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Technical Note

Calibration of Load and Resistance Factors for Reinforced Concrete Beams

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ABSTRACT: Current approach for designing of reinforced concrete members is based on the load and resistance factor. However the load and resistance parameters are random variables, the constant values have been designated for them in the designing procedure. Assuming these factors as the constants, will be led to the unsafe and uneconomical designs. Safe designing of structures requires appropriate recognition of the effective parameters and their uncertainties. Therefore, this achievement is possible through clarifying the effective design parameters and applying risk-based design methods. The main purpose of this paper is reliability based design of the reinforcement concrete structures under bending action. Rectangular sections with tension rebars (singly reinforced), rectangular sections with tension and also compression rebars (doubly reinforced) and T-shape sections are designed based on probabilistic methods. The appropriate tool for reliability calculations is selected based on pros and cons of each method. Evaluation of the load and the resistance factors for all mentioned beams is the next goal of this investigation. In this research, the steel usages for desired safety level are determined through the produced graphs. Using the proposed methodologies, the economic and fully probabilistic design of the concrete beams for bending is now available.

Keywords: Load and Resistance Factors, Monte-Carlo Simulation, Reinforced Concrete Beam, Reliability-Based Design, Safety Factor.

INTRODUCTION

Ultimate limit states approach to design the concrete elements have been widely accepted in most universally standards. This method is a semi-probabilistic method that the margin of safety is indirectly used by load and strength factors. Since these load and resistance factors are constant, the designers are not able to change and manage the safety indexes. Sometimes, it is necessary to reduce or increase the safety level of designs. For instants, to design a nuclear power plant, the safety index usually must be higher than for designing of an ordinary building. Due to constant values of the load and resistance factors, limit state methods are not able to suggest any possibility to assign a specific level of safety to structural designs. In comparison with semi-probabilistic

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approach, fully-probabilistic method is realized about 40-year ago. In the semi-probabilistic method, the load and resistance factors are not applied directly in the designing procedure, instead; the safety indexes clearly utilized. Therefore, reliability-based approach will be led to an economic and safe design. Ravindra et al. (1974) used the so-called safety index method to design reinforced concrete beams and structural steel members. In their studies, the reliability index was calculated by the mean value first order second moment (MVFOSM) method. The method is not invariant with respect to the formulation of the limit-state function, i.e.; equivalent formulations of the limit-state lead to different values of the reliability index. Rapid developments in reliability analysis methods and reliability-based structural design, have been generated a rich research in this subject. Renjian et al. (1994) conducted the reliability based designs of RC beams under the combination of bending-shear. They used the FORM method to evaluate the failure surface. The results showed that safety index is related to the ratio of live load and effective parameters of a problem. It could be obtained from the background literature that the research of Marefat and Vafaei (1999) is the most relevant one in this filed. They evaluated the safety index of a rectangular beam for various values of loading ratios. They compared steel requirements of a beam for the code based and fully probabilistic designs. Terzos and Thomas (2002) carried out a research for calibration of Greek seismic code for beam-column members. Their aim was the evaluation of shear capacity of the beam-column members and the safety index for shear demand. Bentz et al. (2006) applied the new reliability-based method for assessment of shear resistance of concrete beams using ACI regulations. Paik et al. (2008) calibrated the bridge codes and used the probabilistic approach for concrete bridge designing. Nowak and Kaszynska (2011) investigated the reliability-based indexes of concrete beams and frames that designed by ACI-318. They proved that the designed buildings have lower safety indexes. Porco et al. (2013) applied the reliability-based approach for punching in the concrete slabs with emphasis on the compression strength of concrete. Jenson (2014) conducted a research on obtaining the safety index for shear and strengthening of the concrete bridges.

A considerable literature was published on reliability based designing of reinforced concrete members. The investigations were only focused on dead and live loads on members. Moreover, some researchers were focused on the combinations of bending-shear and bending-torsion, but they only were studied the safety indexes in their investigations (Backes et al., 2014). Although extensive researches were accomplished for safety index calculations, no single study exists, which computes the load and resistance factors that related to the specified safety index. Therefore, to our knowledge, detailed studies for all types of the loadings, i.e. dead, live and earthquake loads, and calibration of the loading and resistance factors were not reported throughout the literature. Therefore, in this investigation, the load and resistance, loading factors for all main loadings in the design world are calculated for any desired safety index and any loading ratios. Accordingly, the methodology of the present paper could be used on designing of the beams under bending for a fully probabilistic based approach.

CONCRETE BEAMS DESIGN

In this section, the formulations of designing reinforced concrete beams for bending are presented (McCormac and Brown, 2015). The relations are based on Iranian national building standard (NBCI, 2011). For fully-probabilistic designs, three types of RC beams have been studied here. The first type is rectangular section beams with tension reinforcements only, singly reinforced (SR); the second one is rectangular section with tension and compression reinforcements, doubly reinforced (DR), and the last one is T-shape beam. The schematic figures of these common sections are depicted in the Figure 1.

The geometrical parameters and the abbreviated name of the mentioned sections have been presented in Figure 2.

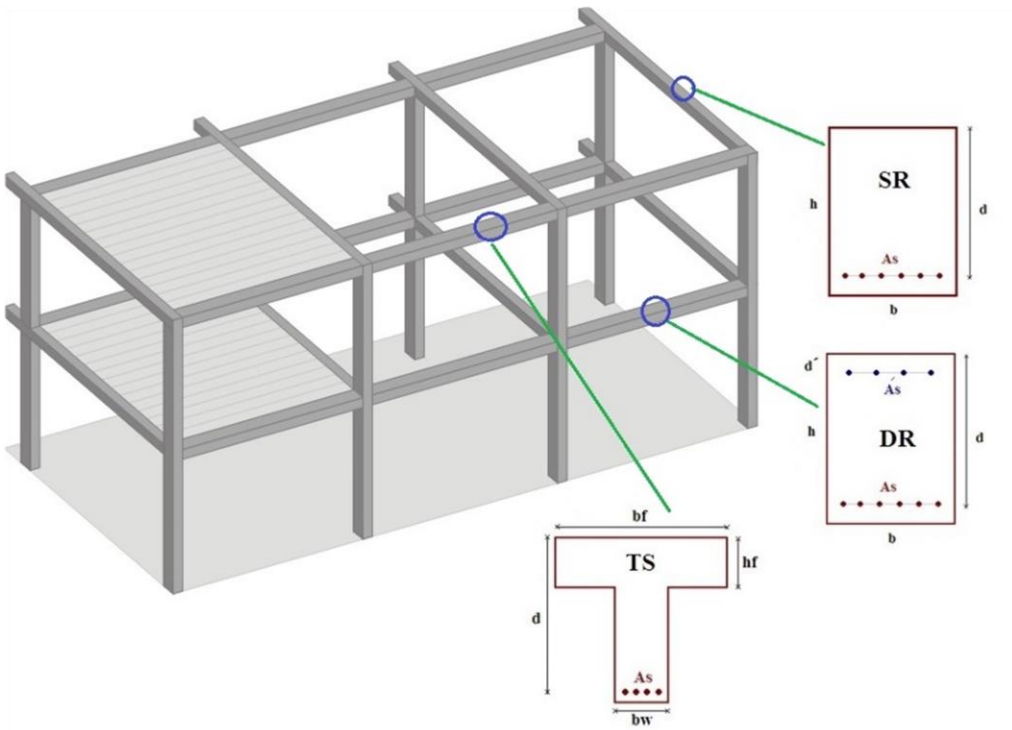

Fig. 1. Concrete building with different types of beams: SR (Singly reinforced rectangular beams), DR (Singly reinforced rectangular beams), TS (T- shape Beam with tension reinforcements)

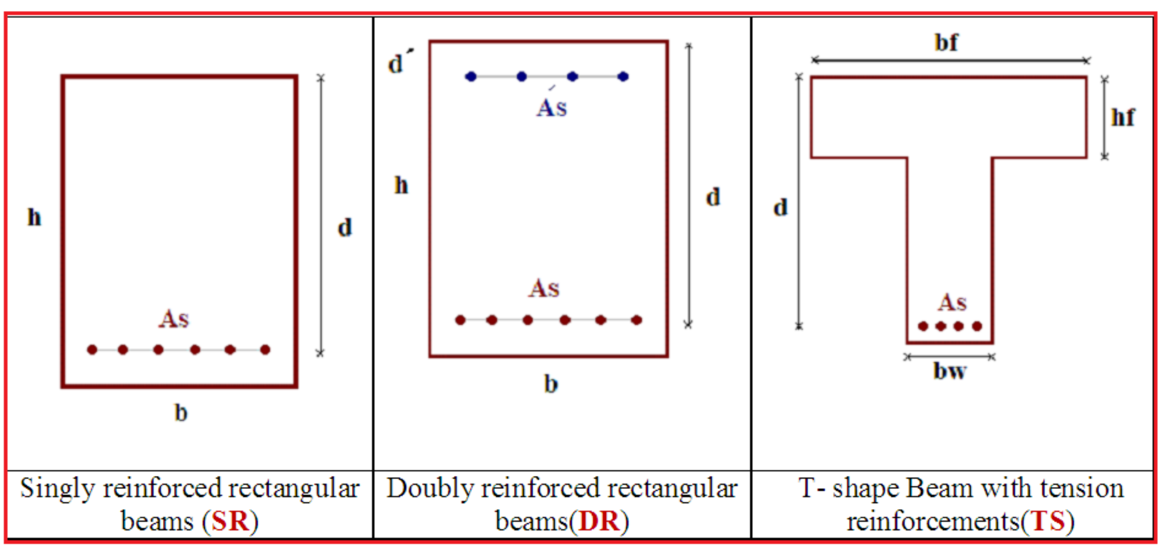

Fig. 2. The parameters of SR, DR and TS reinforced concrete beams

The resistance moments of each sections SR, DR and TS beams have been calculated using Eqs. (1-3), respectively. For instance, In the case of SR section the resistance moment is calculated using Eq. (1)

$$M_R = A_s \varphi_s f_y (d - \frac{A_s \varphi_s f_y}{2b\alpha_1 \varphi_c f_c'}) \quad (1)$$

For DR sections the M_R obtained from Eq. (2)

$$M_R = \varphi_s f_y (A_s - A_s')(d - \frac{\varphi_s f_y (A_s - A_s')}{2b\alpha_1 \varphi_c f_c'}) + A_s' f_y (d - d') \quad (2)$$
$$\text{when } \rho > \rho_{min}, \rho < \rho_b$$

For T-shape beams (TS), the resistance moment M_R is composed of M_{Rf}, M_{Rw} (Moments of flange and web of T-shape beam). Then, in this case $M_R = M_{Rf} + M_{Rw}$

$$M_{Rf} = 0.85 f_c' h_f (b_f - b_w)(d - \frac{h_f}{2})$$
$$M_{Rw} = 0.85 f_c' h_f a b_w (d - \frac{h_f}{2}) \quad (3)$$

where M_R, A_s, f_y, f_c' : are the resistance moment, the areas of longitudinal bars, the yield strength of steel, 28-day strength of concrete. b, d, d' : are width, the effective depth of the beam and the distance between top of the beam and the center of rebars, respectively. φ_s, φ_c : denote to the reduction factors of steel and concrete and M_{Rf}, M_{Rw} : are the resistance moments of flange and web of T-beam, respectively. h_f, b_f, b_w : are the thickness of flange, the width of slab and the width of web, respectively.

RELIABILITY-EVALUATION

With the advancement in computational power, Monte Carlo Simulation (MCS) technique is becoming very attractive to estimate the underlying reliability (Paxton et al., 2001; Sanaeei et al., 2003; Rashki et al., 2012, 2014; Arab et al., 2014). This method does not require the sophisticated mathematical background to reliability evaluation. In addition, calculations of the derivatives for mentioned methods for highly nonlinear problems or for the implicit limit states require carefully error control, and usually, the procedures are time-consuming processes. Simulation techniques provide a cheaper alternative to evaluate risk or the effect of uncertainty in the computer environment than the expensive physical experiments in the laboratory or in the field. With only a little background in probability and statistics, any user could utilize simulation to estimate the reliability. Therefore, in this study Monte Carlo simulation (MSC) method is applied to risk analysis of bending design of the beams (Paxton et al., 2001). To evaluate the probabilistic parameters for all types of beams, the following characteristics of random variables as presented in Table1 have been applied in the simulation procedure.

Failure probability is calculated by $P_f = N_f / N$ where, N_f: is the number of simulation cycles when the limit state function (g) is less than zero, and N is the total number of simulation cycles. In this paper, the number of N is fixed to 10000 cycles. For safety index calculations, the limit state function, $g(\mathbf{X})$, is considered as a difference between capacity of members, M_R , and demand, M_S, as $g(\mathbf{X}) = M_R - M_S$, in which, $\mathbf{X}$: is the vector of random variables. For probability-based design, the value of demand is obtained as $M_S = M_D + M_L + M_E$.

Table 1. Probability data of capacity and demand for the beams

R.V	NOMINAL VALUE	PDF	μ	Cov (δ)
f'_c(Mpa)	21	N^*	19.3	0.18
	28		23.7	0.18
	35		28.2	0.18
f_y(Mpa)	280	N	317	0.12
	300		331	0.12
	420		472	0.12
Dimension (mm)	b	N	b	b/10
	h		h	h/17
	d		d	d/15
Area (mm^2)	As	N	As	0.03
	$A's$		$A's$	0.03
Loading	D	N	1.05D	0.1
	L	EI^{**}	L	0.2-0.4
	E	EI	V^{***}	2.3

N^*: refers to the normal random variables, EI^{**}: refers to the extreme value Type I probability density function and V^{***}: indicates that the loading is variable.

where M_D, M_L, M_E: are the effective moments of dead, live and earthquake loadings, respectively. Total moment for each member is obtained as $M_{Tot} = \gamma_D * M_D + \gamma_L * M_L + \gamma_E * M_E$.

The safety index for load and resistance calibrations and reliability-based design of each beam is calculated as $\beta = (\mu_R - \mu_s) / \sqrt{\sigma_R{}^2 + \sigma_s{}^2}$

NUMERICAL STUDIES

Here, reliability-based designs of flexural beams have been explained. The details of geometries and material properties of each beam are described in section 2. The first case is the SR rectangular beam, and the geometric and material properties are same as Table 1. Data for first and second beam are b = 400 mm, d = 700 mm, d' = 50 mm and M_D = 200 kN.m. The third beam is the T-shape one that its specifications are: b = 2100 mm, d = 900 mm, h_f = 300 mm, b_w = 50 mm, a = 50 mm and M_D = 6500 kN.m. Figure 3 demonstrates the variations of the safety index (β) against loading ratios (t).

Clearly, by increasing the value of loading ratio (t) or reducing the value of live load, the safety index is increased. The reason is that according to the Table 2, the coefficients of variations for live load ($\delta = 0.20 - 0.40$) are more than the coefficients of variations for dead load ($\delta = 0.10$). For all sections, the maximum safety indexes have been occurred at t = 0.7-0.80. As well, according to Figure 3 for specific value of t, the safety indices of DR beams are more than other beams. The reason is that for SR and TS sections, there are not any compressive bars. Presence of compressive bars in the beams has been improved the capacity of them, and therefore, for practical designing using rectangular beams with compressive bars is economic and has better performance.

The required steel for DR beams for various loading ratios (t) against the safety indexes has been depicted in Figure 4. For desired level of the safety index and given loading ratios, these graphs could be directly used for probabilistic designing of the rectangular beams.

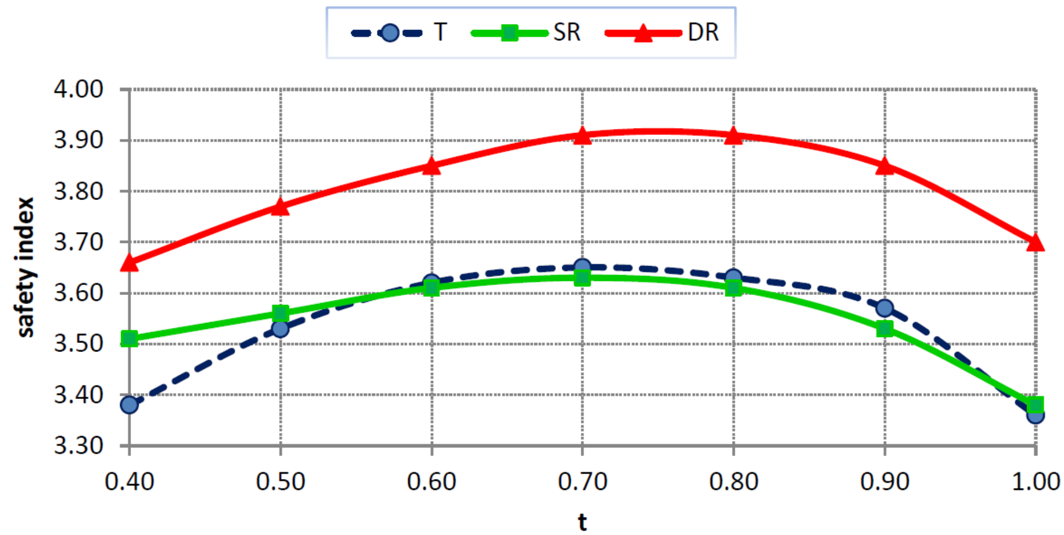

Fig. 3. Safety index variation versus loading ratio for rectangular beam with tension rebars (SR), rectangular beam with tension & compression rebars (DR) and T-shape beam (T) vs $t = M_D/(M_D + M_L)$

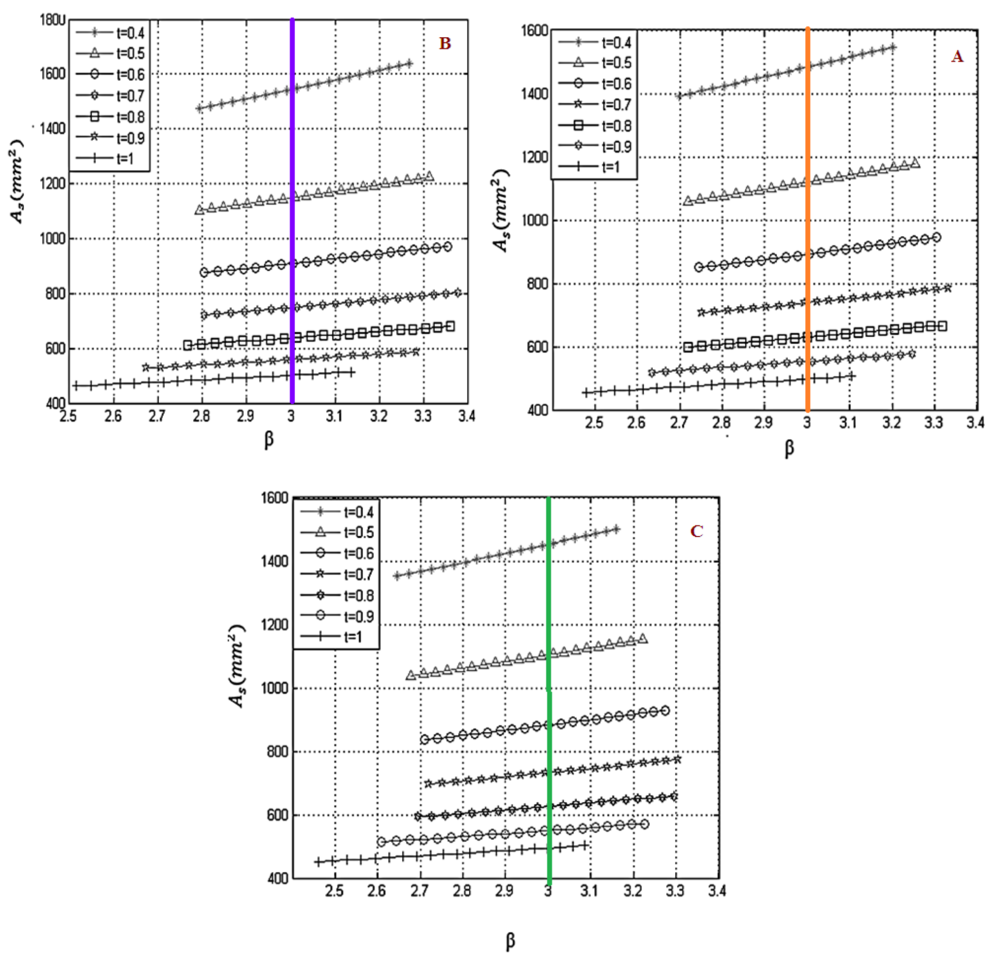

Fig. 4. Required steel for DR rectangular beam for $t = M_D/(M_D + M_L)$: A) (f_y =420 Mpa, $f'_c = 28$ Mpa), B) (f_y =420 Mpa, $f'_c = 21$ Mpa), C) (f_y =420 Mpa, $f'_c = 35$ Mpa)

As shown in Figure 4, the required steels are very sensitive to loading ratios (t) and the sensitivity of A_s with respect to $f_c^{'}$ is insignificant. Therefore, in practice, using high-strength concretes will not necessarily lead to the economic design, and more important parameter is loading ratio. Here, the strength reduction factors of steel and concrete (ϕ_s,ϕ_c) and dead and live factors (γ_D,γ_L) directly have been calculated. For this purpose, instead of using constant coefficients, the ranges of these factors have been introduced to the developed computer program. Then, the factors are computed using the Monte Carlo simulation technique. In Figures 5-9 these factors for mentioned beams have been illustrated. Furthermore, the designers could use these factors for code level safety i.e. $\beta = 3.0$.

For rectangular beams with only tension rebars (SR), designers could directly apply the graphs of Figure 5 for their design purposes at desired safety index and loading ratios. According to the Figure 6, for rectangular beams with tension and compression rebars (DR), designers could directly use the graphs for their purposes for any safety index and loading ratios.

As seen from Figure 7 for DR beams for earthquake loading case, reliability-based design graphs for any desired safety index and loading ratios have been illustrated. The graphs are practically useful for risk-based designs of all DR beams and for any safety index values.

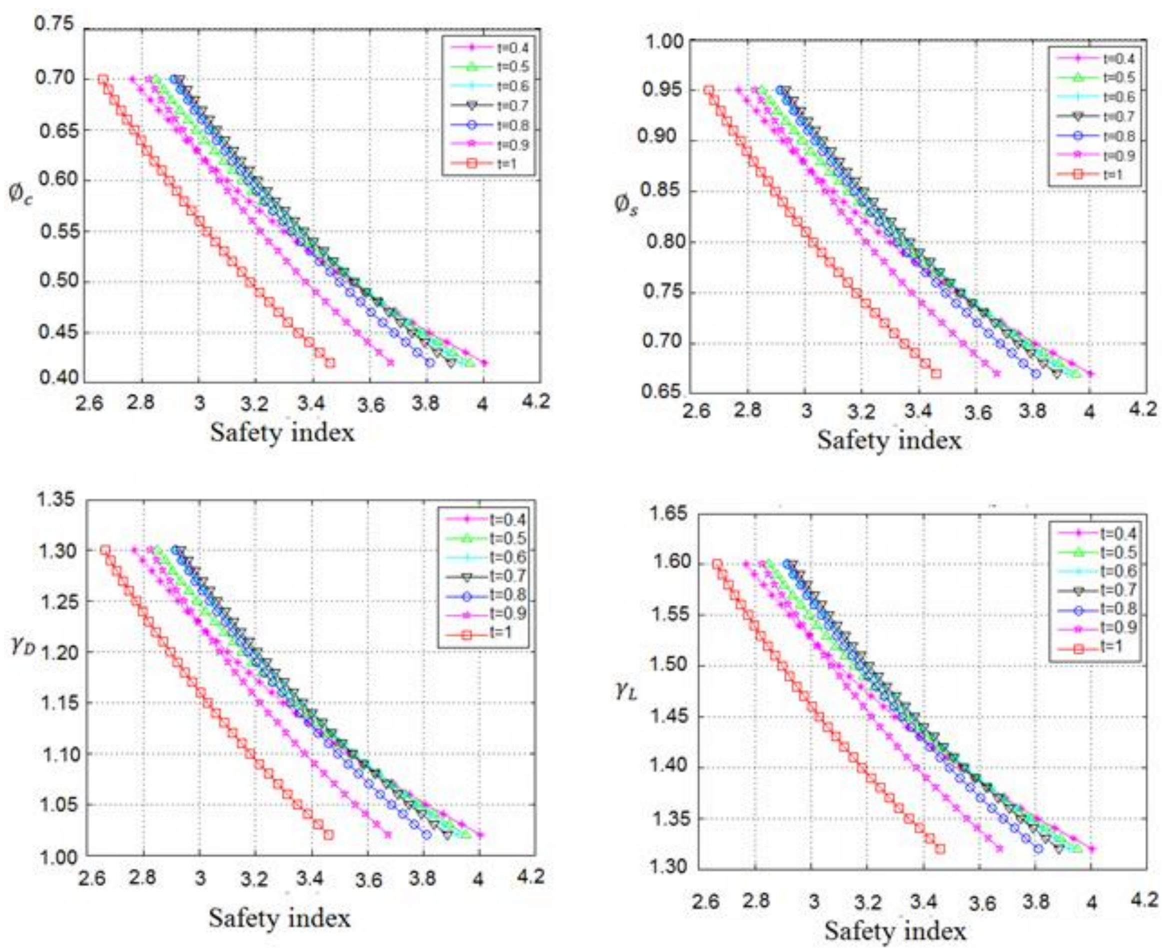

Fig. 5. Variations of $\phi_c,\phi_s,\gamma_D,\gamma_L$, versus β for SR beams for $t=M_D/(M_D+M_L)$

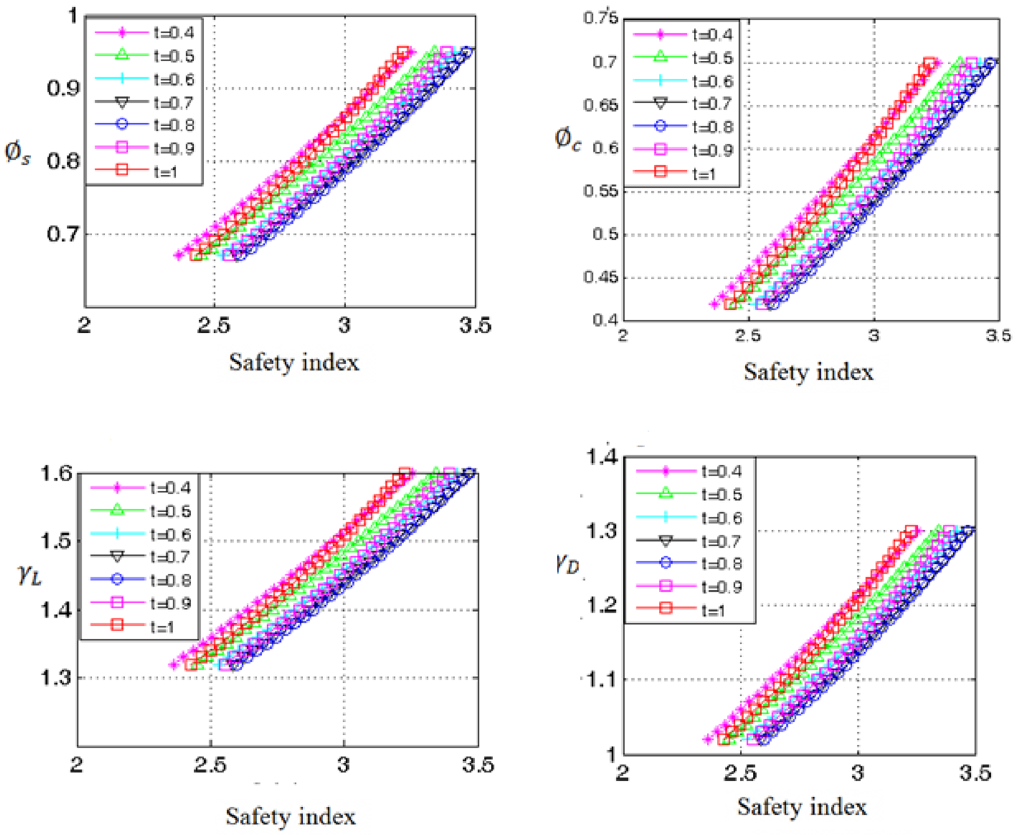

Fig. 6. Variations of $\phi_S, \phi_C, \gamma_L, \gamma_D$, versus β for DR beam, for $t=M_D/(M_D+M_L)$

Similar to the rectangular beams, according with Figure 8, for T-section beams; designers could directly use the graphs for their purposes for any safety index and loading ratios.

According to Figure 9, for T-shape beams, the factors for all types of loadings has been presented for any desired safety index and loading ratios. The graphs are applicable for probabilistic designing of T-sections beams and for any safety index values.

CONCLUSIONS

This study employs a methodology for probabilistic based designing of reinforced concrete beams for bending. For this purpose, a computer program has been developed using the MATLAB and Monte Carlo simulation technique has been utilized for reliability evaluation. Based on the results of the present study, the following conclusions and remarks are drawn:

- Results show that required steels are very sensitive to the loading ratios and the sensitivity of required reinforcements with respect to strength of concrete is negligible. Therefore, in practice, using high-strength concretes will not necessarily lead to the economic design, and more important parameter is loading ratio.
- Clearly, the loads and resistance factors are not constants, particularly for different values of load ratios. Therefore, for an economic design the users could design their projects for specific load ratio and desired safety level.
- Designers could realize their economic designs by selecting the desired safety index for each project for specific load and resistance factors based on the importance of each building.

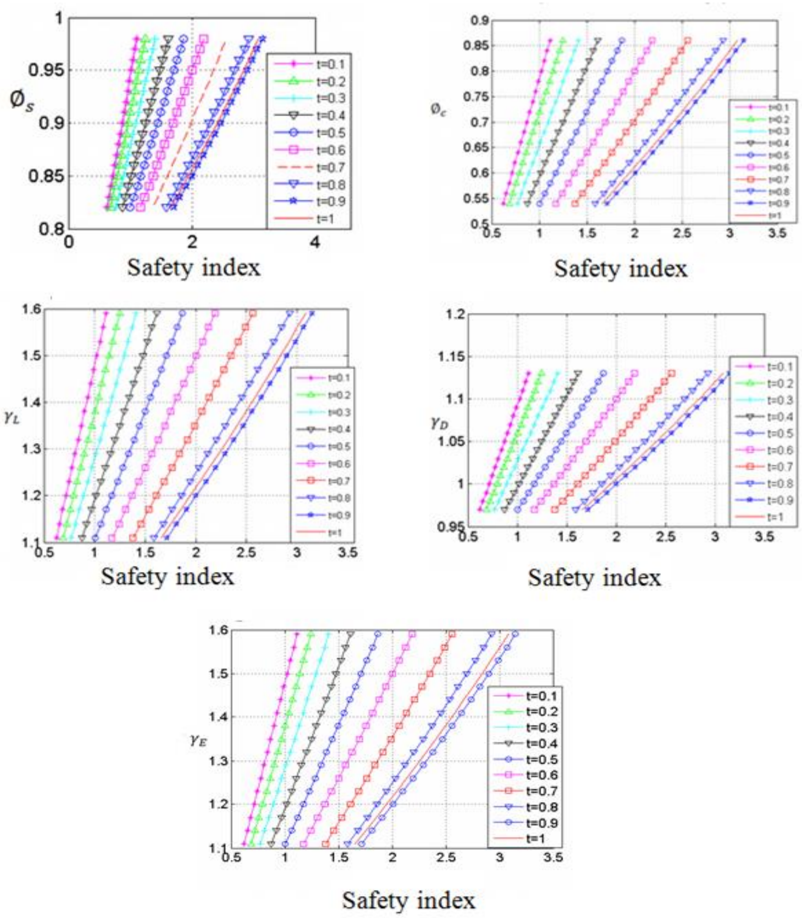

Fig. 7. Variations of $\phi_s, \phi_c, \gamma_L, \gamma_D, \gamma_E$, versus β DR beams for loading ratio $t=M_D/(M_D+M_E)$

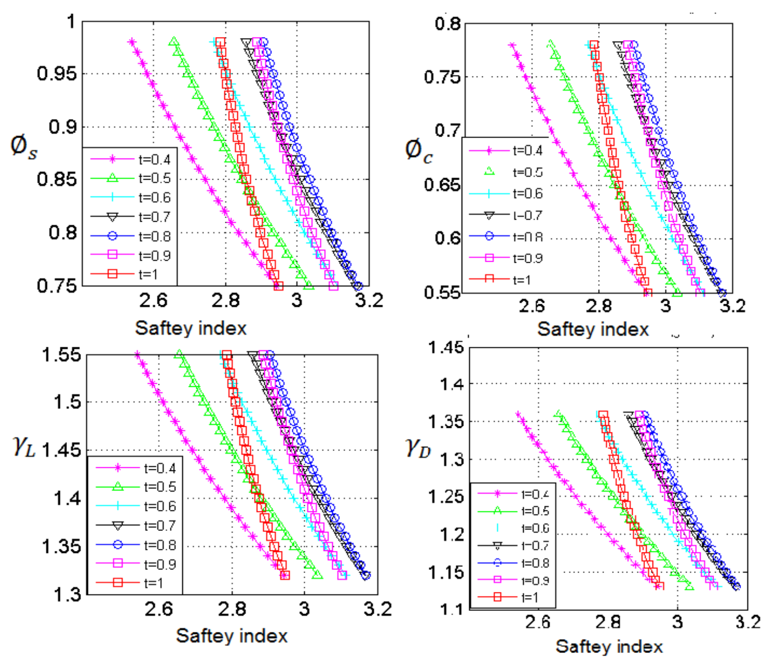

Fig. 8. Variations of $\phi_S, \phi_C, \gamma_L, \gamma_D$, versus β for T-shape beam for loading ratio $t=M_D/(M_D+M_L)$

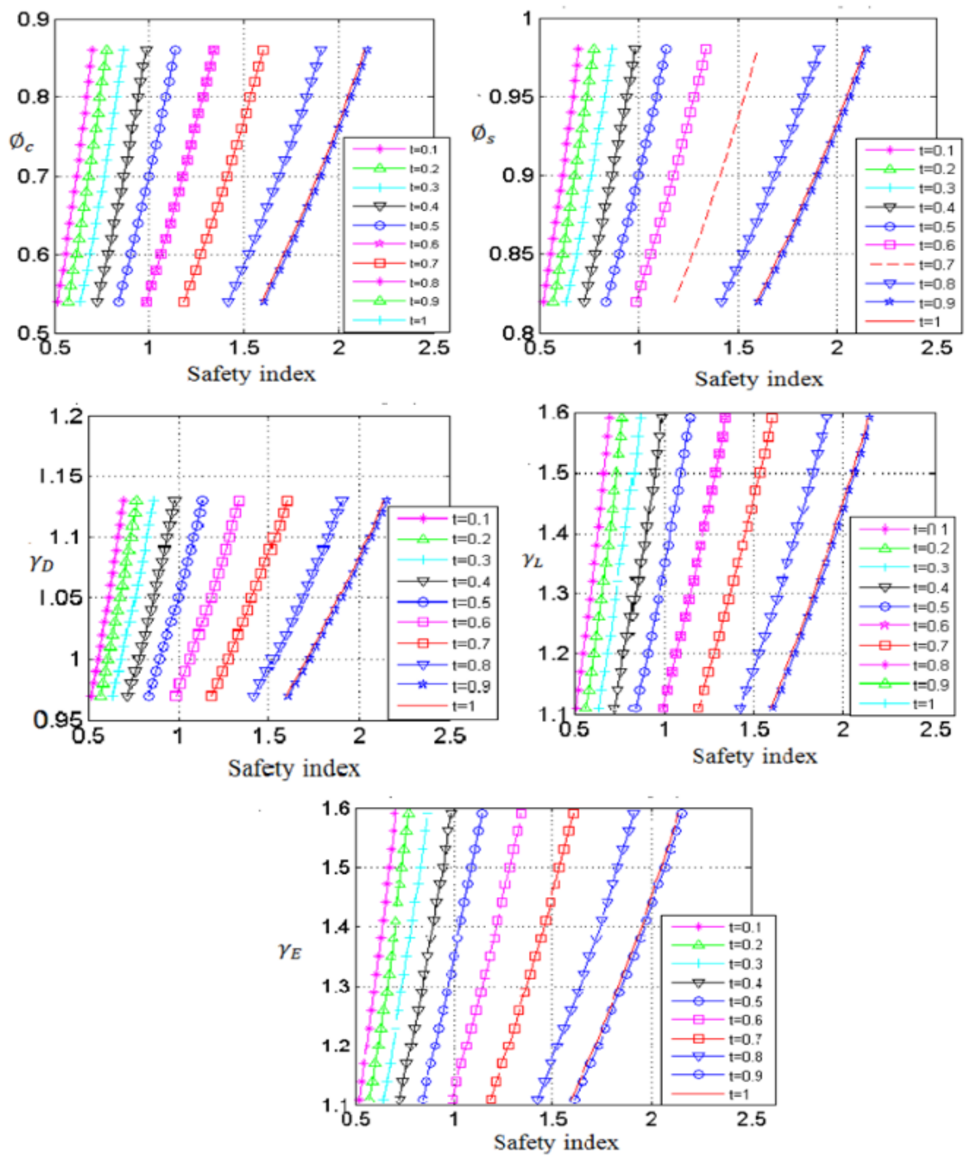

Fig. 9. Variations of $\phi_c, \phi_s, \gamma_D, \gamma_L, \gamma_E$, versus β for T-shape beam for $t=M_D/(M_D+M_E)$

- Using rectangular beams with compressive bars is better than T-shape beams.
- The designers could use the produced graphs for their requirements.
- Future investigations on the current field are therefore recommended. For this purpose, the calibrations of the load and resistance factors for combination of bending-shear, bending-torsion and bending-shear-torsion interactions are recommended to be conducted.

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