Effects of Flow Hydraulics, Pipe Structure and Submerged Jet on Leak Behaviour

Shahangian, S.A.1, Tabesh, M.2* and Mirabi, M.H.3

1 Ph.D. Candidate, School of Civil Engineering, College of Engineering, University of Tehran, Tehran, Iran.
2 Professor, Center of Excellence for Management and Engineering of Civil Infrastructure, School of Civil Engineering, College of Engineering, University of Tehran, Tehran, Iran.
3 Ph.D. Candidate, School of Civil Engineering, University of Qom, Tehran, Iran.

Received: 04 Jul. 2018; Revised: 19 Aug. 2019; Accepted: 14 Sep. 2019

ABSTRACT: The aim of this paper is numerical and experimental study of the effects of flow hydraulics, pipe structure (particularly elastic behaviour) and submerged jet on leak behaviour. In this regard, experimental tests were performed on a high-pressure circulation set up. Experiments were performed on an old steel pipe and a High Density Polyethylene (HDPE) pipe discharged to the atmosphere in a wide range of pressures up to 50 m. To analyze the leak behaviour, the effect of the surrounding environment and the pressure on leak area, the experimental setup was modeled by ANSYS software. Then, the numerical model was validated using experimental results and used to analyze and generalize leakage results in other situations. The results indicated that: 1) Standard k-ε turbulence model showed a better performance and relatively better results in modelling leakage in comparison with the other turbulence models, 2) Combining the Finite Volume and Finite Element methods for taking into account the impact of pressure allowed simultaneous examination of the pipe hydraulics and the structure of the leak area to obtain more reasonable results from hydraulic analysis of the flow and pipe structure, 3) Pressure fluctuations in the submerged jet affect the leakage discharge so that it is reduced compared to discharging to the atmosphere, 4) it was observed that the leakage exponent is close to the theoretical value of 0.5, considering the effect of pressure head on leak area behaviour. Furthermore, there is a linear relationship between pressure head and leak area for elastic pipes.

Keywords: Leak Area, Leak Behaviour, Leakage Exponent, Pipe Structure, Submerged Jet.

INTRODUCTION

Basic Leakage Head- Discharge Relationships

Perhaps the development of the leakage head-discharge relationship can be attributed to the Bernoulli’s equation written for an orifice in the wall of a tank. This ultimately led to Eq. (1) known as the Torricelli’s formula as the basis for the development of leakage head-discharge relationships.

\[ Q_l = C_d A_l \sqrt{2gH} \]

(1)

where \( Q_l \): is the leakage discharge, \( C_d \): is the discharge coefficient, \( A_l \): is the leak area, \( g \): is
the gravity acceleration, and $H$ is the water pressure inside the pipe (Streeter, 1962). The above relation can also be written in the following general form as Eq. (2):

$$Q_l = kH^{0.5}$$

where $k$: is the leakage coefficient equal to $C_d A_d (2g)^{0.5}$, and 0.5 is the same leakage exponent.

According to the field studies in Japan and the United Kingdom from 1979 to 1997, the theoretical leakage exponent equals to 0.5, if the leak area is constant (May, 1994). When variable leakage area changes with pressure, the leakage exponent varies in the range of 0.5 to 2. The concept of Fixed and Variable Area Discharge (FAVAD) was discussed by May (1994). He considered two leakage cross-sections; one with a constant area that acts like an orifice and the other with a variable area which varies with pressure changes. In order to apply Eq. (1) to the leakage in the pipes, the International Water Association (IWA) presented it in a more general form as Eq. (3) (Thornton, 2003):

$$Q_l = kH^n$$

where $n$: is the leakage exponent.

Due to the complexity of the leakage head-discharge relationship and a wide range of its impressive parameters, many previous researches have been conducted in the last decades. According to the literatures, it can be inferred that the results of previous researches relevant to the leakage head-discharge relationship can be influenced by many factors including research methods (experimental, numerical or field research), pipe material (such as rigid or non-rigid pipes), leak area (circular hole, longitudinal and circumferential cracks), and the effect of pipe surrounding environment on the leakage discharge (i.e., water discharged from leak point does not face with barrier, discharging to the atmosphere, or faces with barrier, such as a pipe submerged in water or encompassed by soil). Researches carried out in these fields is briefly reviewed in the following three sections (leakage exponent, leak Area, surrounding environment):

### Leakage Exponent

Given the role of $n$ in Eq. (3) as the leakage exponent, it is a very influential factor in determining discharge from an opening. According to the literature, if the pressure in the water distribution network falls to half its value, discharge from the leakage point decreases by 29%, 50% and 82% for the leakage exponents of 0.5, 1 and 2.5, respectively (Greyvenstein and van Zyl, 2007). Due to such a significant difference in leakage reduction, it is necessary to estimate the exact value of leakage exponent to estimate the leakage discharge of a network. Several field and laboratory studies also show that the leakage exponent can be significantly higher than the theoretical value of 0.5 and typically in a range from 0.5 to 2.79 with an average of 1.15 (Farley and Trow, 2003).

Table 1 compares studies on the leakage exponent including the lowest and highest leakage exponents identified so far. Such a difference in the leakage exponent plays an important role in the leak behaviour from a system leading to important implications for pressure management, material selection and maintenance of existing systems. This subject has led to conduct numerous studies on the relationship between leakage and its effective parameters. In last decades, results of some researches demonstrated that the simple assumption of 0.5 theoretical value for relationship between leakage and pressure head can often lead to unacceptable results (Ferrante et al., 2013), because it has been proved that leakage value especially when leakage area changes with pressure, is more susceptible to pressure variation than the orifice equation (Ssozi et al., 2016). Some previous researchers have
asserted that this subject is due to the structural characteristics of the pipes (such as elastic and viscoelastic behavior). Furthermore, some previous studies have also claimed that the surrounding environment of the leak point can affect leakage discharge and consequently leakage exponent (Fox et al., 2016; Coetzer et al., 2006, 2008; Latifi et al., 2017) and vice versa. These uncertainties or lacks in perception of the accurate definition of the leak law, have propelled many researchers to investigate this, i.e. the relationship between the leakage outflow, the pressure head at the leak area and other related parameters such as the surrounding environment, the pipe material, and etc. In the following sections a comprehensive review related to researches conducted in these domains are presented.

**Leak Area**

With the aim of investigating the behaviour of UPVC pipes with an opening under pressure, Cassa et al. (2006) studied and compared the behaviour of three types of openings including holes, longitudinal cracks and circumferential cracks using Finite Element method. Their results indicated the insignificant effect of pressure head on hole change while a significant impact of pressure on cracks. Cassa and van Zyl (2008) and Cassa et al. (2010) analyzed the effect of pressure head on leak area by Finite Element method for different types of leakage openings in pressurized pipes of different materials. They deduced that the leak area linearly increases with augmenting pressure for both longitudinal and circumferential cracks. According to their results, Eq. (4) is proposed indicating the elastic behaviour of leak area with pressure head based on Eq. (1).

\[
Q_l = C_d(A_0 + mH)\sqrt{\frac{2gH}{c}} = cH^{0.5} + dH^{1.5}
\]  

where \(c=C_d(2g)^{0.5}A_0\), \(d=C_d(2g)^{0.5}m\), \(A_0\): is the initial leak area and \(m\): is leakage head-area slope. Using Finite Element (FE) analysis (ABAQUS software), Eq. (4) were developed by Cassa and van Zyl (2013) in a 110 mm diameter class 6 uPVC pipe with a wall thickness of 3 mm. In this study, impact of the crack types (including longitudinal, spiral and circumferential cracks), loading condition and pipe material properties on the head-area slope \((m\) in Eq. (4)) were investigated and a relationship for each crack types was provided. In continue using combination of Eq. (3) (or power equation) and Eq. (4), Cassa and van Zyl (2014) investigated the link between Eq. (3) and FAVAD concept. The dimensionless leakage number, \(N_L\), was introduced as the ratio between variable and fixed portions of the leakage. Then a relationship between \(N_L\) and \(N_1\) \((n\) in Eq. (3)) was defined. Indeed, the aim of this research was integration the results of this research with previous obtained results from Cassa and van Zyl (2013) to achieve proper leak law in different crack types and materials.

<table>
<thead>
<tr>
<th>Reference and Year</th>
<th>Leakage exp. (n)</th>
<th>Reference and Year</th>
<th>Leakage exp. (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ogura (1979)</td>
<td>1.39- 1.79</td>
<td>Lambert (2001)</td>
<td>0.52- 2.79</td>
</tr>
<tr>
<td>Hiki (1981)</td>
<td>0.36- 0.79</td>
<td>Ghazali and Ardakanian (2003)</td>
<td>1.10- 1.12</td>
</tr>
<tr>
<td>Tests on UK distribution systems (Lambert, 2001)</td>
<td>0.50- 1.50</td>
<td>Farley and Trow (2003)</td>
<td>0.52- 2.79</td>
</tr>
<tr>
<td>Parry (1881)</td>
<td>0.66- 1.26</td>
<td>Thornton and Lambert (2005)</td>
<td>0.50- 1.60</td>
</tr>
<tr>
<td>Takizawa (1997)</td>
<td>0.5- 0.58</td>
<td>Walski et al. (2006)</td>
<td>0.66- 0.76</td>
</tr>
<tr>
<td>Ashcroft and Taylor (1983)</td>
<td>1.23- 1.97</td>
<td>Greyvenstein and van Zyl (2007)</td>
<td>0.41- 2.03</td>
</tr>
<tr>
<td>Sendil and Al Dhowalia (1992)</td>
<td>0.66- 1.76</td>
<td>Noack and Ullaniaki (2007)</td>
<td>0.50- 1.00</td>
</tr>
<tr>
<td>May (1994)</td>
<td>0.50- 2.50</td>
<td>Walski et al. (2009)</td>
<td>0.47- 0.76</td>
</tr>
<tr>
<td>Yeung (1999)</td>
<td>above 1.00</td>
<td>De Paola and Giugni (2012)</td>
<td>0.46- 0.51</td>
</tr>
</tbody>
</table>
Ferrante et al. (2009, 2011), Ferrante (2012), Massari (2012), Massari et al. (2012) and Ferrante et al. (2013) experimentally studied the effects of pipe material properties on the leak head-discharge relationship and leak area behaviour. Their results showed that pipe materials can affect the leakage head-discharge relationship. However, this effect of viscoelasticity on the leakage head-area relationship depends on the history of stress in the pipe.

Fox et al. (2016) investigated the effects of external porous media on the longitudinal cracking behaviour of the Medium Density Polyethylene (MDPE) pipe under the interaction of leakage, structural behavior, and soil hydraulics. Their results showed the direct impact of an ideal external porous medium on the leakage head-discharge relationship leading to a pressure drop at the point of leakage. Consequently, the impact of this factor should also be taken into account. On the other hand, increased loading by increasing the pressure inside the pipe will affect the leak area causing an increase in the leak area.

De Marchis et al. (2016) conducted an experimental study on the HDPE pipe to investigate how leak area and pipe rigidity affect on discharge. The data were analyzed based on the Torricelli’s formula, IWA (Eq. (3)) and Cassa et al. (2010) relationships. According to their results, it follows the Torricelli’s formula for a constant leak area. However, IWA and Cassa et al. (2010) relationships are more consistent with the results when the leak area changes with pressure. The results also confirmed the elastic behaviour of a pipe indicating that the leakage dimensions and upstream pressure affect the strength and toughness of pipe materials.

Using Finite Element method (ABAQUS software), Ssozi et al. (2016) investigated how do viscoelastic behaviors affect on circumferential and longitudinal cracks in class 6 HDPE and Polyvinylchloride (PVC) pipes. In this study, the responses of the leak area behavior to pressure increase and cyclic pressure variations was analyzed by time-dependent behaviors of pipes. The results indicated that viscoelastic deformation of leak area was proportional to their elastic behavior and decreased gradually during the time and fixed after approximately 12 h. Also, an equation for estimate the head-area slope ($m$ in Eq. (4)), represented for pipes with elastic behavior) in pipes with viscoelastic material was presented.

**Surrounding Environment**

Few studies have been conducted on the surrounding environment around the leakage point. According to AWWA (1999), Walski et al. (2009), Guo et al. (2013), Fox et al. (2016) (mentioned in the previous section), surrounding environment around the pipe can affect the leakage discharge. In contrast, some other studies (Franchini and Lanza, 2014; De Paola et al., 2014) show lack of the impact of the surrounding environment on the leakage discharge.

Coetzer et al. (2006, 2008) investigated the behaviour of various leak openings in pipes under pressure, namely circular holes, longitudinal and circumferential cracks. Variables studied included pipe material, leak size, surrounding media, and pressure fluctuations. Their results showed that the pressure exponent is very close to the theoretical value of 0.5 in the case of discharging to the atmosphere and under ideal conditions, while the leakage exponents in networks can be different from the theoretical orifice exponent of 0.5. It means that leakage is more sensitive to pressure than previously believed and has significant implications for pressure and water loss management. This work indicated that the effect of pressure on the leakage exponent of circular holes was not significant, while for longitudinal and circumferential cracks, pressure played
significant role in their leak behaviour. The leak behaviour was also similar for pipes submerged in water and buried in glass beads. However, it differed considerably from discharging to the atmosphere. The pressure exponent was significantly less than 0.5 in the case of discharge in water and in glass beads. Other findings of the study indicated that the pressure fluctuations did not have a significant effect on the leak behaviour. Due to the complex hydraulic behaviour of leakage, it is not often possible to determine a constant discharge coefficient but this coefficient is usually expressed as a function of the Reynolds number.

De Paola et al. (2014) studied the interaction of an HDPE pipe buried in a volcanic soil from both hydraulic and geotechnical point of view, with the aim of investigation the behaviour of a leaking pipe in realistic condition of installation. The obtained results of this paper indicated that there is no significant difference between leakage discharge into the atmosphere and embedding soil.

Latifi et al. (2017) evaluated the effects of soil characteristics, selecting several soils with different specifications, on the leakage equation with the help of experimental modeling of leakage using polyethylene pipes buried in soils of different characteristics ($D_{10}$, $D_{50}$, PL, LL and hydraulic permeability). Accordingly, grain diameter greater than 10% and 50% passing, coefficient of uniformity, coefficient of curvature, liquidity limit, plastic limit, plasticity index, and hydraulic permeability were considered to represent the soil properties. Their results showed the significant impact of $D_{50}$, PL and hydraulic permeability compared to other parameters. There was no significant relationship between leakage and some parameters. They also proposed some relationships to illustrate the effects of soil characteristics on leak behaviour.

Regarding the previous researches conducted on the leakage head-discharge relationship, investigation the simultaneous impacts of flow hydraulic and pipe structure on the leak behaviour and leak opening has paid less attention. Therefore, aforementioned topics are studied further in this research. Both numerical and experimental models are used to analyze the effect of flow hydraulics, pipe structure and submerged jet on the leak behaviour to obtain reasonable and efficient results. For this purpose, the tests are performed on a high-pressure circulation set up at the College of Engineering of the University of Tehran. The tests are performed on a steel pipe and an HDPE pipe in a wide range of pressures up to 50 m (between 5 m and 50 m) while discharging to the atmosphere. The simultaneous effect of flow hydraulics and pipe structure on the leakage exponent and the leak area behaviour are investigated. For this purpose, the experimental setup is first simulated in a numerical model and the numerical model is validated using the experimental results.

Using various turbulence models and by extracting continuous discharge-head data from the model, the numerical results are compared with experimental results from steel and HDPE pipes. The experiments on the HDPE pipe are conducted at low pressures because of the possibility of deformation of the leak area at high pressures. By choosing a model consistent with experimental results, the leak behaviour are modeled and the results are analyzed. Then the model is developed and generalized in different conditions. It should be noted that the combination of Finite Volume and Finite Element methods allows the study of pipe hydraulics and its effect on the leak area. This, in turn, provides reasonable results for modeling and predicting the system behaviour. By modeling leakage in the presence of water and pressure fluctuations in
the submerged jet, the effect of various conditions on the leak behaviour is also investigated.

**EXPERIMENTAL MODELING**

**Experimental Setup**
To investigate the leakage head-discharge relationship at the experimental scale, several tests were carried out on a high-pressure circulation set up with an approximate length of 15 m in the Research Institute of Water Turbomachinery at School of Mechanical Engineering at the College of Engineering of University of Tehran. Figure 1 shows the schematic experimental setup view and its plan.

The beginning of the setup is connected to a high-capacity tank (R). A pump (P) circulates water into the system. Water re-enters the tank from the end of the system. All pipes used in constructing this setup are steel and HDPE with a nominal diameter of 110 mm. All standards and constraints related to measurement equipment have been observed in constructing the experimental setup.

![Experimental setup view](image1.jpg)

**Fig. 1.** Experimental set-up: a) View, and b) Plan
Two flow control valves (BV, UV) are installed in the upstream with a valve (DV) in the downstream to adjust the pressure inside the system. The pressure inside the pipe is measured using two calibrated pressure gages with a precision of 0.1 m at the beginning and the end of the test pipe (UP, DP). The average leakage pressure is also calculated by distance averaging. The flow of water entering the system is measured by a calibrated electromagnet flowmeter in the setup with a precision of 0.1 liters per second (UF). The numbers recorded by the flowmeter are used in the numerical model as input to the system. The setup is installed in such a way that it is easily possible to remove and replace the test pipe for testing pipes of different materials. The tests were performed on an old steel pipe with an outside diameter of 110 mm with artificial leakage holes (with 3.3 and 5 mm diameters) using CNC machine and a new HDPE pipe with an outside diameter of 110 mm with artificial leakage cracks (with 42×1.3 mm$^2$ crack) using CNC machine discharging into the atmosphere. There are air valves along the setup for removing air inside the pipe and the experimental setup.

**Method**

A wide range of pressures from at least 5 m to a maximum of about 50 m with an interval of 5 m was applied to the system. The tests were repeated 3 times to achieve maximum reliability as well as repeatability. The leakage discharged completely into the atmosphere and there was no barrier to water flow. At the beginning of each experiment, a small flow was induced into the system by adjusting the upstream and downstream control valves to gradually remove the air in the pipes from the downstream. Then using the same valves, the pressure was adjusted to the desired value (set point) and after reaching steady state condition, the test was conducted. To calibrate the numerical model and to ensure its proper operation, the pipes pressure was measured at before and after of the test location by using two pneumatic pressure gages and the velocity of water flow was also measured by using a digital flowmeter at the beginning of the test pipes. In fact, the values of pressure and flow velocity are used for calibration of the numerical model.

The pressures before and after the test pipes and the flow rate recorded by a flowmeter were read. After recording the required data, the leakage discharge from the leakage point in the system at a specified time was calculated by volumetric method. For this purpose, the volume of leakage was collected and marked in a container for a specified period of time. The water weight was then measured by a calibrated digital scale. The water volume was calculated using the water density. The average leakage discharge was finally calculated by dividing the volume of leakage by the test time. It should be noted that the water density of 0.998 gr/cm$^3$ was calculated using two cylinders of 100 and 1000 mL and a digital scale.

Furthermore, the viscoelastic behaviour of the pipe was neglected. Two approaches are taken into account to reduce the impact of viscoelasticity on the results: 1) The applied pressure on the system was in a much lower range than the nominal pressure of the pipe, and 2) the time period for each test was considered so that the pipe is not under stress and pressure for a long time.

**Experimental Results**

Figure 2 shows the experimental results for the steel and HDPE pipes with cracks. The vertical and horizontal axes represent the leakage discharge and pressure of the system, respectively. The data were fitted to the exponential function $y=ax^b$ (based on Eq. (3)).

The experimental data were fitted to Eq.
(3) with a significant correlation coefficient:
- The leakage exponent for pipes with a rigid behaviour like the steel pipe is close to the theoretical value of 0.5. Thus, they follow the Torricelli’s formula (Eq. (2)).
- The leakage exponent for pipes with an elastic behaviour like the HDPE pipe is higher than 0.5. According to the research conducted on elastic pipes, the increased leakage exponent in fitted data can be due to the elastic behaviour of the pipe and increased leak area at high pressures according to the FAVAD theory.

**Leakage Mathematical Modeling**

The experimental setup was modelled by 3D construction of the flow geometry, the 3D meshing of the flow geometry by ANSYS ICEM, applying boundary conditions and numerical estimation of flow hydraulic parameters from continuity and momentum equations and an appropriate turbulence model by the Finite Volume method in the ANSYS Fluent.

The three-dimensional flow geometry was modeled in accordance with the geometry of the experimental setup for more precise estimation of the hydraulic parameters of the flow in numerical analysis and for more precise calibration as shown in Figure 3.

**Geometric Flow Modeling**

Three-dimensional configuration of the flow geometry was performed in the ANSYS Geometry environment. Since accurate reporting is required only in some places including the location of pressure gages and the leakage points, higher density meshing was used at these points. An appropriate meshing density was used in other parts. A hexagonal mesh was used for meshing the flow geometry. Orthogonal quality is one of the main characteristics of meshing quality. An orthogonal quality closer to 1 indicates its optimal quality. In this modeling, the average meshing quality is equal to 0.856. Other specifications are as follows:
- Maximum Aspect Ratio = 1.2
- Number of Cells = 380000
- Number of Cell Faces = 700000

![Experimental Results](image)

**Fig. 2.** Results of experimental tests
Boundary Conditions
According to the tests carried out in the experimental setup, boundary conditions plotted in the flow geometry are as follows:
- Velocity inlet boundary condition including the inlet velocity, inlet velocity profile, and estimation of turbulent kinetic energy parameters and the turbulent energy dissipation rate at the entrance.
- Pressure outlet boundary condition including relative outlet pressure, the probability of backflow at outlet and estimation of turbulent kinetic energy parameters and the turbulent energy dissipation rate resulting from the backflow at the outlet.
- Wall boundary condition including a stationary wall with standard conditions, wall roughness and estimation of the mean height resulting from the internal roughness of pipes in the experimental setup.

The Results of Mathematical Modeling with Various Turbulence Equations
Five mathematical turbulence models were used to better estimate the hydraulic parameters of flow in circular holes and cracks. The mathematical turbulence models are as follows:
- Standard k-epsilon
- RNG k-epsilon
- Realizable k-epsilon
- Standard k-omega
- SST k-omega

The SIMPLE algorithm was used for coupling the hydraulic parameters of pressure and velocity during a transient analysis by Finite Volume method. While coding the inlet velocity in C++ and applying it to the velocity inlet boundary with UDF format, the pressure range in the experimental setup was created in the numerical model and 2000 discharge-head data were extracted. In fact, instead of a limited number of discrete data, a large number of continuous data was used to predict results at any pressure.

Comparing the results from the numerical analysis of all five turbulence models and those of experimental tests, the model consistent with the experimental results was selected for modeling and developing the numerical model. Figure 4 compares the results of the experimental tests and numerical analysis of the old steel and HDPE pipes for various turbulence equations. In these charts, the vertical and horizontal axes respectively represent the leakage discharge and pressure of the system.

As seen in Figure 4, the results of numerical modeling using the Standard k-ε turbulence model are closer to those obtained from experiments. Of course, the ability of other turbulence models is undeniable. But the results of this study show that the Standard k-ε turbulence mathematical model provides more reliable results than other turbulence models in leakage modeling. Therefore, it can be concluded that the
Standard $k$-$\varepsilon$ model is an appropriate model for leakage analysis. For this reason, this turbulence model was used for modeling and developing the numerical model. It should be also noted that the results of numerical modeling up to a pressure of about 25 m are presented and compared with the results of the experimental tests in Figure 4c. The reason is the probability of an increase in the leak area of the HDPE pipe at high pressures. Due to the importance of the issue and the need for further analysis to calculate crack deformation, part of this research work focuses on this issue.
NUMERICAL MODELING RESULTS

Evaluation of the Behaviour of the HDPE Pipe by the Numerical Model

Figure 5 shows the results of experimental tests and those obtained from the numerical analysis using the Standard k-ε turbulence mathematical model for an HDPE pipe with a 42 × 1.3 mm crack. The results were obtained by taking into account the rigidity of the wall and the lack of crack deformation. In this Figure, the vertical and horizontal axes represent the leakage discharge and pressure of the system, respectively.

Based on the Figure 5, the results of the numerical analysis by the Standard k-ε turbulence model are very close to the experimental results up to a pressure of 25 m water. At higher pressures, the leakage discharge in the experimental model increases and differs from those obtained from the numerical analysis. To investigate the cause of this phenomenon, referring to the experimental results and knowing the elastic behaviour of the HDPE pipe, crack behaviour and deformation by increasing pressure was studied. Therefore, to include both flow hydraulics and structural behaviour of the pipe, the combination of Finite Volume and two-way Finite Element methods was used. For this purpose, loading due to fluid pressure inside the pipe obtained from ANSYS Fluent transient analysis, was entered into the ANSYS Structural environment using Fluid-Structure Interaction (FSI) method. Then, Finite Element analysis was began for various pressures by loading according to the physical characteristics of the HDPE pipe in accordance with the ASTM standard in the software (ASTM, 2003, 2008, 2010, 2014). Changes in the boundary conditions of the elastic wall and the shape of the crack were again entered in the ANSYS Fluent given the transient analysis of ANSYS Structural. Using dynamic meshing, wall shape changes over time were transformed again into the boundary conditions of the flow and the resulting changes in the hydraulic parameters were again applied to ANSYS Structural. This process continued until the end of the
numerical analysis.

Figure 6 shows the leakage discharge versus pressure for a 42×1.3 mm crack in the HDPE pipe taking into account the elastic behaviour of the pipe and changes in the leak area obtained from the Standard k-ε turbulence mathematical model compared to the HDPE pipe without considering the elastic behaviour of the pipe results. In this figure, the vertical and horizontal axes represent the leakage discharge and pressure of the system, respectively.

**Fig. 5.** Experimental test vs. numerical modeling results without considering of pipe elastic behaviour

**Fig. 6.** The effect of elastic behaviour with k-ε standard turbulence mathematical model
As shown, considering the elastic behaviour of the pipe in the mathematical model and combining the Finite Volume and Finite Element methods provide more reliable results. In fact, by combining these two methods and using some relatively real data from the pressure inside the pipe and around the leak point, the effect of pressure changes on the leakage opening can be analyzed more accurately to achieve more reasonable results.

In Figure 7, Eq. (4) is used for fitting to the numerical and experimental results for a 42×1.3 mm² crack in the HDPE pipe. In these figures, the vertical and horizontal axes represent the leakage discharge and pressure of the system, respectively.

Fig. 7. Using the linear relationship between leak area and pressure head for HDPE pipe 42×1.3 mm² crack: a) numerical model, and b) experimental model
Figure 7 with a significant correlation coefficient shows that a linear relationship between leak area and pressure head in the form of $A_l = A_0 + mH$ exists. This is consistent with the results of Cassa et al. (2008) and Cassa and van Zyl (2010). Therefore Eq. (4) is more appropriate than the conventional orifice equation (Eq. (3)) to analyze the leak behaviour in pipes exhibiting an elastic behaviour.

**Evaluation of the Submerged Jet Using the Numerical Model**

Numerical modeling was performed in submerged conditions in water by studying the behaviour of the submerged jet in water, pressure fluctuations at the opening and submerged leakage. The modeling was performed for examining the effect of the surrounding environment on steel pipes with a hole of 5 mm in diameter. To study the pressure fluctuations in the potential core of the submerged jet and its vicinity, the jet of water in a wide tank was modeled as a free shear layer for different static levels at the time of opening.

The potential core of the jet is a part of the fluid elements whose linear momentum is equal to the linear momentum of the flow inlet boundary. According to studies by Albertson and Dai (1950), the internal convergence angle of the potential core of the submerged jet should be about 4 to 7° ignoring the effect of wall boundary conditions. The velocity profile along the flow must follow the normal probability function. Due to the use of RANS (Unsteady Reynolds Averaged Navier Stokes) turbulence mathematical models, the above criteria are also considered. It should be noted that the use of the Large Eddy Simulation (LES) turbulence model for sufficiently accurate estimation primarily requires the use of a high-density meshing. Second, if this turbulence model is used, after extracting fluctuating hydraulic parameters such as velocity and pressure, given the time-series nature of hydraulic parameters (i.e. their values change at any time), an appropriate average function must be used in the given time series. Therefore, the results are very close to those estimated by the RANS turbulence models. Also, the use of Direct Numerical Simulation (DNS) modeling requires a very dense meshing as Kolomogrov micro-scales. Considering the volume of calculations and computational time, it requires supercomputers which are currently not available for the authors.

By studying average pressure fluctuations at the leak point within the potential core of the jet and its adjacent areas, the average pressure at the leakage point can be predicted. Figure 8 shows the results of numerical modelling of leakage for pipes submerged in water for different static water levels versus experimental results for discharging into the atmosphere. In these Figures, the vertical and horizontal axes represent the leakage discharge and pressure of the system, respectively.

Comparing the graphs in Figure 8, it can be concluded that the static height of overhead water at the leak point and consequently pressure fluctuations in the potential core of the submerged jet (leak point) affect leakage discharge so that it reduces in comparison to discharging to the atmosphere. Discharge reduction is dependent on the static level of water on the pipe and the pressure applied to the system. The higher static level of water on the leakage point causes a significant difference between the leakage discharging to the atmosphere and submerged in water. With increasing pressure, this difference is gradually reduced to the point where it becomes almost negligible at very high pressures. To compare the relationship between leakage in submerged water condition and discharging to the atmosphere, the ratio of leakage discharge in the submerged water condition
to the leakage discharging to the atmosphere is plotted against pressure for different water levels on the pipe in Figure 9.

Fig. 8. a) Comparison of results for 1.5 m water level and b) Comparison of results for 2.5 m water level
Fig. 9. a) The ratio of leakage discharge in two different situations for 3.3 mm hole and b) The ratio of leakage discharge in two different situations for 5 mm hole

According to Figure 9, it can be found that there is a relationship between the water level on the pipe, the pressure at leakage point (outlet jet) and discharging to the atmosphere and the leakage discharge in submerged water condition. This relation is presented in the form of Eq. (5).
\[
\frac{Q_{\text{sub}}}{Q_{\text{atm}}} = 1 - \frac{D_s}{H^{n_s}}
\]  \hspace{1cm} (5)

where \( Q_{\text{sub}} \): is leakage discharge in submerged water condition, \( Q_{\text{atm}} \): is leakage discharge to the atmosphere, \( n_s \): is leakage exponent for submerged mode in water, \( D_s \): is a parameter dependent on the water level on the pipe (\( D_s \) increases with increasing water level on the pipe. The corresponding values for the static levels of 1.5 and 2.5 m are shown in Figure 9), and \( H \): is static pressure head. The above equation can be written as Eq. (6).

\[
Q_{\text{sub}} = Q_{\text{atm}} - \frac{D_s}{H^{n_s}} \times Q_{\text{atm}}
\]  \hspace{1cm} (6)

According to Eq. (6), leakage discharge in submerged water condition is different in comparison with leakage discharge to the atmosphere, which arises from the second term in the right hand side in the equation. Based on this equation, if the static level of water on the pipe increases, ratio of \( \frac{D_s}{H^{n_s}} \) decreases which makes \( Q_{\text{sub}} \) close to \( Q_{\text{atm}} \). Of course in reality the static level of water on the pipe cannot be more than 3 meters, but as an exaggeration this assumption and by considering increasing of the static level of water on the pipe and consequently \( D_s / H^{n_s} \) gradually increases until \( Q_{\text{atm}} - \frac{D_s}{H^{n_s}} \times Q_{\text{atm}} \) approach to zero. Ultimately it can be theoretically seen that a reverse flow into the inside of the pipe will take place, if \( D_s \) reaches the infinity.

**CONCLUSIONS**

In this paper, experimental and numerical modeling results were presented to analyze the effects of flow hydraulics, pipe structure and submerged jet on the leakage head-discharge and leak area behaviour. Through tests on steel and HDPE pipes with different leakage holes, the leakage hydraulics, leakage head-discharge relationship parameters including leakage exponent and leak area, the effect of pressure variations as well as the effect of surrounding environment in submerged jet were studied. The main findings of this study are summarized as follows:

1) The Standard k-e turbulence model outperformed the other two-equation turbulence models in the experimental conditions. The numerical model with relatively acceptable results was able to model leak behaviour in experimental conditions.

2) According to the literature, the leakage exponent for leakage discharge against the system pressure from pipes with a rigid behaviour is equal to the theoretical value of 0.5. The model follows the Torricelli’s formula. In contrast, the leakage exponent for pipes with an elastic behaviour is greater than 0.5. The reason for this increase in the leakage exponent is variations of the leak area at high pressures and thereby increased leakage discharge.

3) Increased pressure can increase the leak area and consequently, leakage discharge.

4) In this research, the effect of pressure on the changes in leak area and hydraulics was investigated using the experimental results, numerical modeling by ANSYS and simultaneous analysis of flow hydraulics and pipe structure by combining the Finite Volume and Element methods. The results showed that the changes in the leak area with pressure follows the linear equation \( Q_l = A_0 + mH \) proposed by Cassa and van Zyl (2010). Therefore, the leakage discharge for elastic pipes such as HDPE pipes is obtained from \( Q_l = C_d (A_0 + mH)^{\sqrt{2gh}} = cH^{0.5} + dH^{1.5} \).

5) The presence of water and pressure fluctuations near the submerged jet at the opening location affect leakage discharge such that it reduces compared to discharging to the atmosphere. Of course, this reduction is...
dependent on the static level of water at the leak point and the pressure inside the pipe. A higher static level of water on the pipe causes a higher difference between the leakage discharge to the atmosphere and in submerged mode. With increasing the pressure inside the pipe, this difference is gradually reduced so that it approaches a negligible value at very high pressures.

ACKNOWLEDGMENTS

The authors wish to thank Dr. Nourbakhsh, the director, and staff of the Research Institute of Water Turbomachinery at School of Mechanical Engineering at the College of Engineering of the University of Tehran.

NOMENCLATURE

$C_d$: discharge coefficient
$g$: gravity acceleration
$A_0$: initial leak area
$A_l$: leak area
$k$: leakage coefficient
$Q_l$: leakage discharge
$Q_{sub}$: leakage discharge in submerged mode in water
$Q_{atm}$: leakage discharge to the atmosphere
$n_1$: leakage exponent for the submerged mode in water
$m$: leakage head-area slope
$D_s$: parameter dependent on the water level on the pipe
$n$: pressure exponent
$H$: water pressure inside the pipe (static head)

REFERENCES


Ferrante, M., Massari, C., Brunone, B. and Meniconi,


