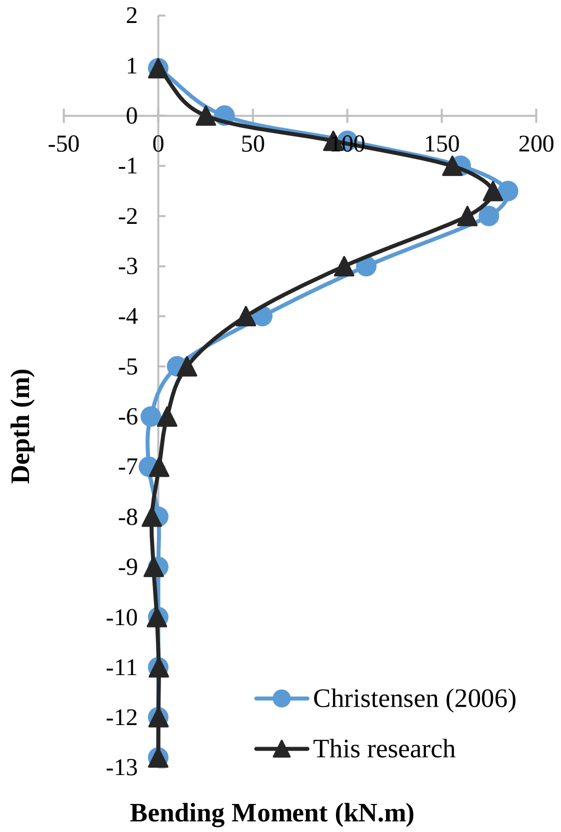

Fig. 6. Computed versus observed total group bending moment profile

(a) **(b)**

Fig. 7. Types of pile sections: a) Pipe; and b) Square tube

Table 2. Variations of model parameters to study the group reduction factor

Section of the piles in the group	Pile arrangement	S/D	φ (°)
Pipe	3 × 3	3	25
			30
			35
			40
Square tube	3 × 3	3	25
			30
			35
			40

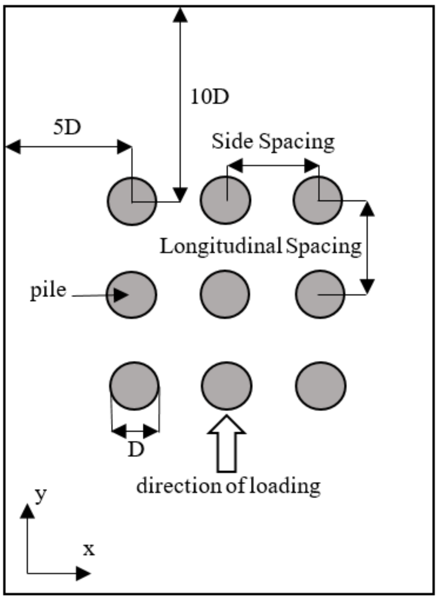

Fig. 8. Continuum model plan

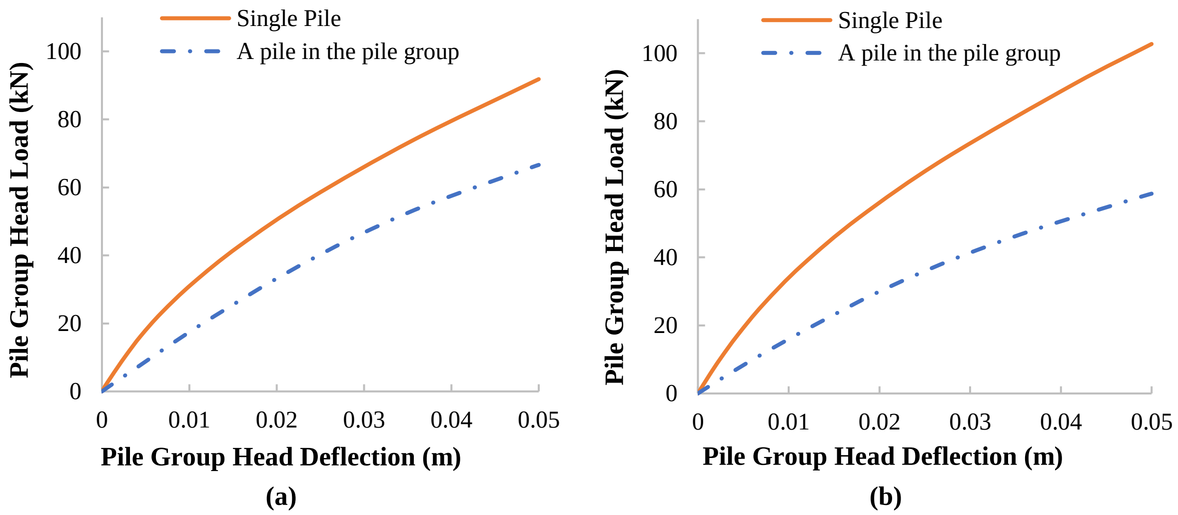

Fig. 9. Load vs. deflection of pile group head for: a) Pipe; and b) Square tube sections (φ = 25°)

As seen in this figure, when the internal friction angle of soil is constant, by changing the section type, the group behavior varies. When the pile section changes from pipe to square tube, the difference between the lateral force of the single pile and that of the middle pile in the group increases. In Figure 10, the distribution of lateral force is demonstrated along the pile. As shown, the lateral load carried by the single pile compared to the pile in the middle of the group is greater in case of the square tube section in contrast to pipe section.

To study the group effects in various conditions of pile section and soil types, the pile group lateral resistance can be plotted against the pile group head deflection with the help of numerical modeling outputs. For instance, the curves of Figure 11 are presented for different pile sections in a soil with friction angle of 25°.

Pile group behavior in the soils of different friction angles is similar to that of the piles located in the soil with friction angle of 25° as depicted in Figure 11. Figure 12 illustrates the lateral force of the pile group head with pipe and square tube sections, at maximum deflection, for different internal friction angles of the soil. As can be seen, the lateral load of the pile group with pipe section is greater in comparison with the case of square tube section.

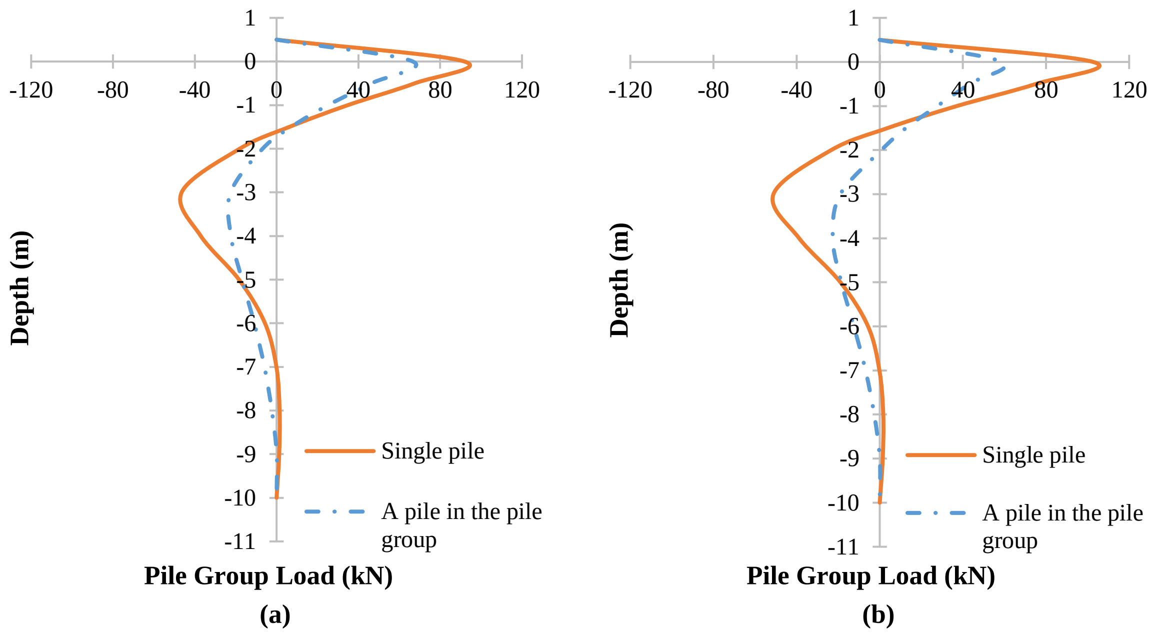

Fig. 10. Lateral load variations of the pile groups with depth for different sections (φ = 25°): a) Pipe; and b) Square tube

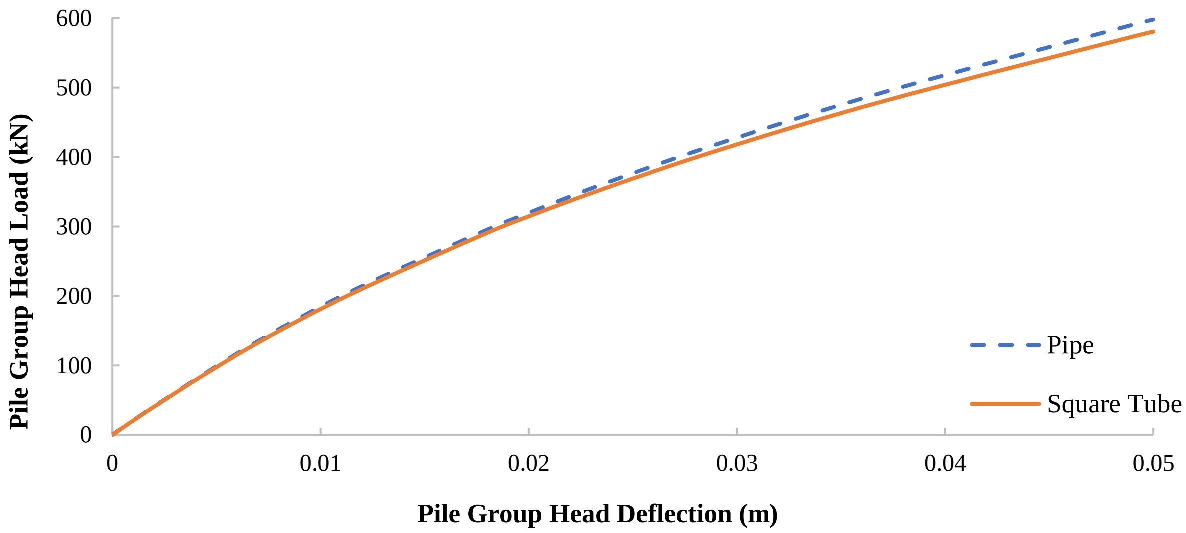

Fig. 11. Lateral load versus deflection of pile groups with different sections (φ = 25°)

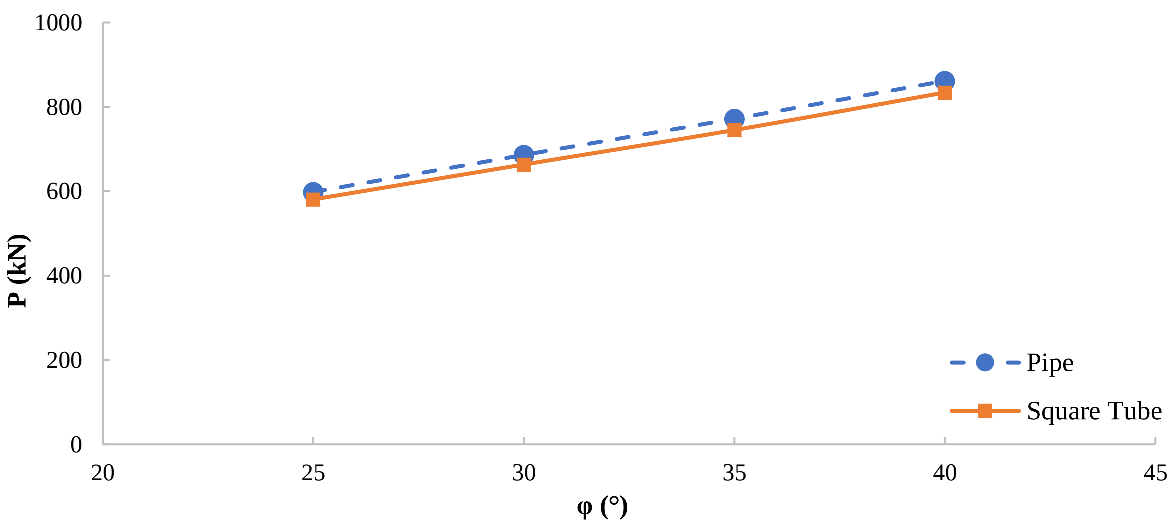

Fig. 12. Pile group head load vs. soil friction angle for different pile sections

3.2. Effects of Pile Section Type on the Group Reduction Factor

In this section, the results from different models built based on Table 2 is evaluated. To investigate the effects of pile sections on the group reduction factor in soils with different friction angles, the continuum modeling was carried out and group reduction factors were obtained and given in Figure 13.

When the internal friction angle increases, the group reduction factor decreases for all pile sections. In fact, by increasing the soil friction angle, the soil shear strength and therefore its bearing capacity increases. According to the results of all pile sections, when the soil has more resistance, with a friction angle of 40°, the lowest group reduction factor is attained, and when the soil is weaker, with a friction angle of 25°, a greater group reduction factor is obtained.

By comparing the results of different pile sections, it is found that the Square Tube section has less group reduction factor than Pipe section. The use of square tube section piles could extend the area affected by the piles in the soil and as a result, the shadowing effect and edge effect increase, so the group reduction factor decreases.

4. Continuum Model Results vs. Design Guidelines and Experimental Studies

4.1. Comparison with Available Recommendations for Practice

Two important methods for estimating the P-multipliers of the group piles are proposed by AASHTO (2012) and FEMA (2012) regulations. Both of these references provide simple suggestions based on the results of a number of laboratory studies to predict the P_m for the rows of pile groups. To obtain the group reduction factor, one can calculate the average of the P_m values of different pile rows.

The values of proposed P-multipliers by the AASHTO are presented in Table 3. The weak point of this recommendation is that the effect of pile section type and the soil type in which the pile group constructed, are not taken into account.

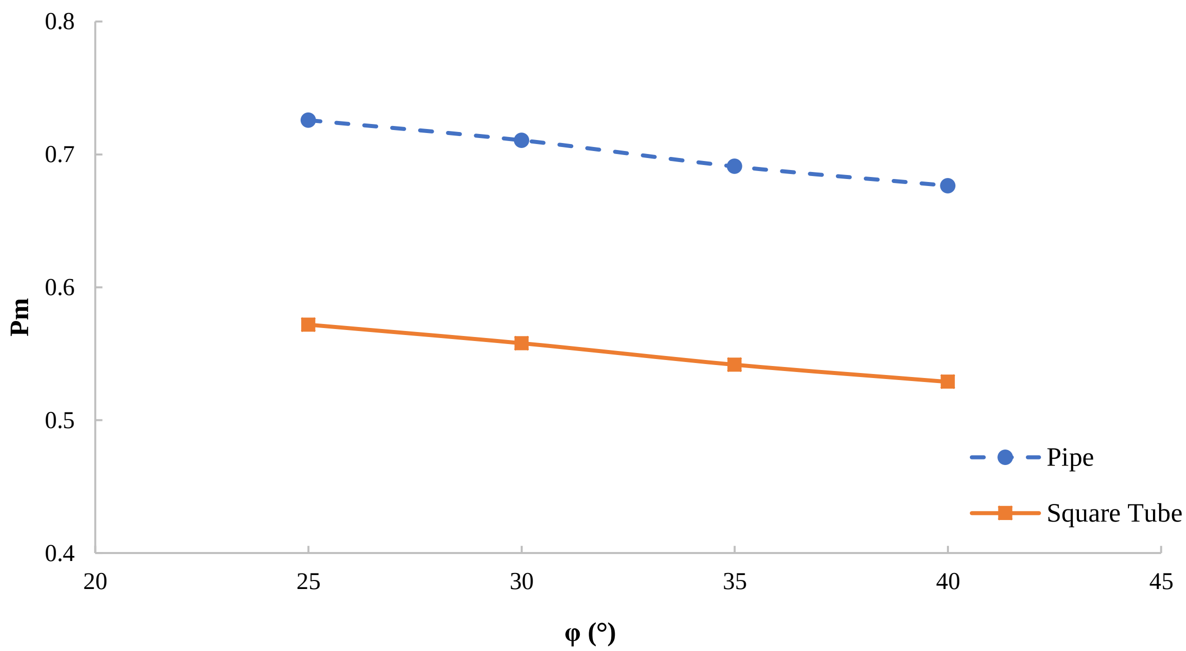

Fig. 13. Group reduction factor for different pile sections versus soil friction angle

Table 3. Proposed P-multipliers by AASHTO guidelines

Pile spacing in the direction of loading	Pm Row 1	Pm Row 2	Pm Row 3
3D	0.8	0.4	0.3
5D	1	0.85	0.7

The FEMA regulations provide the relationships presented in Table 4 to calculate the P-multipliers. Similar to AASHTO, this regulation only considers the effects of row number and ratio of pile spacing to pile diameter in the pile group for determining the P-multipliers and it does not pay attention to the other parameters involved.

The results of numerical modeling are compared with predictions of the existing regulations in Figure 14. In this figure, the group reduction factors for piles of different sections with S/D of 3, are plotted versus soil internal friction angle.

Considering the curve of pile group with pipe section, the reduction factors obtained from continuum models are higher than the estimations by the ASSHTO and FEMA guidelines, so, they suggest conservative values. However, the curve of pile group with square tube section shows more conservative values in contrast to FEMA recommendation, and less conservative estimates compared to the AASHTO values.

In fact, these regulations provide a constant value for several different sections and soil types. The estimates of the regulations were close to the values obtained from numerical analysis for pile group with square tube section. In case of pile group with pipe section, when the internal friction angle is greater, the reduction factors obtained from numerical modeling are closer to the regulations' recommendations. In general, it is found that the regulations are more conservative concerning weaker soils. However, in stronger soils, estimations of the guidelines are closer to the predicted values through numerical modeling.

4.2. Comparison with Pervious Experimental Data

Comparison of the continuum model results with the experimental data is demonstrated in Figure 15. The experimental data points provided in this figure are obtained from the centrifuge tests conducted by McVay et al. (1995) and the full-scale tests performed by Brown et al. (1988) and Morrison and Reese (1988). These experimental studies were selected in a way that their condition is similar to that of the continuum models as much as possible, however, some features of the physical and numerical models could be slightly different. As shown in Figure 15, the group reduction factors obtained from the experiments are close to the data points calculated based on the continuum model results.

Table 4. Proposed P-multipliers by FEMA guidelines

First (leading) row piles	$P_m = 0.26 \mathrm{Ln}(S/D) + 0.5 \leq 1$
Second row piles	$P_m = 0.52 \mathrm{Ln}(S/D) \leq 1$
Third or higher row piles	$P_m = 0.6 \mathrm{Ln}(S/D) - 0.25 \leq 1$

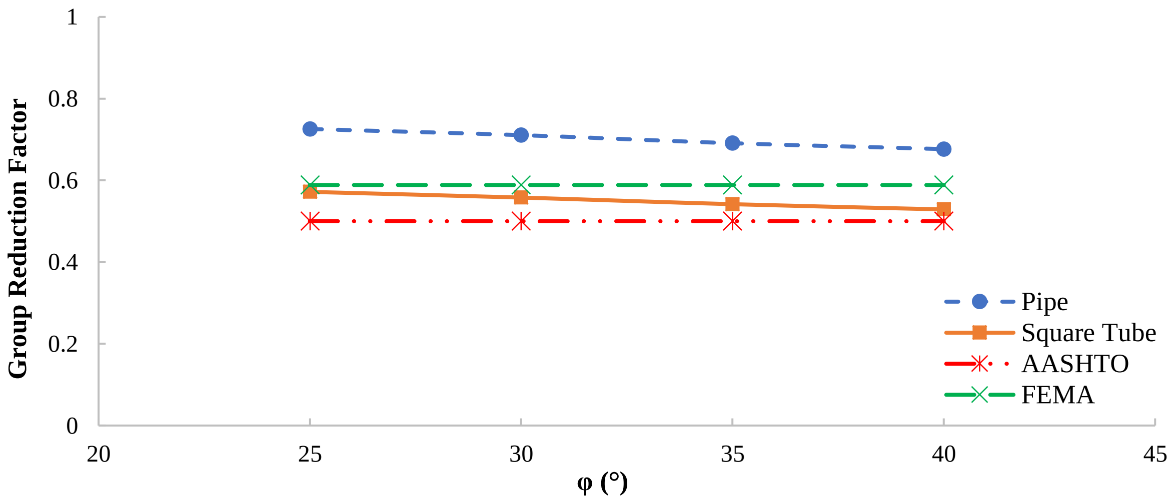

Fig. 14. Comparison of continuum model results with recommendations of guidelines

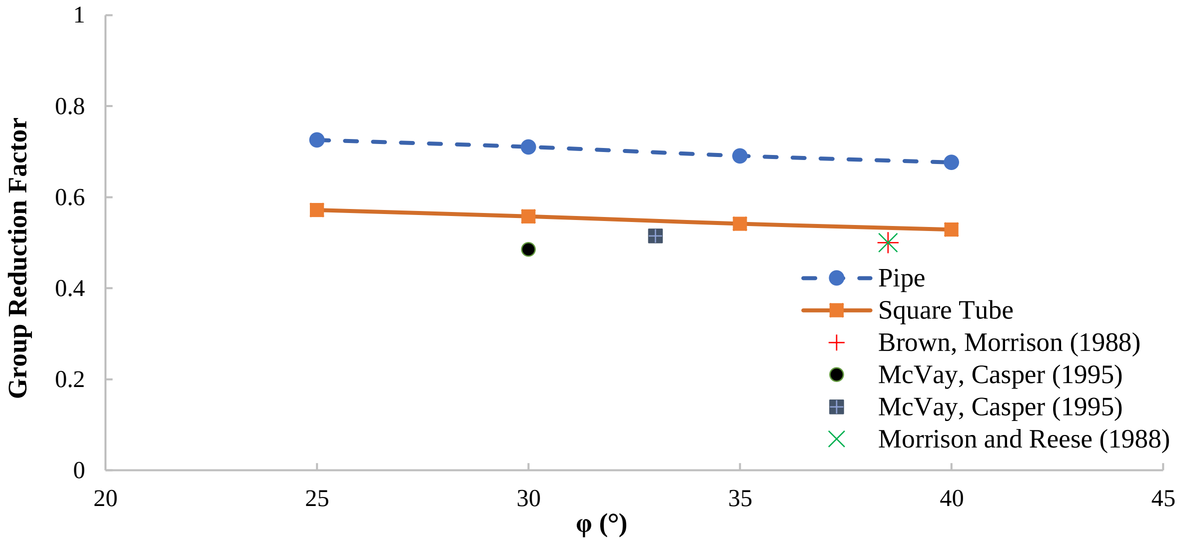

Fig. 15. Comparison of continuum model results with experimental data

5. Conclusions

The use of p-y curves is one of the conventional approaches for evaluating the pile behavior under lateral loads. Considering the group effects, when using this method to analyze the pile group under lateral load, the p-y curves should be modified. This modification is done by applying a constant coefficient smaller than one (i.e. P-multiplier) to the whole p-y curve. For cyclic and seismic loadings where the direction of loading changes repeatedly, the average of P-multipliers of the pile rows, called the group reduction factor, is used for pile group analysis.

Various factors affect the value of reduction factor, however, design guidelines recommend its value just on the basis of pile row number and spacing to diameter ratio of the piles. In this study, the influence of pile section and soil friction angle on the group reduction factor were investigated. To this end, continuum models of a 3 × 3 pile group with spacing to diameter ratio of 3 with different sections were built in soils of different friction angle. By using the p-y model, the group reduction factor was calculated for each continuum model. According to the results of this study, the following conclusions are drawn:

- Group reduction factor for square tube piles is less than the pipe piles in different soils. This means that the group effect is greater in case of square tube section. As a result, using the pipe piles in a group is more efficient than the square tube piles.
- By increasing the soil shear strength due to the increase of friction angle, soil among piles transfers the effects of piles on each other better. In other words, the overlapping areas affected by the piles are increased, so, the pile-soil-pile interactions increase. Subsequently, greater group effects lead to lower group reduction factor.
- The modeling results were compared with the results of AASHTO and FEMA guidelines. Since these regulations only consider the effects of row number and the ratio of spacing to diameter of piles, they suggest the same amount of reduction factors for all the pile sections and soil properties. By comparing the group reduction factors based on recommendations of these guidelines, it can be inferred that these regulations offer conservative values for pipe section. On the other hand, the group reduction factors of the pile groups with square tube section are close to the values recommended by the guidelines. In addition, it was shown that the group reduction factors obtained from the numerical modeling are close to those computed based on measurements through the physical modelling.

It is worth noting that this study is limited to the group piles located in the sandy soils and further research should be conducted to account for those constructed in the clay. Moreover, this study is restricted to specific steel pile profiles and pile group configuration, hence, other types of pile sections and group arrangements might be investigated in future research programs. Accordingly, the results of the current study may not be generalized to all different groups of piles without the mentioned considerations.

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