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RESEARCH PAPER

Damage Detection in Double Layer Grids with Modal Strain Energy Method and Dempster-Shafer Theory

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ABSTRACT: Change in modal strain energy is one of the indicators used to detect damage in structures. However, in structures with high degrees of freedom, such as double-layer grids, this method requires a relatively large number of mode shapes which in practice is difficult to determine. Therefore, it is necessary to reduce the number of required mode shapes. In this study, a damage detection technique based on modal strain energy and Dempster-Shafer evidence theory is presented for locating damage in double layer grids using only a few number of mode shapes. First, by calculating mode shapes of the grid in undamaged and damaged states, the modal strain energy based index for each mode shape is determined. Then, the results obtained from separate mode shapes are combined using Dempster-Shafer theory to achieve better results. In order to investigate the effect of noise on damage detection, 3% random noise is added to mode shapes. To demonstrate the performance of the proposed method, different single and multiple damage cases with different damage intensities are considered. Numerical results show that using 5 mode shapes, the presented technique can detect up to 3 damaged elements with different damage intensities in different parts of the grid with good accuracy (probability of 92.3%). Considering the fact that the classical modal strain energy method fails to distinguish even 1 damaged element in the double layer grid, the result shows significant improvement.

Keywords: Damage Detection, Dempster-Shafer Theory, Double Layer Grids, Modal Strain Energy.

1. Introduction

Double layer grids constitute a significant part of the space structure's family and are utilized for covering large spans without internal supports or with a few of them (Arekar et al., 2016). Elements in double layer grids are dominated by axial forces and these structures usually are treated like space trusses. Double layer grids, like every other type of structure, may encounter damage due to natural or artificial reasons

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during their service life. Early damage detection for these structures is important since it can prevent progressive or overall collapse of the structure.

Damage in a structure changes physical properties (stiffness, mass and damping) of structure. Since the physical properties of a structure are equivalent to its dynamic properties (natural frequencies, mode shapes and damping ratios), usually changes in dynamics properties are used to detect damage in structures. These dynamic properties can be determined in practice by experimental modal analysis (Mahdavi et al., 2012; Davoodi et al., 2012; Mostafavian et al., 2012). Many damage detection techniques based on dynamic properties have been developed in recent decades (Dawari and Vesmawala, 2016; Ding et al., 2017; Wei et al., 2017; Rezaifar and Doost mohammadi, 2016; Yasi and Mohammadizadeh, 2018). One of these methods which uses dynamic properties to identify damage is modal strain energy technique. Damage location in this method can be detected by comparison of modal strain energy of elements in undamaged and damaged states.

Carrasco et al. (1997) used modal strain energy method to detect damage in a space truss structure with 300 elements. They were able to detect damaged elements when one or two elements were at least 50% damaged. Shi et al. (1998) used modal strain energy methods to detect damage in structures. They found that modal strain energy change index is at its highest in the damaged element and that the value becomes much smaller at elements far from the damaged elements. But, elements located at nodal points of the mode shape do not follow this rule and have unusually small or large values, at times leading to wrong detection of damage location. To overcome this problem, they proposed using multiple modes in calculating modal strain energy. Srinivas et al. (2011) used a method based on modal strain energy to detect damage in a 2D truss structure. In the damage localization stage they observed that even though the damage index has its largest value at the damaged element, the value is sometimes too close to that of adjacent elements.

Ma et al. (2014) used modal strain energy method to detect damage in a seven story frame structure. Since damage index for some of the healthy elements was considerable, they proposed to use a threshold level to separate damaged and healthy elements, which means that only elements with damage index greater than threshold level were considered as damaged elements. Seyedpoor (2012) proposed a two-step damage detection technique on the basis of modal strain energy to find damage in structures. He used this method to identify damage in a 2D truss with 25 degrees of freedom. Despite using five mode shapes in damage detection procedure, still some healthy elements had considerable damage index; therefore he used a threshold level of 0.05 to separate damaged and healthy elements.

Wei et al. (2016) used modal strain energy technique to identify damage in plate like structures. At first, damage was localized by means of modal strain energy change ratio. Then a method was suggested to reduce the number of suspicious damaged elements entailed by the ‘Vicinity Effect’, in order to better results. Torkzadeh et al. (2013) presented a two-step damage detection technique on the basis of modal strain energy to indicate damage in double layer grids. They needed a relatively high number of mode shapes to detect damage. In practice determining many mode shapes is unfeasible and thus solution requiring fewer number of mode shapes is urged.

Data fusion is a technique that combines data from different information sources to achieve better results. Recently, data fusion has appealed increasing consideration in structural health monitoring due to its abilities in taking out information from various sources and merging them into a consistent, precise and apprehensible data set (Zhao et al., 2014). Guo (2006) used three fusion approaches, including

Dempster-Shafer evidence theory, Bayesian fusion and fuzzy fusion method to detect the damage in a two dimensional truss structure. The numerical results showed that the Dempster-Shafer evidence theory is the most efficient fusion method between the three. Guo and Li (2011) used the Dempster-Shafer evidence theory to detect damage in 2D and 3D truss structures. Grande and Imbimbo (2014) used Dempster-Shafer evidence theory to amalgamate information which was acquired from different mode shapes in modal strain energy method to detect damage location in a fixed end beam. Grande and Imbimbo (2016) proposed a technique based on flexibility method and Dempster-Shafer theory to detect damage in structures. The efficacy of this technique has been shown with reference to a fixed end beam and a 3D structure. Guo et al. (2019) used an enhanced Dempster-Shafer theory and a model in time domain to locate nonlinear damages in structures. They indicate that the enhanced Dempster-Shafer theory has great recognition accuracy and decent performance. Cheng et al. (2019) proposed a new damage detection technique based on the flexibility identification principle and Dempster-Shafer theory. They showed that the proposed method is more suitable to detect damage in beam structure in noisy environments in comparison to traditional methods. Ding et al. (2019) used an improved Dempster-Shafer data fusion algorithm to detect damages in a spatial truss structure.

As mentioned above, the results obtained solely from classic modal strain energy methods cannot clearly distinguish damaged elements from healthy ones and therefore demand further enhancement. This problem is exacerbated in high degrees of freedom structures which require many mode shapes for reliable damage detection. In this study, modal strain energy method and Dempster-Shafer evidence theory are used to detect damage location in a double layer grid. By performing modal analysis and calculating vibration mode shapes of the grid in undamaged and damaged states, the modal strain energy based index for each mode shape is determined. The indices are then combined with Dempster-Shafer evidence theory to improve the results. In practice, measurements are corrupted by noise and therefore we added random noise to mode shapes. A set of 1000 damage detection runs is performed for each damage case and the mean of results is considered. The complete process of damage detection has been implemented in MATLAB.

2. Modal Strain Energy Based Damage Localization

The modal characteristics of an undamaged structure are described by the eigenvalue equations (Ren and Roeck, 2002):

$$K\varphi_i = \omega_i{}^2 M\varphi_i, \qquad i = 1, \ldots, n \tag{1}$$

where K and M: are stiffness and mass matrices, respectively; ω_i and φ_i: are the ith natural frequency and mode shape of the structure, respectively.

The Modal Strain Energy of element j in mode i before damage and after that is denoted as (Shi et al., 1998):

$$\begin{aligned} MSE_{ij}^{u} &= \varphi_i^{u^T} K_j \varphi_i^{u} \\ MSE_{ij}^{d} &= \varphi_i^{d^T} K_j \varphi_i^{d} \end{aligned} \tag{2}$$

where MSE_{ij}^{u} and MSE_{ij}^{d}: are the undamaged and damaged modal strain energy of the jth element in ith mode respectively; φ_i^{u} and φ_i^{d}: are the partial mode shape vectors before and after damage respectively, containing the ith mode shape elements related to the degrees of freedom of jth element. As the damage locations are unknown, the undamaged elemental stiffness matrix K_j is used for estimating MSE_{ij}^{d} (Shi et al., 2000).

The whole modal strain energy of ith mode of the structure is calculated by the summation of modal strain energy of all structural elements as follows:

$$MSE_i = \sum_{j=1}^{ne} MSE_{ij} \quad (3)$$

where MSE_i: is the total MSE of the structure in mode i and ne: is the total number of elements. The normalized modal strain energy of jth element for mode i will be determined by dividing MSE_{ij} by MSE_i as:

$$NMSE_{ij} = \frac{MSE_{ij}}{MSE_i} \quad (4)$$

where $NMSE_{ij}$: is the normalized modal strain energy of jth element in mode i. Since damage changes the dynamic properties of the structure, normalized MSE of an element in each mode will change after damage occurs. Modal Strain Energy Based Index (MSEBI) is an efficient indicator for damage localization which can be determined as:

$$MSEBI_{ij} = max\left[0, \frac{NMSE_{ij}^d - NMSE_{ij}^u}{NMSE_{ij}^u}\right] \quad (5)$$

where $NMSE_{ij}^u$ and $NMSE_{ij}^d$: are the undamaged and damaged normalized modal strain energy of jth element in ith mode, respectively. $MSEBI_{ij}$: is the modal strain energy based index of jth element in ith mode. $MSEBI_{ij}$ will be zero for undamaged elements and greater than zero for damaged ones. Elements with higher $MSEBI$ are thus more likely to be the damaged ones.

$MSEBI$ for an element can be evaluated with several first mode shapes as follows:

$$MSEBI_j = max\left[0, \frac{\sum_{i=1}^{nm} NMSE_{ij}^d - \sum_{i=1}^{nm} NMSE_{ij}^u}{\sum_{i=1}^{nm} NMSE_{ij}^u}\right] \quad (6)$$

where $MSEBI_j$: is the modal strain energy based index of jth element evaluated by considering nm first mode shapes.

3. Dempster-Shafer Evidence Theory

Information fusion methods can incorporate data from numerous information sources and corresponding information from dependent databases, to attain better accuracies and more particular deductions than could be attained by using only one source (Guo, 2006). Dempster-Shafer evidence theory is a data fusion technique based on mathematical theory first suggested by Dempster (1967) and then developed by Shafer (1976).

Considering a finite set θ of mutually exclusive and exhaustive proportions, the power set 2^θ is the set of all the subsets of θ containing itself and an empty set $\emptyset$. For instance, if $\theta = \{a, b, c\}$, then $2^\theta = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. Dempster-Shafer theory is based on probability and allocates a Basic Probability Assignment (BPA) function to any subset of 2^θ. m which represents the BPA, describes a map of the power set to the range [0, 1], so that BPA of the empty set is equal to 0 and BPAs of all the subsets of the power set have a total of 1. This can be mathematically represented by:

$$\begin{aligned} &m: 2^\theta \rightarrow [0,1] \\ &m(\phi) = 0 \\ &\sum_{x \subset 2^\theta} m(x) = 1 \end{aligned} \quad (7)$$

Since the subset X has 2 data sources, s_1 and s_2, let $m_1(s_1)$ and $m_2(s_2)$ be BPAs given by the sources s_1 and s_2, respectively. The composition, namely the joint m_{12} is computed by the orthogonal sum of 2 BPAs $m_1(s_1)$ and $m_2(s_2)$:

$$\begin{aligned} &m_{12}(\phi) = 0 \\ &m_{12}(X) = \frac{\sum_{s_1 \cap s_2 = X} m_1(s_1). m_2(s_2)}{1 - K'} \\ &K' = \sum_{s_1 \cap s_2 = \emptyset} m_1(s_1). m_2(s_2) \\ &when \quad X \neq \emptyset \end{aligned} \quad (8)$$

where K': is a basic probability mass related to conflicts which is evaluated by summing the products of the BPAs of all the sets in which the intersection is null (Chen and Xia, 2011).

4. Application of Modal Strain Energy Method and Dempster-Shafer Theory in Damage Localization

Dempster-Shafer theory and modal strain energy method can be used to detect damage in structures. Let $\theta = \{D_1, D_2, \dots, D_{ne}\}$ be a set which shows damage condition of a structure with ne elements. The power set 2^{θ} can be shown as follows:

$$2^{\theta} = \{\emptyset, \{D_1\}, \{D_2\}, \{D_3\}, \dots, \{D_n\}, \dots, \{D_1, D_2\}, \{D_1, D_3\}, \dots\} \quad (9)$$

Since modal strain energy method gives only damage index of each element, only n members of 2^{θ} which have one element e.g. $(\{D_1\}, \{D_2\}, \{D_3\}, \dots, \{D_n\})$ are considered.

Let $s_1$and $s_2$be two information sources which containing MSEBI evaluated by considering mode shapes 1 and 2, respectively.

$$\begin{aligned} s_1 &= \{MSEBI_{11}, MSEBI_{12}, \dots, MSEBI_{1j}\} \\ s_2 &= \{MSEBI_{21}, MSEBI_{22}, \dots, MSEBI_{2j}\}' \\ j &= 1, \dots, ne \end{aligned} \quad (10)$$

BPA can be determined by dividing MSEBI of each element in ith mode by the summation of the MSEBI of all elements in that mode.

$$\begin{aligned} m_1(s_1) &= \left\{\frac{MSEBI_{11}}{\sum_{i=1}^{n} MSEBI_{1i}}, \frac{MSEBI_{12}}{\sum_{i=1}^{n} MSEBI_{1i}}, \dots, \frac{MSEBI_{1j}}{\sum_{i=1}^{n} MSEBI_{1i}}\right\} \\ m_2(s_2) &= \left\{\frac{MSEBI_{21}}{\sum_{i=1}^{n} MSEBI_{2i}}, \frac{MSEBI_{22}}{\sum_{i=1}^{n} MSEBI_{2i}}, \dots, \frac{MSEBI_{2j}}{\sum_{i=1}^{n} MSEBI_{2i}}\right\} \end{aligned}, \quad j = 1, \dots, ne \quad (11)$$

Each member of $m_1(s_1)$ and $m_2(s_2)$ shows damage probability of elements determined from modes 1 and 2, respectively. s_1 and s_2 can be merged with Dempster's combination rule as follows:

$$m_{12}(s_{12}) = \frac{\sum_{s_1 \cap s_2 = s_{12}} m_1(s_1).\, m_2(s_2)}{1 - \sum_{s_1 \cap s_2 = \emptyset} m_1(s_1).\, m_2(s_2)} \quad when \quad s_{12} \neq \emptyset \quad (12)$$

where $m_{12}(s_{12})$: is the combination of $m_1(s_1)$ and $m_2(s_2)$. To improve the results, $m_{12}(s_{12})$ can be combined with damage probability of elements obtained from mode 3, represented by $m_3(s_3)$, as follows:

$$m_{12}(s_{123}) = \frac{\sum_{s_{12} \cap s_3 = s_{123}} m_{12}(s_{12}).\, m_3(s_3)}{1 - \sum_{s_{12} \cap s_3 = \emptyset} m_{12}(s_{12}).\, m_3(s_3)} \quad when \quad s_{123} \neq \emptyset \quad (13)$$

where $m_{12}(s_{123})$: is the combination of $m_1(s_1)$, $m_2(s_2)$ and $m_3(s_3)$ which are first, second and third information sources, respectively. Data fusion can be continued until reaching desirable results.

5. Damage Detection in a Double Layer Grid

The feasibility of the Dempster-Shafer theory and modal strain energy method in damage detection is demonstrated by detecting damage in a double layer grid. The model chosen for this study is a double layer grid with 32 nodes and 96 elements as shown in Figure 1.

Width, length and height of grid are 7.5 m, 10 m and 1.77 m, respectively. Each element is 2.5 m long with 0.004 m^2 cross section. In the grid, the Young's modulus is 200 GPa and mass density of each element is 7850 kg/m^3, respectively. In Figure 1 the top, bottom and middle elements are shown with thick, thin and dash lines, respectively and the grid has 4 supports.

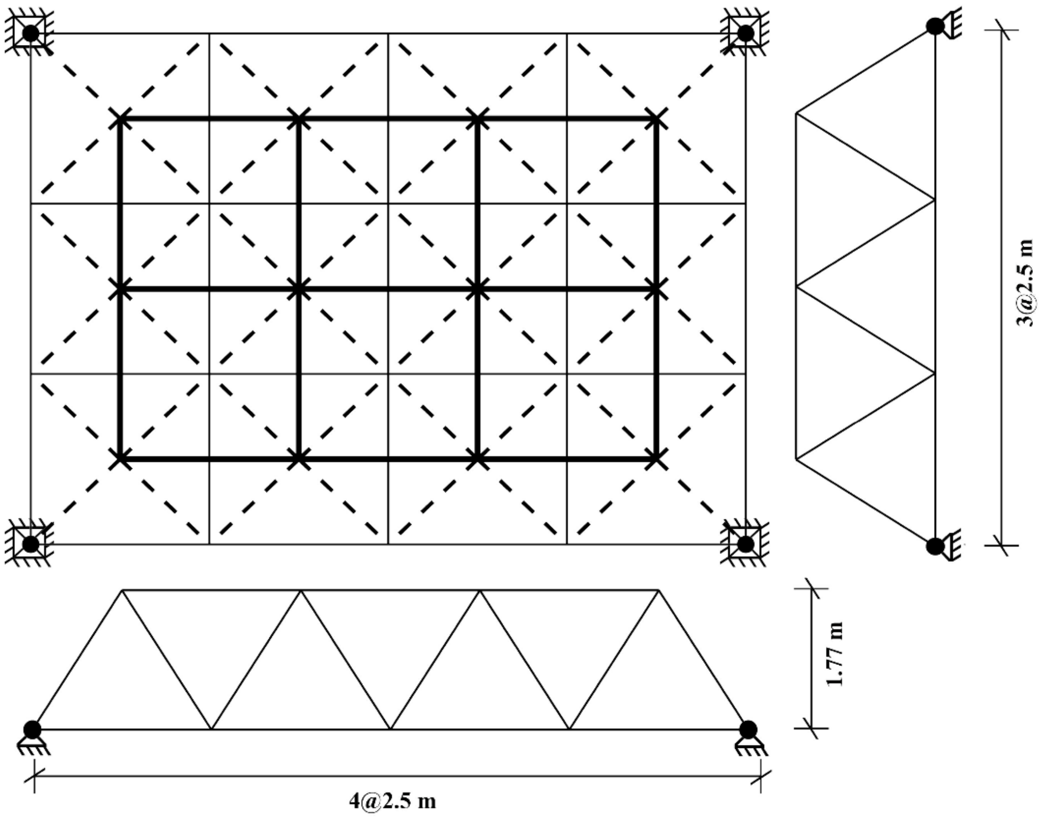


Fig. 1. Double layer grid chosen for study

To simulate a damaged grid, Young's modulus of material is decreased as follows:

$$E_j^d = (1 - \alpha_j)E_j^u \quad (14)$$

where E_j^u and E_j^d: are the undamaged and damaged Young's modulus of jth element, respectively; and α_j: is the damage percentage of jth element.

To study the effect of noise on the efficiency of the suggested damage detection technique, 3% random noise (Messina et al., 1998) is added to mode shape as follows:

$$\overline{input} = input(1 + 0.03 * rand) \quad (15)$$

where $rand$: is a random number which is normally distributed with mean 0, variance $\sigma^2 = 1$, and standard deviation $\sigma = 1$. $input$: is each component of mode shape matrices and $\overline{input}$: is the noise polluted input. Due to the random characteristics of noise, a set of 1000 damage detection runs is performed for each damage case and the average is considered.

Three single and four multiple damage cases shown in Table 1 are considered. For all cases, damage simulation is done by reducing the Young's modulus of damaged elements. Damaged elements are shown in Figure 2 with thick lines. For all damage cases 3% random noise is added to mode shapes and a set of 1000 damage detection runs is performed.

Table 1. Damage cases

Case	Damage type	Damaged element and intensity (%)
D1	Single	Element 16 with 40%
D2	Single	Element 48 with 40%
D3	Single	Element 81 with 40%
D4	Multiple	Element 16 with 50% and Element 48 with 40%
D5	Multiple	Element 48 with 50% and Element 81 with 40%
D6	Multiple	Element 16 with 50% and Element 81 with 40%
D7	Multiple	Element 16 with 50% and Element 48 with 40% and Element 81 with 50%

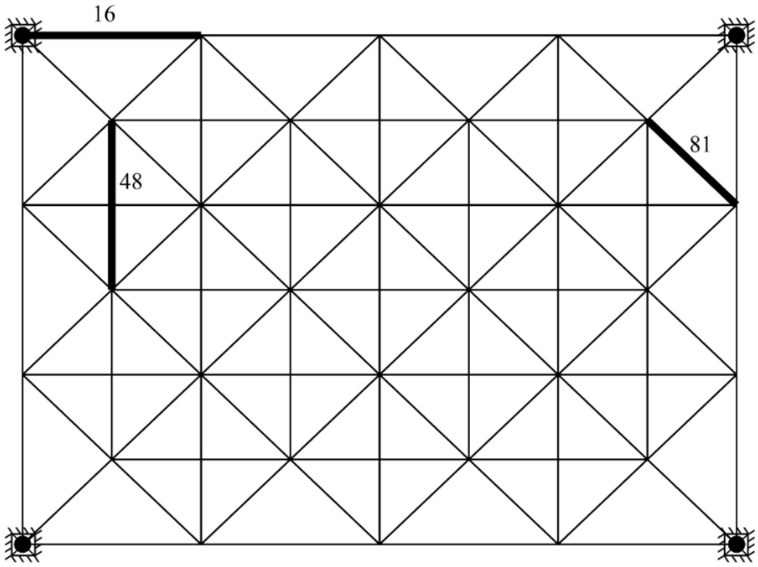
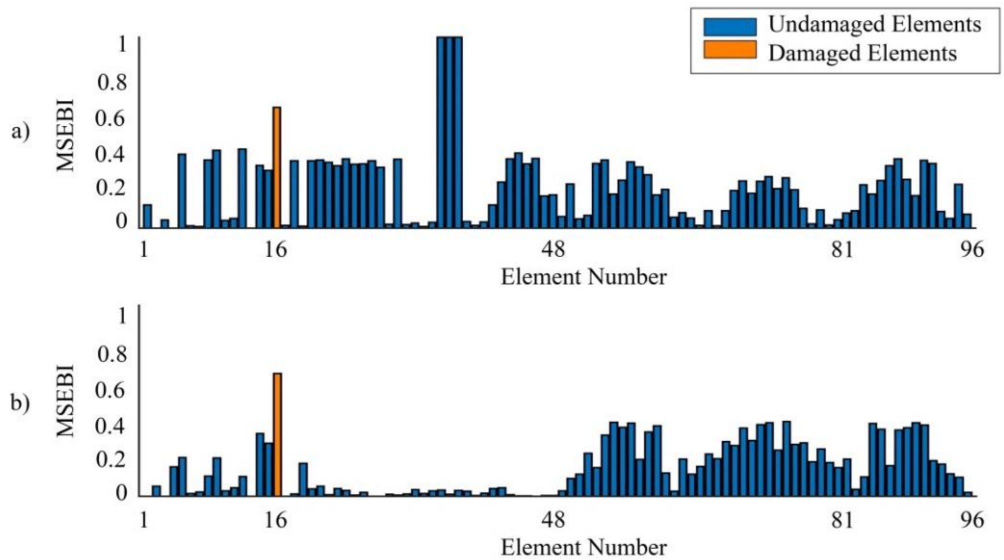


Fig. 2. Damaged elements

In the first damage case D1, damage occurs at element 16 from the bottom layer. The value of MSEBI is evaluated by considering only the first mode shape. Figure 3a shows the mean value of MSEBI for a set of 1000 runs evaluated by considering first mode shape. As it is shown, by considering only first mode shape, a lot of healthy elements have high value of MSEBI which leads to misdiagnosis in damage detection procedure. The value of MSEBI can also be evaluated by considering other mode shapes. Figure 3b-3e show the mean value of MSEBI for a set of 1000 runs when mode shapes 2-5 are considered, respectively. According to Figure 3a-3e, by considering only one mode shape to evaluate the value of MSEBI, a lot of healthy elements are introduced as potentially damaged elements. The figure clearly shows that MSEBI for the damaged element depends on the mode shape used; MSEBI for the damaged element is highest using mode 5 and lowest using mode 4. The result also depends on the location of the damaged element in the double layer grid. Moreover, the value and distribution of MSEBI for healthy elements are different among mode shapes; some mode shapes yield higher MSEBI values for fewer elements (Figure 3e), and other mode shapes give lower MSEBI value for more elements (Figure 3a).



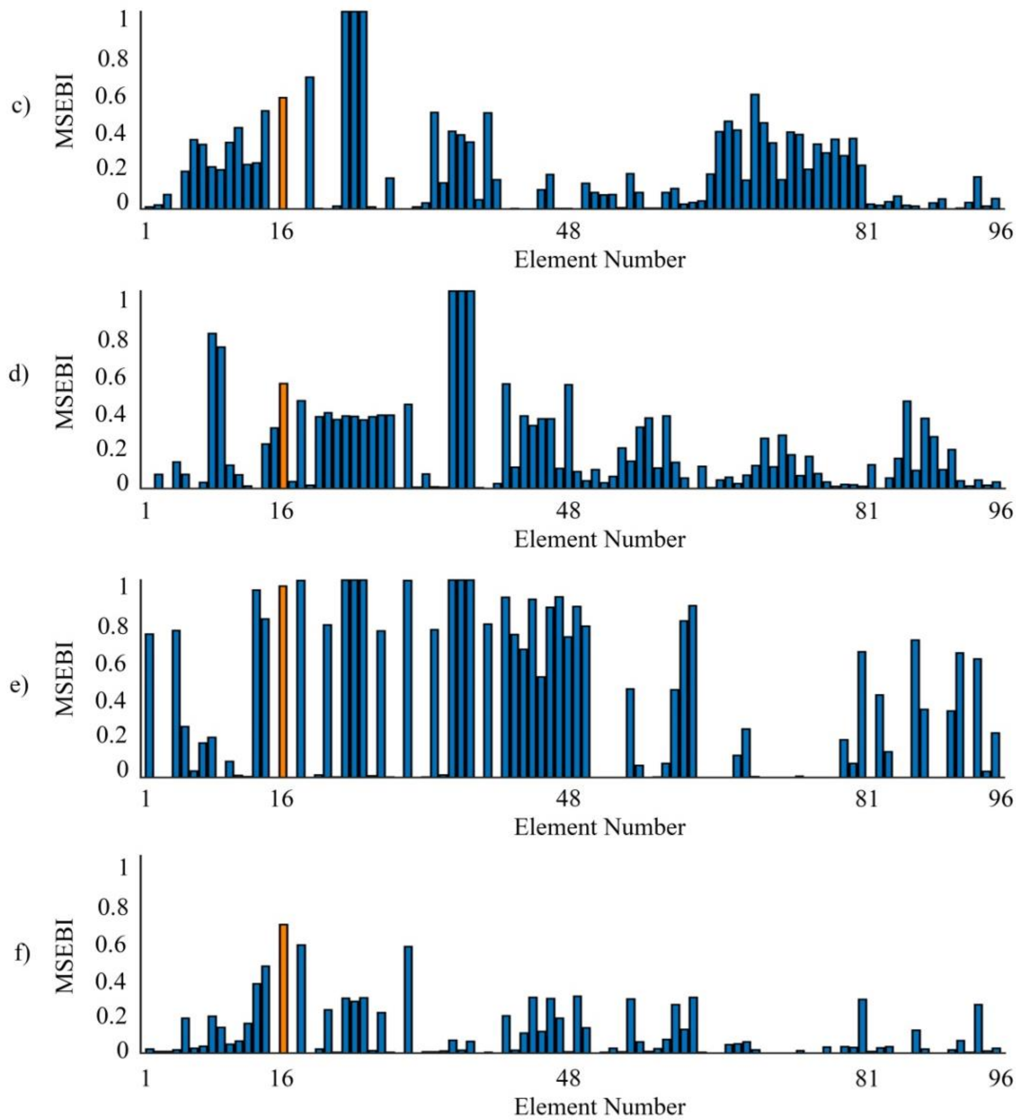


Fig. 3. MSEBI values for damage case D1 considering: a) Mode1; b) Mode 2; c) Mode 3; d) Mode 4; e) Mode 5; and f) Modes 1 to 5

The MSEBI value can also be evaluated by considering several first mode shapes according to Eq. (6). Figure 3f shows the mean value of MSEBI for a set of 1000 runs evaluated by considering five first mode shapes. As shown in Figure 3f, considering the first five mode shapes to evaluate the MSEBI gives better results, however still some healthy elements have considerable MSEBI which may lead to misdiagnosis in damage localization.

Dempster-Shafer theory is used to improve damage identification results. In the first stage, the value of MSEBI evaluated by mode shapes 1 and 2 are combined by Eq. (12). The result of the combination is damage probability and is shown in Figure 4a. As can be seen in thid figure, damage probability for element 16 is 18.61% and summation of the damage probability of other 95 elements is 81.39%. In this case, the damage probability of element 16 is not sufficiently high to be considered as a damaged element. The value of MSEBI evaluated by mode shapes 1 and 2 are combined with the value of MSEBI evaluated by mode shape 3 and the result is shown in Figure 4b. Combining the MSEBI evaluated by mode shapes 1-4 using Dempster-Shafer theory leads to better results, as shown in Figure 4c. Finally, Figure 4d shows damage probability obtained from combining the MSEBI evaluated from mode shapes 1-5.

Comparing Figures 4a-4d show that as the number of used mode shapes increases, the damage probability of the damaged element increases and the number of healthy elements with a damage probability decreases. As shown in Figure 4d the damaged element is distinct and no more elements have considerable damage probability, therefore no more data combination is needed. Here, damage probability of element 16 is 77.19% and summation of the damage probability of other 95 elements is 22.81% which is not a considerable value.

Element 48 from the top layer is considered as the second damage case D2. The mean value of MSEBI for a set of 1000 runs by considering first five mode shapes is evaluated and is shown in Figure 5a. It can be seen that, damaged element is detected but still some healthy elements have considerable value. The damage probability obtained from combining the MSEBI evaluated from mode shapes 1-5 is shown in Figure 5b. As can be seen in this figure, damage probability of element 48 is 81.54% and the summation of the damage probability of other 95 elements is 18.46% which is not a considerable value.

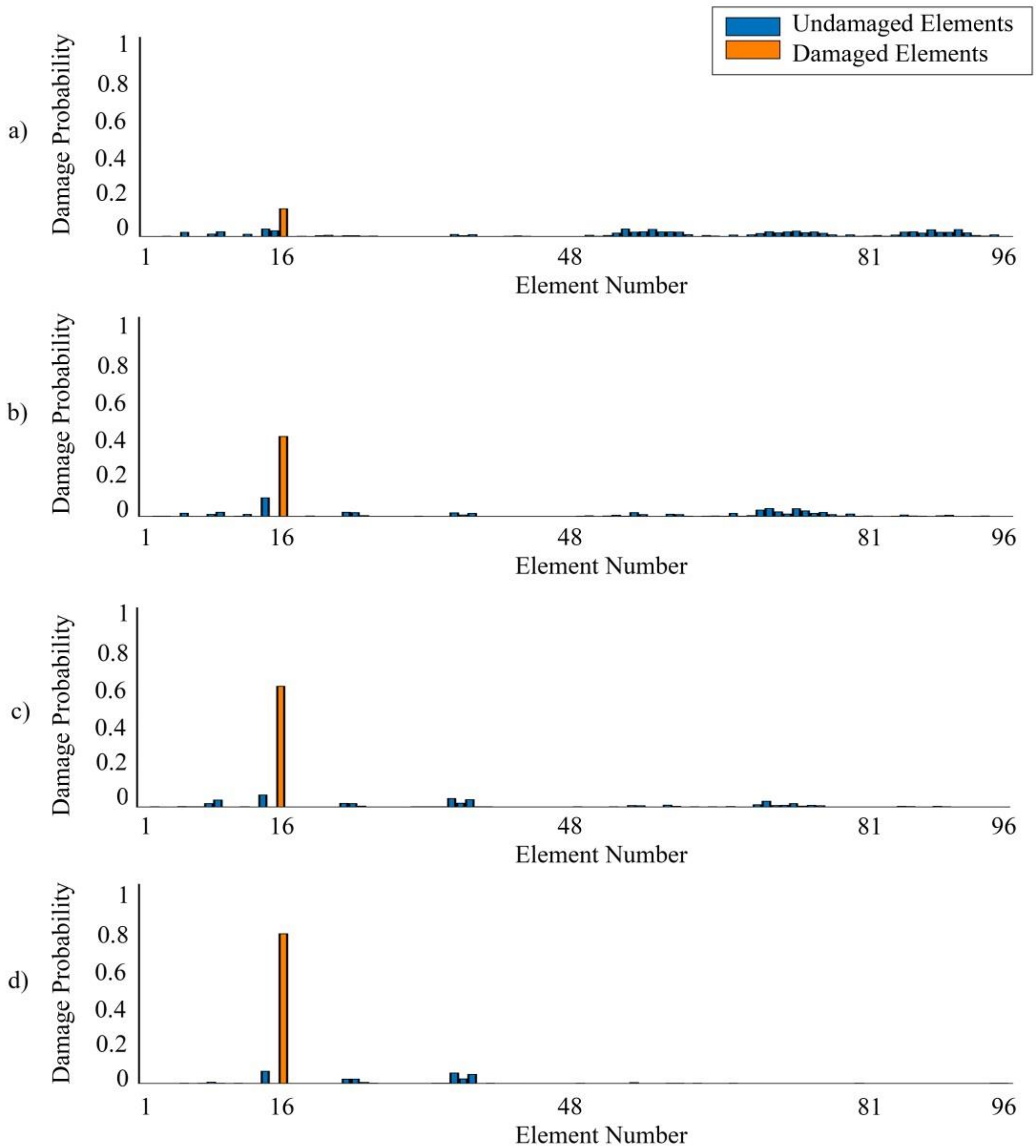


Fig. 4. Dempster-Shafer theory damage probabilities for damage case D1 considering: a) Modes 1 and 2; b) Modes 1 to 3; c) Modes 1 to 4; and d) Modes 1 to 5

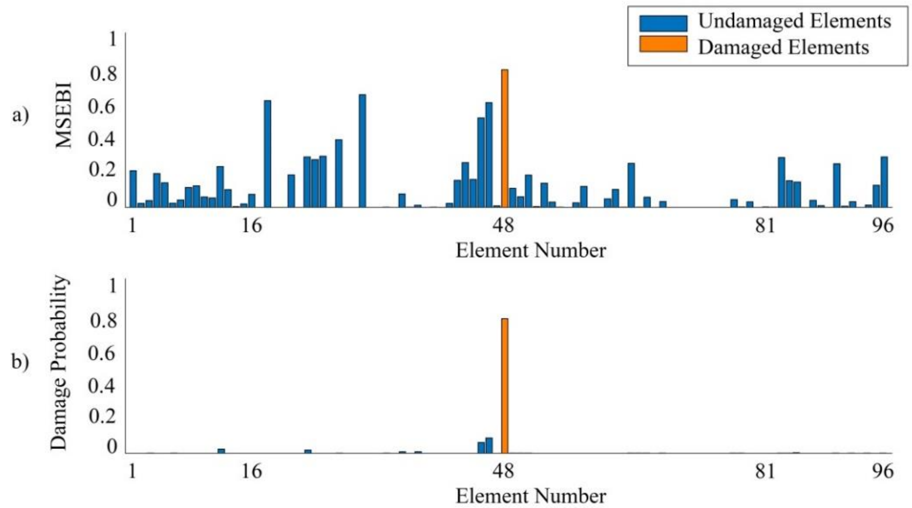


Fig. 5. Damage case D2 considering modes 1 to 5: a) MSEBI values; and b) Dempster-Shafer theory damage probabilities

In damage case D3, element 81 from the middle layer is considered as a damaged element. The mean value of MSEBI for a set of 1000 runs is shown in Figure 6a and damage probability obtained from Dempster-Shafer theory is shown in Figure 6b. First five mode shapes are considered in both MSEBI evaluation and Dempster-Shafer combination. As shown in Figures 6a-6b, by using Dempster-Shafer theory, the results are improved compared to the classical modal strain energy method. Damage probability of element 81 is 84.77% and the summation of the damage probability of other 95 elements is 15.23% which is not a considerable value.

Comparing the damage probability obtained for the damaged element in damage cases D1, D2 and D3 (respectively 77.19%, 81.54% and 84.77%) shows that the technique is relatively insensitive to the location of damaged element in the double layer grid and can find the damaged element anywhere in the grid with a sufficiently high damage probability.

D4, D5 and D6 are multiple damage cases where two elements are damaged. Figures 7a-7b, 8a-8b and 9a-9b show the mean value of MSEBI and damage probability obtained from Dempster-Shafer combination for damage cases D4, D5 and D6, respectively. Similar to the other damage cases, a set of 1000 damage detection runs and first five mode shapes are considered in both MSEBI evaluation and Dempster-Shafer combination.

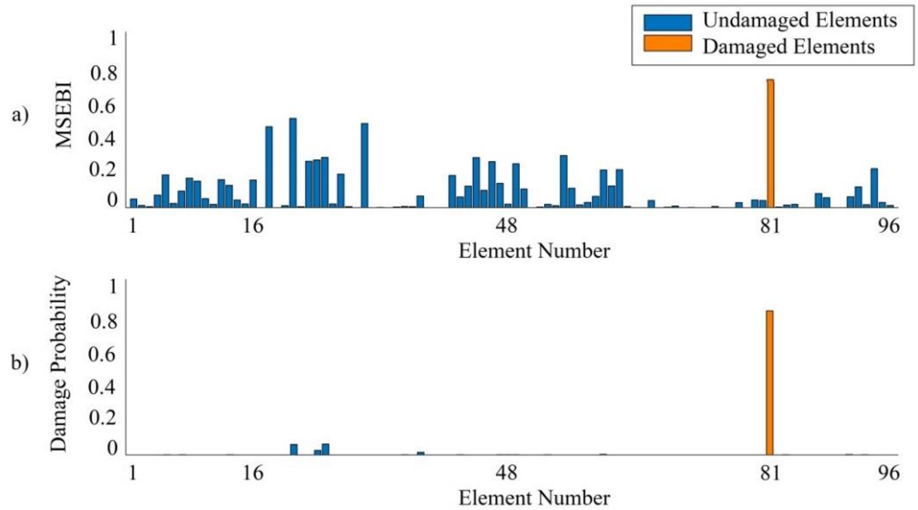


Fig. 6. Damage case D3 considering modes 1 to 5: a) MSEBI values; and b) Dempster-Shafer theory damage probabilities

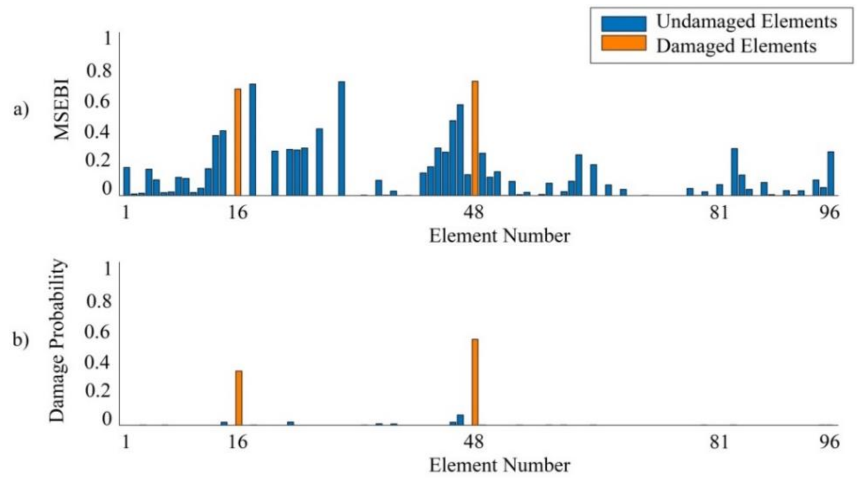


Fig. 7. Damage case D4 considering modes 1 to 5: a) MSEBI values; and b) Dempster-Shafer theory damage probabilities

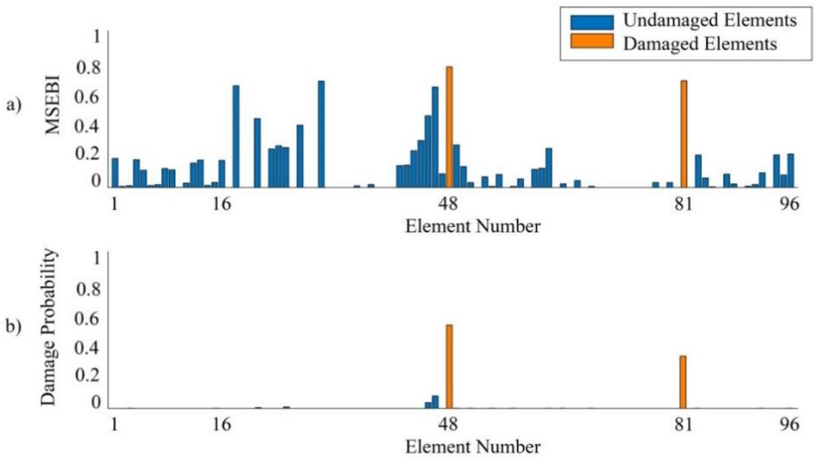


Fig. 8. Damage case D5 considering modes 1 to 5: a) MSEBI values; and b) Dempster-Shafer theory damage probabilities

As can be seen in Figures 7-9, the MSEBI value of some of the healthy elements is near that of the two damaged elements and therefore it is not possible to identify damaged elements only based on the MSEBI values. However, Dempster-Shafer damage probabilities clearly distinguished damaged elements from other healthy elements. In these two damaged elements cases, the probability of damaged elements are not related to the values of damage intensity. Sum of the probability of damaged elements are 88.11%, 89.57% and 90.23% for cases D4, D5 and D6 respectively, which indicate a high probability of damage has been obtained for damaged elements. These values are higher than the values obtained for single damage cases.

Damage case D7 is a multiple damage case where elements 16, 48 and 81 have 50%, 40% and 50% stiffness reduction, respectively. Figures 10a-10b show the mean value of MSEBI and Dempster-Shafer combination results for a set of 1000 damage detection runs. First five mode shapes are considered in both MSEBI evaluation and Dempster-Shafer combination. As shown in Figure 10a, separating damaged and healthy elements is not possible, because MSEBI values for some of the healthy elements is at the same level as the damaged elements. Figure 10b shows that by using Dempster-Shafer combination, the summation of the damage probability of damaged elements is 92.31% and the summation of the damage probability of other 93 elements is 7.69%, which means the combination was able to detect three simultaneously damaged elements with different damage intensities with good accuracy.

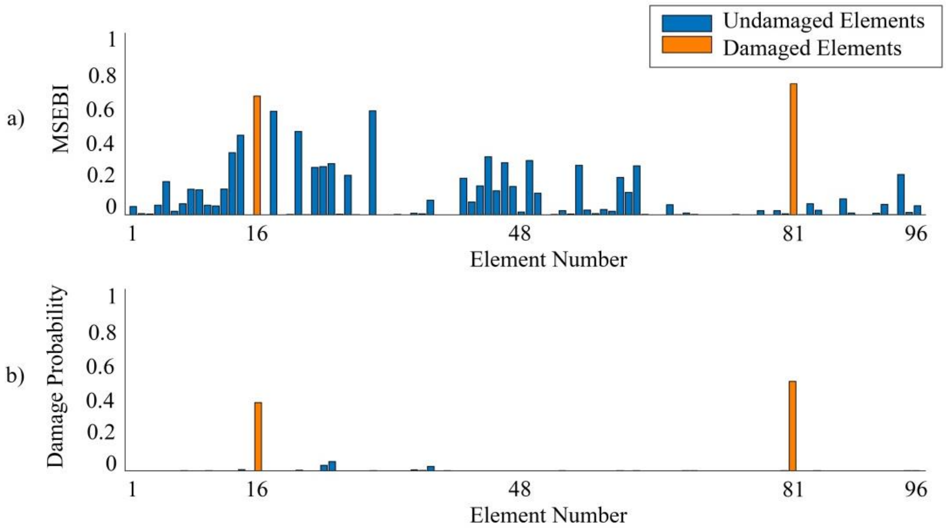


Fig. 9. Damage case D6 considering modes 1 to 5: a) MSEBI values; and b) Dempster-Shafer theory damage probabilities

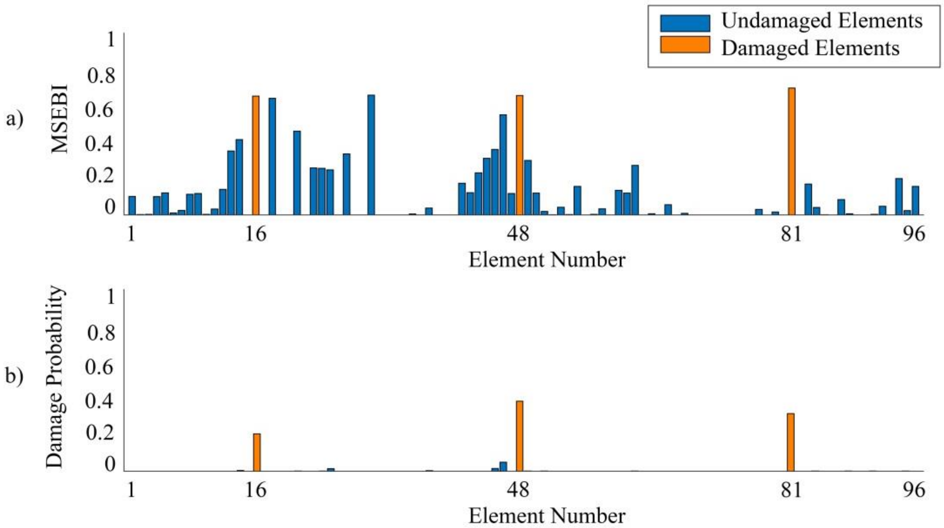


Fig. 10. Damage case D7 considering modes 1 to 5: a) MSEBI values; and b) Dempster-Shafer theory damage probabilities

6. Conclusions

Damage detection methods for structures with high degrees of freedom, usually requires a large number of mode shapes to locate damage. In practice, however, only a few first mode shapes of structures can be measured. In this paper, a damage detection technique based on modal strain energy and Dempster-Shafer theory has been presented for detecting damage in double layer grids using only a few number of mode shapes. To study the effect of noise on the efficiency of the suggested damage detection technique, 3% random noise was considered. The modal strain energy based index was calculated for each mode shape. Then, Dempster-Shafer theory was used to combine the value of indices obtained from each mode. Three single and four multiple damage cases were considered with different damage intensities in different elements of the grid. Using the first five mode shapes of the grid, the modal strain energy based index could not distinguish the damaged element(s) in any of the damage cases. This was mainly because in addition to the damaged element(s), many of the other healthy elements showed a considerable index value. However, combining the value of the indices with Dempster-Shafer theory and calculating damage probabilities of the elements, gave far better results compared to the classical modal strain energy method. In the cases with one damaged element, damage probability of 77.19%, 81.54% and 84.77% was obtained for the damaged element. In the cases with two damaged elements, the sums of the damage probabilities were 88.11%, 89.57% and 90.23%. In the case with three damaged elements, the sum was 92.31%. From this point of view, the Dempster-Shafer combination performed better with a larger number of damaged elements. Unlike classical modal strain energy methods, damaged elements were distinct and none of healthy elements had considerable damage probability. The presented damage detection technique performed very well for a relatively high degree of freedom structure with multiple damaged elements using a limited number of noise contaminated vibration mode shapes.

7. References

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