

## **Behavior of Piled Raft Foundation on Heterogeneous Clay Deposits Using Random Field Theory**

**Jamshidi Chenari, R.<sup>1\*</sup>, Ghorbani, A.<sup>1</sup>, Eslami, A.<sup>2</sup> and Mirabbasi, F.<sup>3</sup>**

<sup>1</sup> Associate Professor, Department of Civil Engineering, University of Guilan, Guilan, Iran.

<sup>2</sup> Ph.D. Candidate, Department of Civil Engineering, University of Guilan, Guilan, Iran.

<sup>3</sup> M.Sc., Department of Civil Engineering, University of Guilan, Guilan, Iran.

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**ABSTRACT:** In the case of problematic soils and tall buildings where the design requirements cannot be satisfied merely by a raft foundation, it is of common practice to improve the raft performance by adding a number of piles so that the ultimate load capacity and settlement behavior can be enhanced. In this study, the effect of spatial variability of soil parameters on the bearing capacity of piled raft foundation is investigated based on the random field theory using the finite difference software of FLAC<sup>3D</sup>. The coefficient of variation (COV) of the soil's undrained shear strength, the ratio of standard deviation to the mean, was considered as a random variable. Moreover, the effect of variation of this parameter on the bearing capacity of piled raft foundation in undrained clayey soils was studied taking the Monte Carlo simulation approach and the normal statistical distribution. According to the results, taking into account the soil heterogeneity generally results in more contribution of the raft in bearing capacity than that of the homogenous soils obtained by experimental relationships, which implies the significance of carrying out stochastic analyses where the soil properties are intensively variant.

**Keywords:** Bearing Ratio, Piled Raft, Random Field Theory, Spatial Variation, Undrained Shear Strength.

## **INTRODUCTION**

Piled raft foundations have been developed and widely used within the recent decades, as they are capable of carrying extreme loads and preventing excessive settlement in superstructures including high-rise buildings, bridges, power plants, etc. This type of composite foundation is a geotechnical construction, consisting of three elements of pile, raft and soil. The raft is commonly designed to be rigid so that it can withstand high amounts of moment and differential

settlement, which are a function of the load intensity and relative stiffness of the raft and the soil. The adoption of piled raft foundations in the design of pile groups is by no means new, and has recurrently been studied by many researchers. One of the main purposes of using piled raft foundations is to act as settlement reducer, in which the settlement is reduced to an allowable amount by redistribution of the load, in part to the piles and in part to the raft (Patil et al., 2014). This allows the piled raft design to be optimized and the number of piles be cost-

\* Corresponding author E-mail: jamshidi\_reza@yahoo.com

effectively reduced as a result of transferring a part of the building load into the deeper and stiffer layers of the soil.

Generally, when the piled raft system is used to transfer the load from the structure to the ground such that the contact between the soil and the raft is not disrupted under no circumstances, the load is distributed between the raft and the piles within a ratio called the “*bearing ratio*”. This parameter depends on some factors such as the material and stiffness of the raft, the material, diameter and length of the piles, the geometrical arrangement of piles and finally the constitutive model considered for the elements of the system including the soil, the raft and the piles. Being aware of the load sharing between the raft and the set of piles would be of particular importance as the key criterion in the piled raft design is the settlement of the system, comprising of two components associated with the contribution of the raft and that of the piles in the bearing capacity. Determination of such parameter can be really helpful for estimation of the foundation settlement and the design factor of safety (Saeedi and Fakher, 2014; Lee et al., 2014, 2015).

On the other hand, the spatial variability of the soil, as the main element of the load bearing mechanism, can clearly affect the relative rigidity of the soil and the piles, suggesting a dramatic effect on the load distribution between the raft and the piles. A great number of studies have been carried out regarding the problem of piled raft foundation, mainly focused on the bearing ratio parameter in homogenous soil, putting into practice various methods and approaches including analytical, physical and numerical modeling. However, in realistic conditions, the soil properties can be spatially variant due to different mechanisms of geological and environmental (Dasaka and Zhang, 2012; Ching and Phoon, 2013; Lloret-Cabot et al., 2014), which might lead to decrease of the bearing capacity, as a consequence of

changing the failure plane to asymmetric and following the weakest path (Popescu et al., 2005; Ahmed and Soubra, 2012). Therefore, it would be necessary to consider the spatial variability of the soil, so that a rational and economical design can be achieved (Salgado and Kim, 2014; Fan et al., 2014).

Poulos (2002) was first who provided a simple method for design of a piled raft system in clayey soils, including estimation of both overall and differential settlement of the foundation. Reul and Randolph (2003), using numerical analysis verified by field data, made comparison between the overall settlement, differential settlement and the bearing ratio carried by piles for a number of three case studies of piled raft foundations resting on over-consolidated clay and introduced these parameters to evaluate the performance of the piled-raft system.

In another study, they also performed numerical analysis of piled rafts system in over consolidated clay and showed that the interaction between piles and the raft plays a vital role in bearing capacity of this foundation system (Reul and Randolph, 2004). Lee et al. (2010), using 3D FEM, appraised the bearing capacity of square piled raft under vertical loading to evaluate the load bearing ratio and settlement of the raft and piles at the ultimate state. They finally concluded that both bearing capacity and settlement performance of the raft foundation could be improved using even a limited number of piles, provided that they are efficiently located. Bajad and Sahu (2008) investigated the influence of pile parameters such as length and number of piles on load distribution and settlement reduction through 1-g model tests on piled raft in soft clay. A series of numerical analyses were also conducted by Cho et al. (2012) to investigate the behavior of a square piled raft under vertical loading to evaluate the validity of 3D elasto-plastic FEM analysis with slip interface model at the pile-soil contact.

On the other hand, the heterogeneity of the soil, as the main essence of this paper, has rarely been studied so far. Niandou and Breyse (2007) conducted a reliability analysis taking the approach of plate on springs and Monte Carlo simulations to show how the horizontal soil variability can affect the soil-structure interactions in a piled-raft foundation. Haldar and Babu (2008a) also investigated the bearing capacity of piles based on CPT tests results considering vertical heterogeneity using finite difference method. In another study, they also studied the allowable capacity of laterally loaded pile in undrained clay with spatial variability of undrained shear strength, as a random variable. The analysis was performed in the framework of random field theory and the Monte Carlo simulation, considering the soil medium as two-dimensional non-Gaussian homogeneous random field (Haldar and Babu, 2008b). Besides, in the most recent and related study, Elahi (2011) investigated the effects of spatial variability of undrained shear strength and soil stiffness of naturally deposited clay on the bearing capacity of piled-raft foundations using the finite difference code of FLAC 2D, in three states of soft, medium and stiff textures in undrained condition. The analyses revealed a minor effect of the spatial variability of the soil properties on the pile and raft bearing ratios, as the load bearing is considered as an accumulation and integration of the stress elements in a stationary random field. However, a three dimensional investigation of such intrinsically three-dimensional structure is not found to consider the effects of different material and geometrical parameters on the load bearing behavior of the system.

In this study, numerical analyses in the form of Monte Carlo simulations (Kalos and Whitlock, 2008) are carried out using FLAC<sup>3D</sup>, to consider the effect of stochastic variability of the soil properties on the

bearing ratio of the raft and the piles through the random field theory, considering the coefficient of variation (COV) of undrained shear strength as a random variable. Besides, the results obtained by numerical analyses are compared to those of experimentally derived relationships in the literature, including PDR (Poulos and Davids, 2005; Randolph, 1992; Fleming, 2008).

## EXPERIMENTAL RELATIONSHIPS

Poulos and Davids (2005) and Randolph (1992) proposed a simplified analytical method called PDR with the assumptions of rigid cap and linear behavior of the soil, resulting in an approximately linear behavior of the system for loads exceeding the working load. Baziar et al. (2009) using small-scale model test and three-dimensional analysis of pile-raft foundation on medium-dense sand concluded that the obtained results indicated a better performance comparing to PDR method to predict real bearing capacity of piled-raft foundation on medium sandy soils for loads higher than the working load.

In the PDR method, the proportion of the total applied load carried by the raft can be estimated using Eq. (1). The contribution of the piles can then be calculated by subtracting the load portion of the raft from the total load.

$$\frac{P_r}{P_t} = \frac{K_r (1 - \alpha_{cp})}{K_p + K_r (1 - \alpha_{cp})} \quad (1)$$

$$\alpha_{cp} = 1 - \frac{\ln(r_c/r_0)}{\xi} \quad (2)$$

$$r_c = \sqrt{\frac{B \times B}{\pi n}} \quad (3)$$

where  $P_t$  and  $P_r$  are the total applied load and the load bearing contribution of the raft,  $K_p$  and  $K_r$  are the stiffness of the pile group and the raft, respectively. Besides,  $\alpha_{cp}$  is the raft–pile interaction factor in which  $r_c$  and  $r_0$  denote the average radius of pile-cap,

(corresponding to an area equal to the raft area divided by number of piles) and radius of the pile, respectively.  $\xi$ : is the ratio of the soil's young modulus at the level of pile tip to that of the bearing stratum below the pile tip.  $B$  is the dimension of the cap (raft) and  $n$  is the number of piles.

In the Fleming method, a similar approach to that of PDR is followed, except that the coefficient of pile-raft interaction ( $\alpha_{cp}$ ) is determined using Eq. (4). Besides, the contribution of the raft in the bearing capacity can be estimated as a fraction of the total load using Eq. (6).

$$\alpha_{cp} = \frac{\ln\left(\frac{r_m}{r_c}\right)}{\ln\left(\frac{r_m}{r_o}\right)} \quad (4)$$

$$r_m = 2.5L_p(1 - \vartheta) \quad (5)$$

$$\frac{P_r}{P_p + P_r} = \frac{K_r(1 - \alpha_{cp})}{[K_p + K_r(1 - 2\alpha_{cp})]} \quad (6)$$

where  $r_m$  and  $L_p$ : are the effective diameter and length of the pile, respectively, and  $\vartheta$ : is the poisson's ratio of the soil (Poulos, 2002; Poulos et al., 2005).

## UNCERTAINTY HETEROGENEITY

## AND

In most studies, soil is considered either homogeneous or layered. In the case of layered soil, the mean value of the parameter through each layer is uniformly assumed as the geotechnical properties of the whole layer. However, in some cases, the geotechnical properties within “so-called” homogeneous layer of the soil are so variant that they change over short distances. Under such circumstances, prediction of the soil-structure behavior in real conditions requires accurate modeling of geotechnical properties of the system (Phoon and Kulhawy, 1999; Haldar and Babu, 2008; Kenarsari et al.,

2011).

In geotechnical problems, uncertainties can be generally divided into two main categories; 1) the inherent uncertainties which cannot be avoided, 2) the extrinsic uncertainties which consist of statistical, computational and estimation errors due to the lack of information on a variable or a system. Usually, the soil properties are affected by both uncertainties in a “so-called” uniform layer. On the other hand, considering engineering problems, it is essential to take into account the effects of uncertainties in the analysis so that a slapdash design would be avoided (Morse, 1971). The mechanical properties of the soil are typically inherently heterogeneous and non-deterministic. The variation of the soil characteristics in depth, can be presented by fitting a deterministic function (linear, parabolic or exponential) with having the residual components fluctuating around the trend in depth. This kind of heterogeneity is commonly investigated by Mont Carlo simulation, as the case of this study.

Theory of random fields can be used to model uncertainties in geotechnical engineering problems. A random field is a generalization of a stochastic process in which the parameters are not necessarily a simple real or integer, as they can also be multidimensional vectors (Vanmarcke, 2010). This principle is of great use in studying natural processes by the Monte Carlo method, in which the random fields are in accordance with the naturally spatially varying properties, such as soil permeability within the scale of meters, or concrete strength over the scale of centimeters. To have a glimpse of this theory, suppose a parameter, say the temperature  $Y$  in a room at position  $x$  and time  $t$ , is to be measured. The temperature can be described by Eq. (7), due to the fact that every measurement will be error-prone.

$$Y(x, t) = \mu(x, t) + \varepsilon(x, t) \quad (7)$$

where  $\mu$ : is the unknown signal of temperature and  $\varepsilon$ : is the measurement error. The measurement error  $\varepsilon(x, t)$  can be modelled as a random variable. So, at each point  $(x, t)$ , measurement error is a random variable. A *stochastic process* is a collection of random variables, and a stochastic process indexed by a spatial variable is called a *random field* (Adler, 2010).

Statistically speaking, it is necessary and sufficient to have three parameters to describe the stochastic characteristics of a soil, including the mean value, the coefficient of variation (COV), and the scale of fluctuation. The COV is defined as the ratio of standard deviation to the mean value, while the fluctuation scale refers to the distance in which soil parameters are significantly correlated. The following simple model, defined based on the random fields theory, can be used to describe variation of behavioral parameters with depth (Phoon and Kulhawy, 1999):

$$k(z) = t(z) + w(z) + e(z) \quad (8)$$

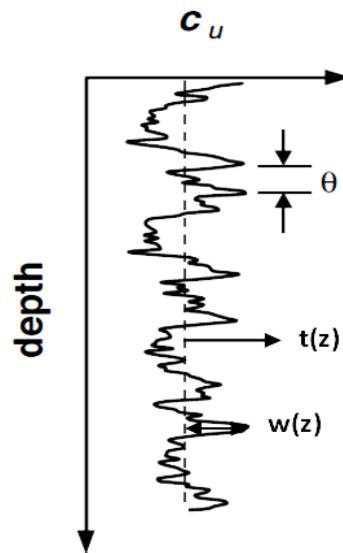
where  $t(z)$ : is the deterministic trend,  $w(z)$ : is

the stochastic component, and  $e(z)$  is the measurement errors. Figure 1 depicts an example of this model regarding the variation of undrained shear strength ( $C_u$ ) by depth in which the principal components to describe soils heterogeneity are shown, ignoring the measuring errors  $e(z)$ . The parameter  $\theta$  is called the scale of fluctuations and is defined as the distance through which the target parameter, say  $C_u$ , is varied. In this model,  $t(z)$  is assumed to be constant with depth and the stochastic component,  $w(z)$ , is normally distributed with a constant mean value to simplify the analysis (DeGroot, 1996).

## MONTE CARLO SIMULATION

As mentioned earlier, the finite difference method (FDM) in conjunction with Monte Carlo simulation were put into practice in this study to investigate the effects of variability of geotechnical properties on the bearing capacity of piled raft foundations.

There are numerous methods for analysis of uncertainties in engineering problems. In a general classification, these methods can be categorized into three groups: analytical methods, approximate methods and Monte Carlo simulation (Tung and Yen, 2005).



**Fig. 1.** Variation of undrained shear strength in depth (Phoon and Kulhawy, 1999)

Due to making mathematical assumptions to simplify the problem, analytical methods are considered to be more computationally effective, although the analysis might become extremely difficult or unrealistic when multiple input variables in a complex system are correlated (Papadopoulos et al., 2001). The approximate methods such as the first-order second-moment method (FOSM) and point estimate method (PEM) are usually based on making an approximate description of the statistical properties of output random variables.

Monte Carlo simulations are called to a set of algorithms based on random sampling to solve problems associated with uncertainty analysis, especially in physical and mathematical problems. This method has widely been used in geotechnical engineering where probability analysis is required. Zhang et al. (2011), Jamshidi and Mahigir (2014), Jiang et al. (2014), Husain et al. (2016) and Jamshidi and Behfar (2017) are just some examples of using Monte Carlo simulations in geotechnical engineering practice. Monte Carlo methods can be varied in algorithm, but tend to be performed under the following general pattern (Kalos and Whitlock, 2008):

- a) Determining the domain of possible inputs.
- b) Generating random inputs based on a probability distribution function (PDF) over the domain.
- c) Carrying out a deterministic computation on the inputs.
- d) Combining the results.

The first step in Monte Carlo simulation in this study is to determine the stochastic properties of the soil including the coefficient of variation, correlation structure, and finally the probability distribution function of the soil's property under study.

In this study, elasticity modulus was taken as representative of the soil's stiffness to be used in the elastic and Mohr-Colomb models. The assumption is that the elasticity modulus

is fully correlated to the undrained cohesion of the soil, obtained for each element by multiplying the random values of cohesion by a constant value, say  $\alpha$ , as follows:

$$E = \alpha c_u \quad (9)$$

Popescu et al. (2005) suggested the value of  $\alpha$  varying between 300 and 1500 for clays. In the current study, the  $\alpha$  coefficient was taken constant at 850 for all analyses. Besides, the values of 700, 900 and 1100 for  $\alpha$  were considered for constant COV = 50% to study the effect of this parameter on the results.

It is worth mentioning that carrying out stochastic analyses, it is concluded that the COV of undrained shear strength varies in the range of 30-50 % and 60-85 % for clayey deposits with regular and intensive variability of properties, respectively. A range of 28 to 96% is also reported for the COV of undrained shear strength (Matsuo and Kuroda, 19740).

The internal friction angle of the soil was assumed zero ( $\phi = 0$ ) to take into account the undrained behavior of clay, implying consideration of the undrained cohesion of the soil in the analysis procedure. The dilation angle was assumed to be zero ( $\psi = 0$ ) as well, since clays do not show tendency to dilate except for over consolidated soils, not the case of this study. Furthermore, the tensile strength of the soil was conservatively considered as zero ( $\sigma^t = 0$ ).

The last but not least to be determined, is the scale of fluctuation ( $\theta$ ) or the correlation length (L), defined as the average distance between two successive peaks of variations of the soil's undrained shear strength, for which some research have already been conducted via in situ and laboratory tests. Such research indicated that  $\theta_v$  and  $\theta_h$ , the fluctuation scale in the vertical and horizontal directions, vary between 0.5 m to 6 m (with main contribution of variations within 1 to 2 m) and 40 to 60 m,

respectively (Morse, 1971). Similarly, the results obtained by in situ tests of CPT suggested a range of 1-3 m and 5-38 m for  $\theta_v$  and  $\theta_h$ , accordingly (Matsuo and Kuroda, 1974). Besides, Jamshidi Chenari and Alaei (2015) using the finite difference program of FLAC 5.0 along with the random field theory investigated the effect of variations of undrained shear strength on the slope stability analysis and showed that the coefficient of variation and anisotropy of undrained shear strength could majorly affect the reliability of design in terms of factor of safety. Table 1 summarily presents the employed parameters along with their range of variation considered in this study.

**Table 1.** Variation range of modelling parameters

Parameters	Variation Range
Mean undrained shear strength, $\mu_{cu}$ (kPa)	25
Coefficient of Variation, COV (%)	10, 50, 90
Scale of fluctuation, $\theta$ (m)	1, 10 , 100
Poisson's ratio, $\nu$	0.45

### Geometry of Model

The geometry of the model is initially drawn and the elements are selected to build the desired model. Clearly, the mesh size should be fine enough to meet the basic requirements of the correlation structure so that the accuracy of the analysis is assured. On the other hand, the element size is dependent on the correlation length of the soil properties, taken 1 meter in this study. The elastic model for the soil and the Mohr-Colomb model for the foundation system were employed as the governing rule of material behavior.

Soil-structure interaction (SSI) effect might be considered as the most complex and controversial issue in the behavior of piled raft foundations (Bourgeois et al., 2012; Nguyen and Kim, 2013; Li et al., 2014; Albusoda and Salem, 2016). In order to take into account the different aspects of the SSI

effect, FLAC<sup>3D</sup> provides interfaces characterized by Coulomb sliding and/or tensile and shear bonding via a set of triangular elements (interface elements), each of which defined by three nodes (interface nodes).

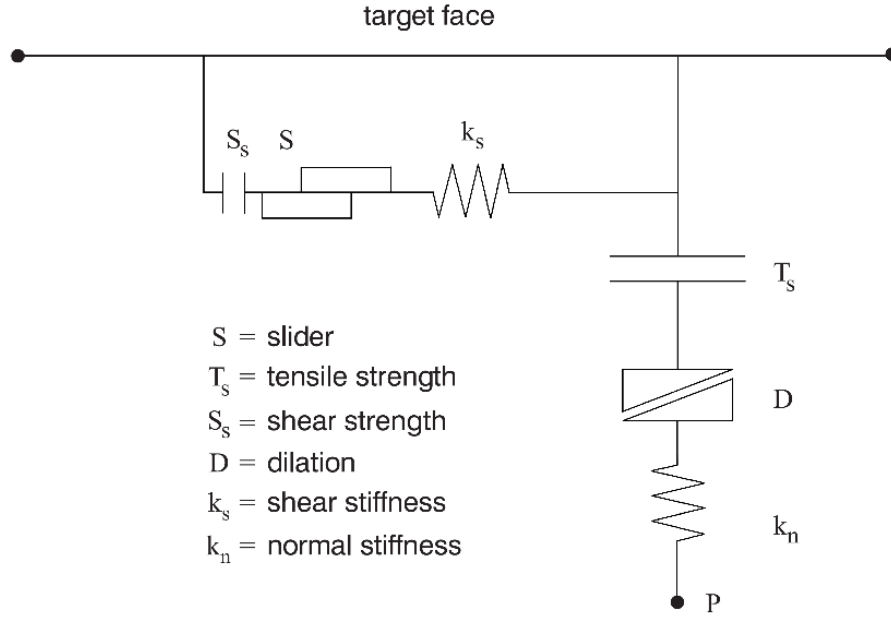
The fundamental contact relation is defined on the “target face”, i.e. the plane between the interface node and a zone surface face, by which the normal direction of the interface force is specified. For each interface node and its contacting target face, the absolute normal penetration and the relative shear velocity are calculated during each time-step to be used by the interface constitutive model, so that the normal force and the shear force vector can be determined.

Linear Coulomb shear strength criterion is used as the constitutive model to bound the shear force acting at an interface node, normal and shear stiffness, tensile and shear bond strengths, and the dilation angle that give rise to the effective normal force on the target face, followed by reaching the shear-strength limit (Itasca, 2009). Figure 2 demonstrates the schematic diagram of this constitutive model.

The contact surface, detected at the interface node, is characterized by normal and shear stiffnesses,  $k_n$  and  $k_s$ , respectively. As recommended by the Flac3D manual,  $k_n$  and  $k_s$  are set to ten times of the equivalent stiffness of the stiffest neighboring zone (Itasca, 2009). The apparent stiffness (expressed in stress per distance units) of a zone in the normal direction is defined as:

$$k_n = \max \left[ \frac{\left( K + \frac{4}{3} G \right)}{\Delta z_{\min}} \right] \quad (10)$$

where  $K$  and  $G$ : stand for the bulk and shear modulus, respectively and  $\Delta z_{\min}$ : is the smallest width of an adjoining zone in the normal direction (Itasca, 2009).

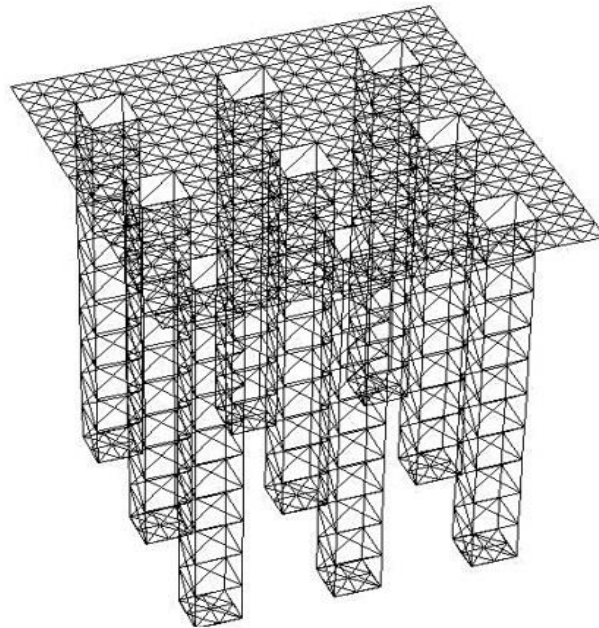


**Fig. 2.** Components of the interface constitutive model (Itasca, 2009)

Figure 3 shows the interface elements around the piles and under pile tips, considered to take into account the effects of interactions between the soil, pile and raft, by considering two sets of interface elements for each pile; one for the pile skin and the other for the pile tip.

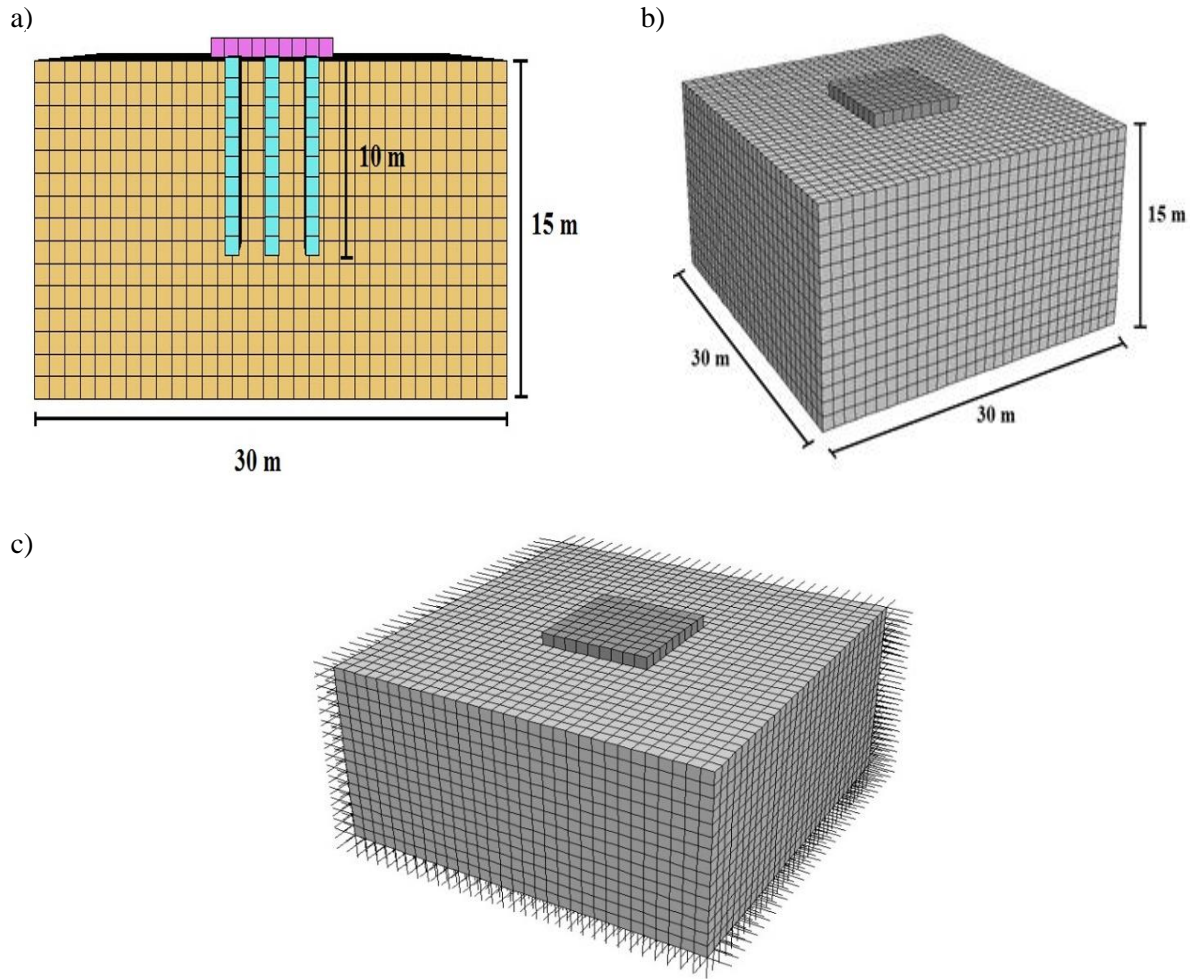
The overall dimension of the model is 30 m  $\times$  30 m  $\times$  15 m along with nine pile

elements of 10 m length and cross section of 1 square meter. The raft dimension was taken as 9 m  $\times$  9 m  $\times$  1 m, placed above the piles (Figure 4a,b). The lower boundary was constrained to the horizontal and vertical directions while the lateral boundaries were allowed to move only in the vertical direction, as shown by Figure 4c. Table 2 presents the model parameters considered in this study.



**Fig. 3.** Interface elements of the proposed model





**Fig. 4.** Numerical model: a,b) Geometry of the model, c) Applied boundary conditions

**Table 2.** Model parameters in this study

	Cohesion (kPa)	Elasticity Modulus (MPa)	Poisson's Ratio	Shear Modulus (MPa)	Bulk Modulus (MPa)	Dimension (m)			Interface Normal Stiffness (MPa)	Element Shear Stiffness (MPa)
						Length	Width	Depth		
Soil	25	21	0.45	7.2	70	30	30	15	1.4 e8	1.4 e8
Pile	-	25000	0.2	10400	13900	10	-	-	-	-
Raft	-	25000	0.2	10400	13900	9	9	1	-	-

## Analysis and Results

### Deterministic Analysis

As the next step in Monte Carlo simulations, a deterministic simulation of the piled raft model resting on homogeneous soil layer with zero COV was carried out. The unbalanced forces of grid points were considered as the failure criterion by reaching

values less than  $10^{-6}$  after some specific steps. Figure 5 presents the load-settlement curve of the piled raft system in the deterministic case, which levels off at an ultimate total load of 1000 kPa, and reaches plastic flow at settlement of about 0.5 m. One third of the total bearing capacity of this case, equals to 333 kPa, was considered as the service load.

The contribution of each pile in the

bearing capacity is presented in Table 3. It should also be noted that the load bearing contribution of the raft in such deterministic case is approximately 28%, distinguished by a Fish code written in FLAC<sup>3D</sup>.

### Random Field Model

The spatial random field model of undrained shear strength can be expressed by log-normal distribution and point statistical analysis, in which the position and width of such distribution are indicated by the mean value ( $\mu_{cu}$ ) and the standard deviation ( $\sigma_{cu}$ ) of cohesion. The choice of log-normal distribution is considered in this study due to the fact that the undrained shear strength is a strictly non-negative quantity, making it easily transformed to the normal distribution. The COV of undrained cohesion is defined as follows:

$$COV = \frac{\sigma_{cu}}{\mu_{cu}} \quad (11)$$

Having determined the properties of the random field, the correlated fields are produced based on the algorithm of matrix decomposition. The random field of log-normally distributed,  $c_u(\tilde{x})$  can be defined as the following equation:

$$c_u(\tilde{x}) = \exp\left(L \cdot \mathcal{E}_{inc_u}(\tilde{x}) + \mu_{inc_u}(\tilde{x})\right) \quad (12)$$

where  $\tilde{x}$  : is the spatial position of the random variable,  $\mu_{inc_u}(\tilde{x})$  : is the mean of the logarithm of the undrained shear strength field (Eqs. (13) and (14)),  $\mathcal{E}_{inc_u}(\tilde{x})$  : is the uncorrelated standard normal random field. Besides,  $L$ : is the lower-triangular matrix, found by decomposition of the covariance matrix using Cholesky decomposition technique (Eq. (15)).

$$\sigma_{inc_u}^2 = \ln \left( 1 + \left( \frac{\sigma_{cu}}{\mu_{cu}} \right)^2 \right) = \ln(1 + COV^2) \quad (13)$$

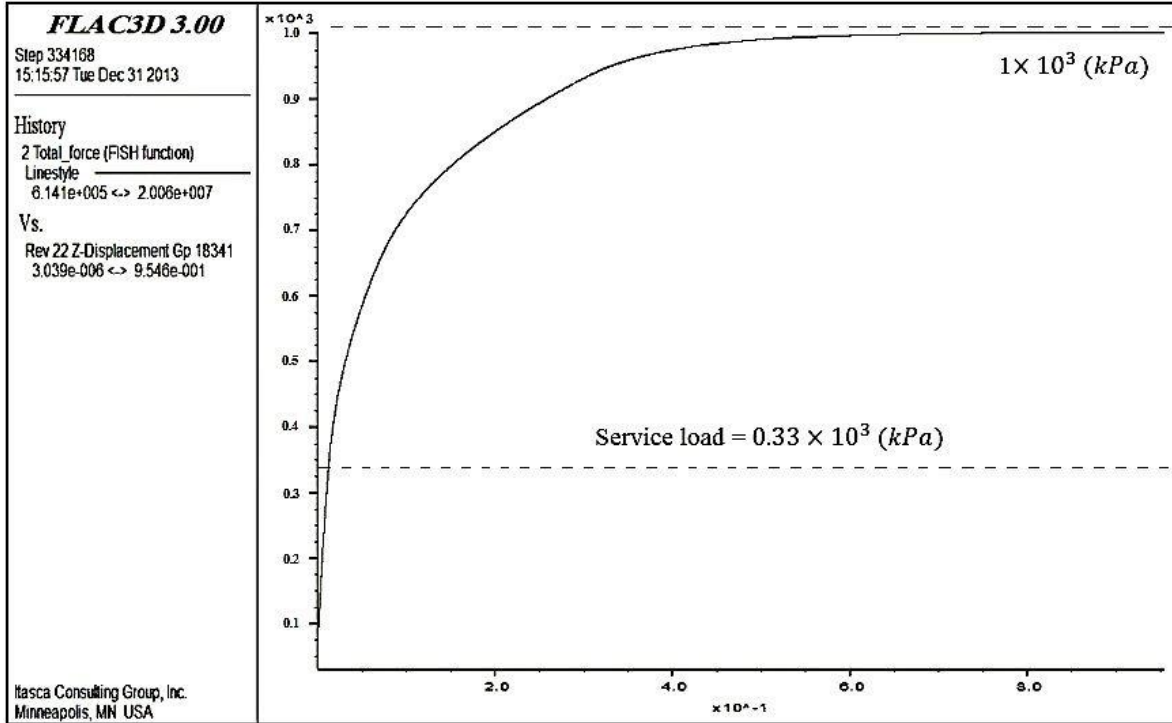


Fig. 5. Bearing capacity of piled raft resting on a homogeneous soil stratum

**Table 3.** Load bearing contribution of piles in deterministic case

No. of Pile	1	2	3	4	5	6	7	8	9
Load bearing contribution (%)	8.68	7.75	8.73	7.52	6.81	7.52	8.73	7.75	8.72

$$\mu_{lnc_u} = \ln(\mu_{c_u}) - \frac{1}{2} \sigma_{lnc_u}^2 \quad (14)$$

$$A = L.L^T \quad (15)$$

where A: is the covariance matrix, stated as below:

$$A = \sigma_{lnc_u}^2 \rho_{lnc_u}(\tilde{x}) \quad (16)$$

$\rho_{lnc_u}(\tilde{x})$ : is the spatial autocorrelation function, presented in the covariance matrix by Gauss-Markov exponential decay correlation function, formatted as the following equation:

$$\rho_{lnc_u}(\tilde{x}) = \exp\left(-\frac{2|\tilde{x}|}{\theta_{lnc_u}}\right) \quad (17)$$

where,  $\tilde{x}$ : is the distance vector between two given spatial points and  $\theta$ : is the scale of fluctuation of the undrained cohesion field. Eq. (17) indicates the correlation with regard to the normal distribution field. Therefore,  $\rho_{lnc_u}(\tilde{x})$ : is the correlation coefficient between  $lnc_u(x)$  and  $lnc_u(x')$  in two given points of the normal field with spatial distance vector of  $\tilde{x}$ . The real correlation between different points in the field of  $c_u(x)$  can be attained by the following transfer function:

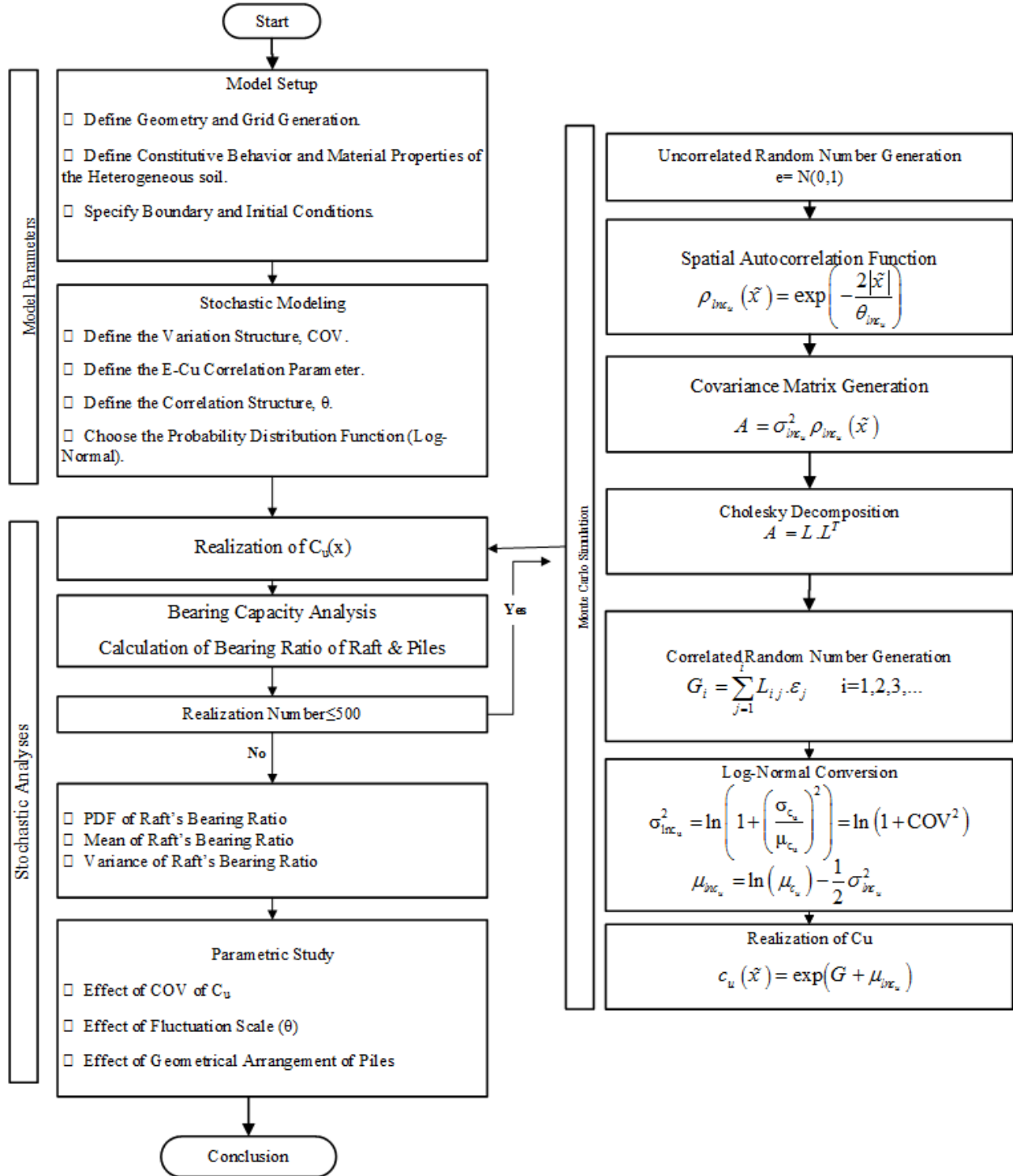
$$\rho_{c_u}(\tilde{x}) = \frac{\exp(\rho_{lnc_u}(\tilde{x}) \sigma_{lnc_u}^2 - 1)}{\exp(\sigma_{lnc_u}^2)} \quad (18)$$

To implement the random field model, the covariance matrix (Eq. (15)) should be initially produced considering Eq. (17), which  $\tilde{x}$ : is the center to center spatial

distance of the elements. Having produced and decomposed the covariance matrix into two lower-triangular and upper-triangular matrices using Choleski technique of matrix decomposition, the random field of  $c_u$  can be produced using Eq. (12). In this study, the whole aforementioned procedure was carried out by programming in Fortran-90 due to the massive calculation required.

At first, the coordinates of each element was determined in FLAC-3D and then the input parameters including the COV, the poisson's ratio, the correlation length (assumed equal in three directions), cohesion (the undrained shear strength) were assigned to calculate the bulk and shear modulus for each zone. The random cases were produced using MATLAB program to have a number of 500 random values of bulk and shear modulus for each element. Finally, the output of Fortran-90 was imported to FLAC-3D to carry out the bearing ratio calculation for the raft and the piles taking into account the heterogeneity of the soil. Figure 6 presents the flowchart of the whole procedure performed in the random field modeling via Mont Carlo simulation in this study.

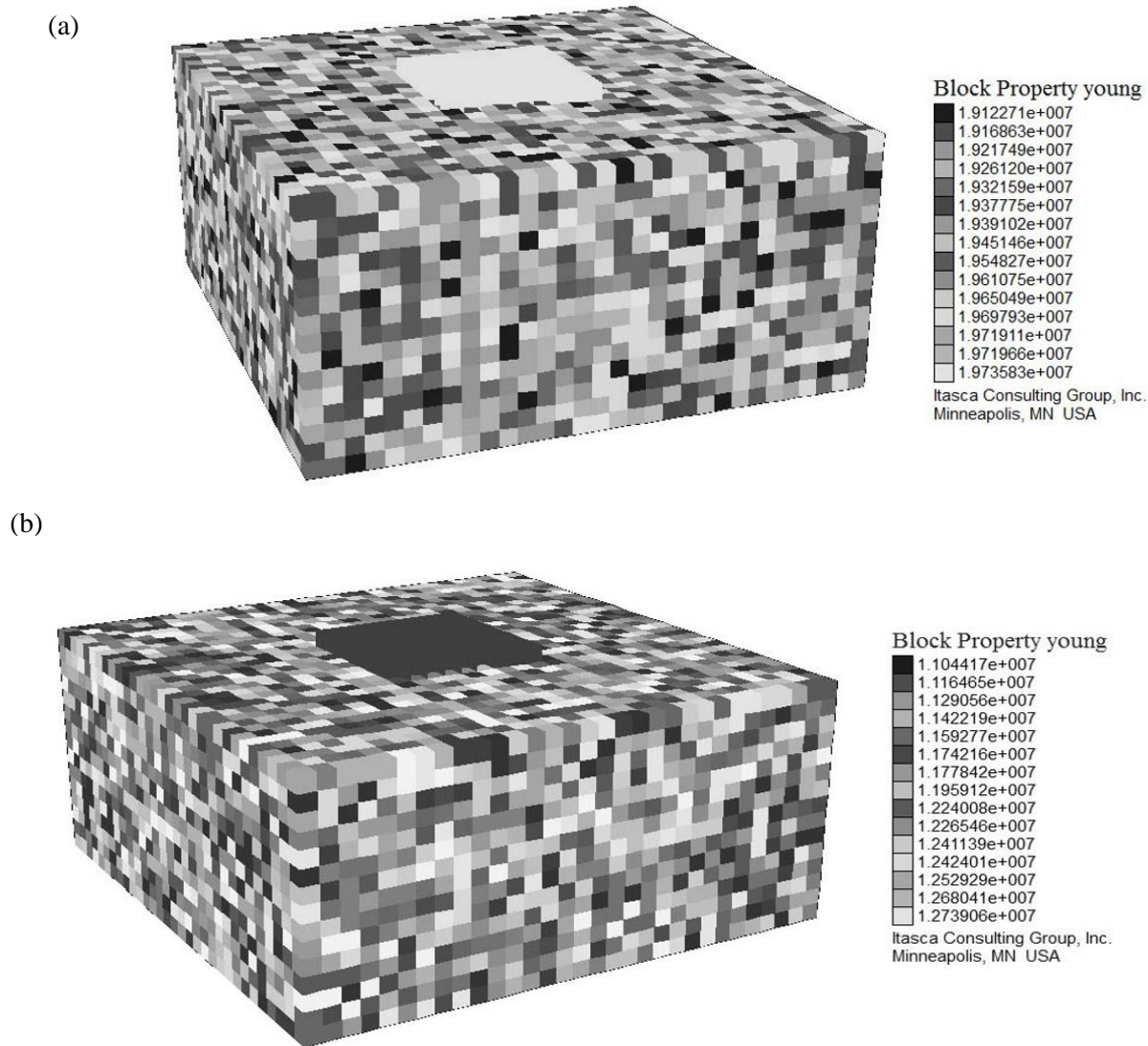
It should be noted that the result of one realization would not give an exact solution, as the soil parameters are randomly distributed through the zones, leading to change of the bearing ratio. Therefore, it would be rational to produce a large number of realizations and consider the average value as the final answer to the problem (Monte Carlo method). On the other hand, increasing the number of realizations should be limited to keep the calculation time reasonable. In this study, a number of 500 realizations were considered through a trial and error procedure, so that the desired precision can be assured.



**Fig. 6.** Flowchart of random field analysis using Monte Carlo simulation

Figure 7, illustrates an example of a proposed model with a mean undrained shear strength of 25 kPa, manifesting the variation of Young modulus in three-dimensional space. In this figure, the darker parts represent

lower values while the brighter ones represent higher values. As can be clearly noticed, the distribution and scattering of colors, i.e. the variation of the Young modulus within the soil body, increases with increase of  $COV_{Cu}$ .



**Fig. 7.** Spatial variability of Young's modulus for correlation length of  $L = 5$  m: a)  $COV = 10\%$ , b)  $COV = 90\%$

## PARAMETRIC STUDY

A parametric study was carried out to investigate the effect of stochastic parameters including the correlation length, the  $COV$  of undrained shear strength, and the geometrical arrangement of the piles on the probability distribution of the raft's bearing ratio as well as the piles' contribution.

a) Effect of  $COV$ : Figure 8 presents the Probability Distribution Function (PDF) of the raft's bearing ratio in a constant correlation length so that the effect of the variation of  $COV_{Cu}$  on the distribution

function can be observed. Accordingly, increasing the  $COV_{Cu}$  generally leads to an increase of the width of PDF, i.e. the scattering of the values around the mean value, for a constant scale of fluctuation. As can be observed, the cap's bearing ratio (CBR) generally decreases with increase of  $COV$ , with the maximum and minimum peaking at  $COV$  equal to 10% and 90%, by giving a mean CBR of 29.5 and 28.4, respectively. However, the decline ratio of the trend is slowed down after  $COV$  of 50.

On the other hand, the results of calculations using experimental relationships



including the PDR and Fleming et al. (2008) for homogenous soil are superimposed to this figure for the sake of comparison. As can be observed, a remarkable discrepancy is clear between the results of this study for the heterogenous soil with that of calculated by both aforementioned relationships for homogenous soils, manifesting the significance of taking into account the heterogeneity of the soil properties. However, Fleming method gives more consistent results to the heterogenous soils compared to PDR with a discrepancy of 18 and 33 percent, respectively.

b) Effect of the scale of fluctuations ( $\theta$ ): Figure 9 presents the probability distribution function (PDF) of the raft's bearing ratio for different values of scale of fluctuation ( $\theta$ ) and constant COV of 10%. It suggests that the mean cap bearing ratio declines with the scale of fluctuations up to 10 m ( $\theta/L_p = 1$ , where  $L_p$  is the pile length), a trend which reverses afterwards, such that it stands at 29.5, 28.9 and 29.2 for the scale of fluctuations of 1, 10 and 100 m, respectively. It can be said that this parameter generally shows a minimal effect on the mean CBR, although a turning point is observed.

Besides, the results of calculations by conventional method are superimposed to Figure 9, revealing a discrepancy of 15 and 34 percent for Fleming and PDR, respectively. Similar to the effect of COV on the cap bearing ratio, Fleming method shows a closer consistency with the results of this study for heterogeneous clay rather than PDR, for the effect of scale of fluctuation.

c) Effect of geometrical arrangement of piles: The bearing ratio of each pile is depicted by Figure 10 to investigate the load bearing contribution of different piles according to their geometrical arrangement. Clearly, the corner piles have the most load bearing contribution while the central pile plays the minimum role in undergoing the load which can mainly be put down to the axis-symmetrical nature of the model. Furthermore, piles in the same geometrical position have the same bearing ratio, implying that heterogeneity of the soil properties does not lead to heterogeneous load distribution between piles. This behavior might be attributed to the integration effect of overlaying cap, which redistributes and balances the applied load within piles according to their geometrical position.

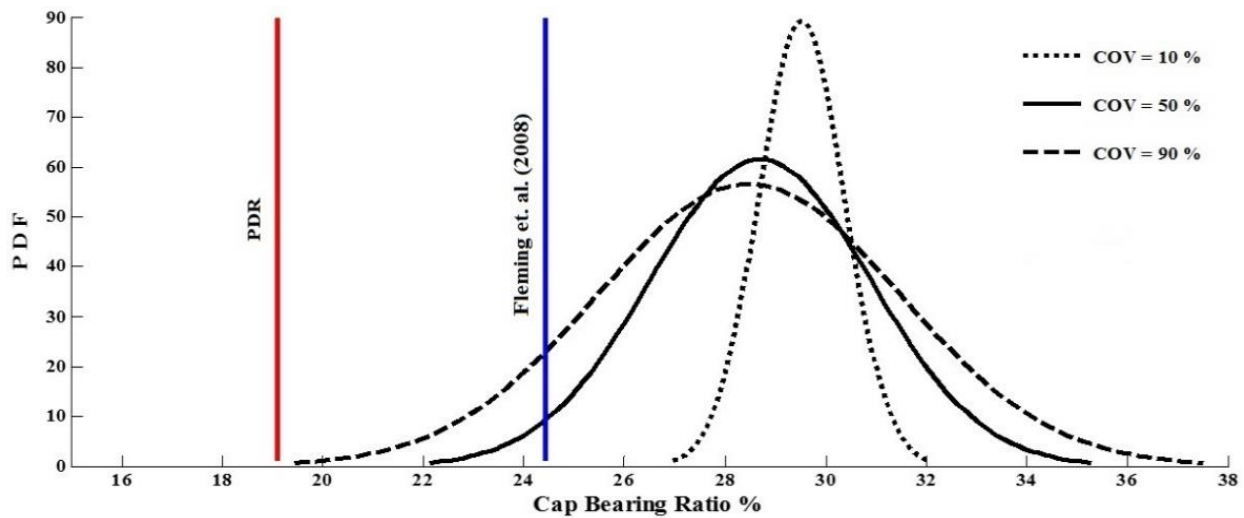


Fig. 8. Probability distribution function of the cap's bearing ratio,  $\theta = 2$  m

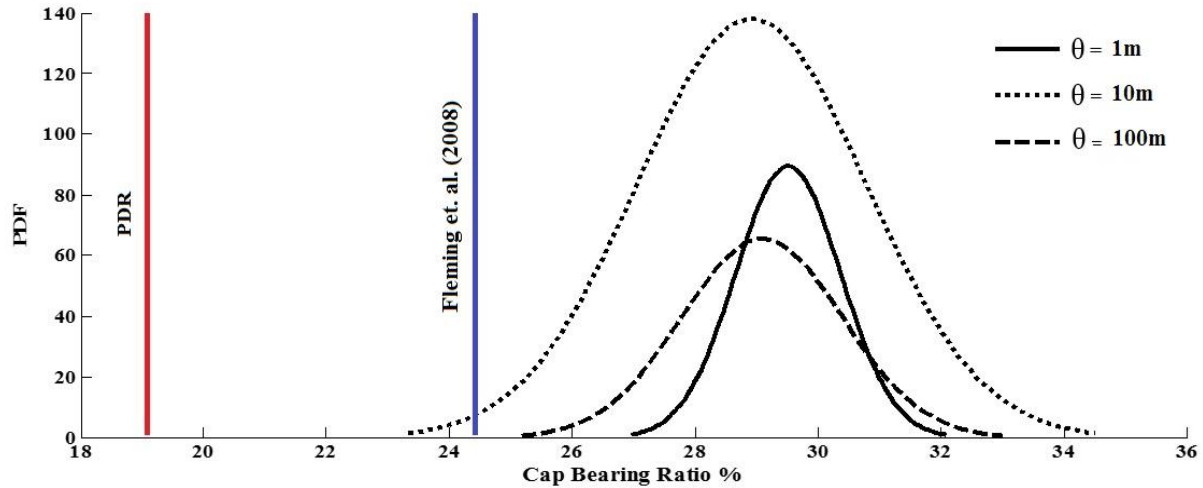


Fig. 9. Probability distribution function of the cap's bearing capacity ratio,  $COV_{cu} = 10\%$

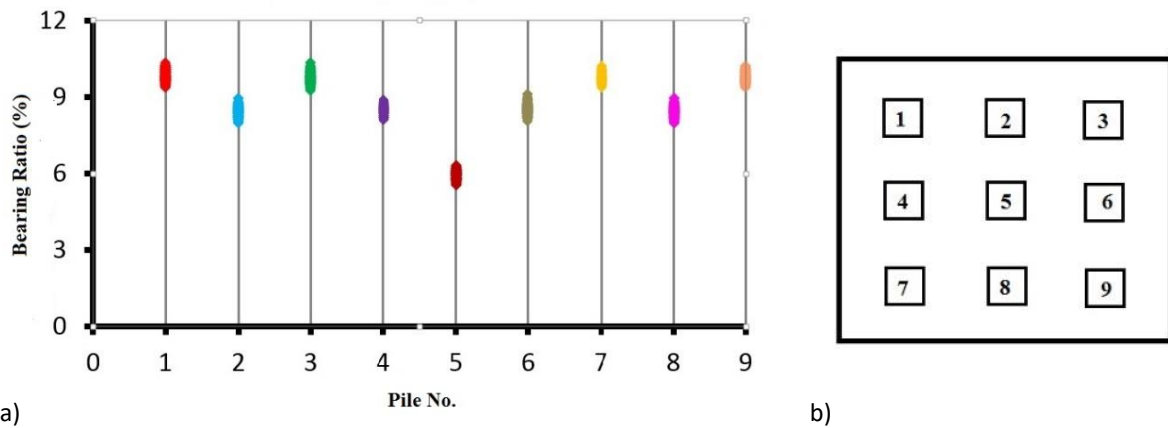


Fig. 10. a) Load bearing contribution of different piles for  $COV_{cu} = 10\%$ ,  $\theta = 2$  m, b) pile's ID number

## COMPARISON AND DISCUSSION

In this study, the obtained results of the analysis were compared to that of calculated by Fleming et al. (2008) and PDR (Poulos and Davids, 2005) and Randolph (1992) relationships for homogenous soils, as presented in Table 4. High importantly, it can be observed that taking into account the soil heterogeneity generally results in more contribution of the raft in bearing capacity than that of the homogenous soils obtained by experimental relationships, although the Fleming method seems to have more consistency to heterogeneous soil. This denotes the significance of carrying out stochastic analyses particularly where the soil

properties are extremely variable.

Effect of variation of two statistical parameters of undrained shear strength of clayey soil, including COV and scale of fluctuation on the cap's bearing ratio was studied. It was observed that increase of coefficient of variation (Figure 8), i.e. increased contribution of stochastic component, leads to declined raft's impact on the bearing capacity, which can be associated with the effect of soil-pile interaction. This is because increase of the soil stiffness in the underlying soil layers, resulting in variation of the raft-pile interaction factor ( $\alpha_{cp}$ ), has more effect on the load sharing of the piles than that of the raft, as the soil-structure contact area of the piles are much higher than

that of the raft (Lee et al., 2010; Zhang et al., 2016). Therefore, it would be even rational to expect more effect of stochastic properties on the load sharing by increase of the piles length or diameter, leading to increase of the contact area between the soil and the piles.

However, effect of the scale of fluctuation on the CBR as manifested in Figure 9, showed a turning point, such that CBR declined with the scale of fluctuation up to 10 m, and increased afterwards. This behavior, as in line with Niandou and Breysse (2007), can be explained as follows:

a) When the scale of fluctuation ( $\theta$ ) is very small, i.e. lower than 10 meters, the rapid fluctuations in soil properties are averaged out, and the soil behaves as a homogenous soil.

b) When  $\theta$  is very large (compared to the foundation size), the soil properties vary very slowly below the structure, and the soil behaves as homogeneous, implying that for large scales of fluctuation, the soil properties are fully correlated.

c) When  $\theta$  is intermediate, which is the general case, the foundation behavior is sensitive to the fluctuations in the soil properties.

Figures 11 and 12 are presented to manifest the effect of variation of soil properties in terms of both COV of the soil's shear strength ( $C_u$ ) and the distance through which it varies (the scale of fluctuation,

normalized by the piles length,  $L_p$ ) on the mean cap bearing ratio ( $\mu_{CBR}$ ). It can be generally inferred that variability of input parameters induces reduction in the cap's bearing ratio. Although, it is notable that a rather minimal reduction is observed in the results by varying either the COV or the fluctuation scale of the soil in a heterogeneous deposit. This can be due to the local averaging of the random undrained cohesion field. Indeed, the property under study is averaged and integrated through the space underneath the raft. This means that the contributions of different elements are not so much affected by the stochastic variations of stiffness and strength parameters and therefore the parameters can be considered in average sense and deterministic analyses are sufficient in such cases.

On the other hand, according to Table 4, the raft's bearing ratio in Fleming method was calculated as 24.4%. A new parameter of Probability of Failure (POF) can be defined as the ratio of the number of cases for which the raft's bearing ratio is less than or equal to 24.4% ( $N_f$ ) to the total number of random cases ( $N$ ), (i.e.  $POF = N_f / N$ ), as depicted in Figure 13. Considering this figure, it can be concluded that the probability that the bearing ratio of the raft would be less than the empirical model increases with increasing  $COV_{cu}$ , implying that the probability of failure increases by increase of  $COV_{cu}$ .

**Table 4.** Comparison of the bearing ratio in heterogeneous soil and experimental results

Methods	COV (%)	$\theta$ (m)	Raft's Contribution (%)	Pile's Contribution (%)
Heterogeneous models by Random Finite Difference Method (RFDM)	10	2	29.5	70.4
		10	28.9	71.0
		100	29.2	70.7
	50	2	28.9	71.0
		10	29.1	70.8
		100	28.9	71.0
	90	2	28.4	71.2
		10	29.1	70.9
		100	28.4	71.5
Experimental methods	Feleming PDR		24.4	75.6
			19.1	80.9



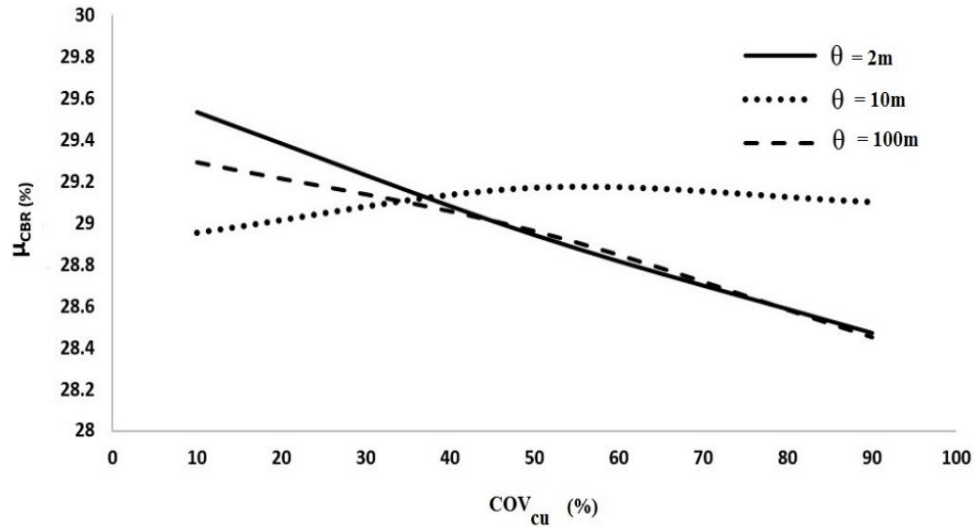


Fig. 11. Variation of mean cap bearing ratio against  $COV_{cu}$  and different scales of fluctuations

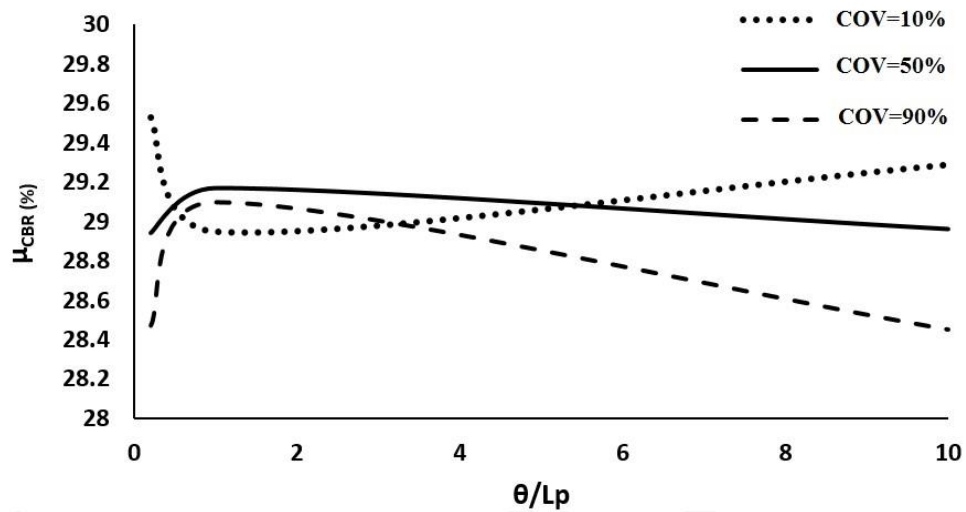


Fig. 12. Variation of mean bearing ratio against normalized scale of fluctuation and different COVs

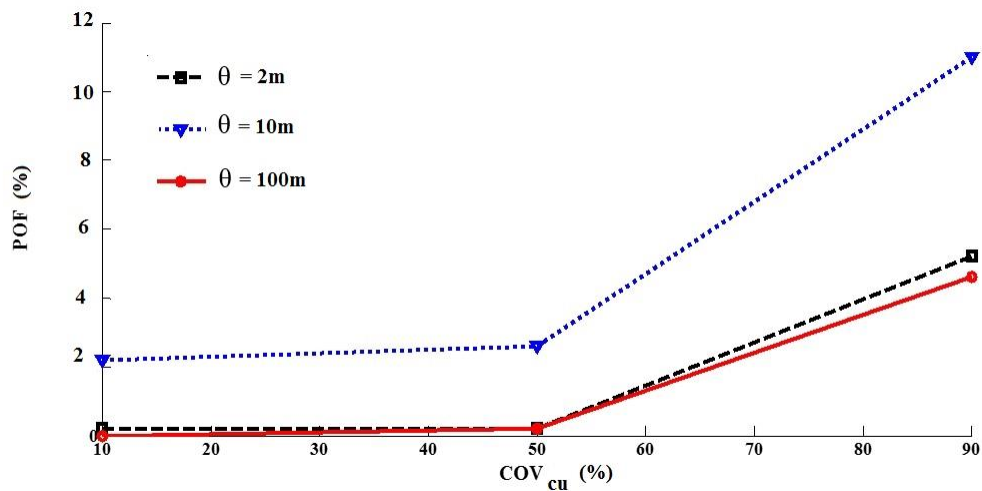


Fig. 13. Probability of failure (PoF) for the raft bearing ratio (Fleming method)

## CONCLUSIONS

The effect of heterogeneity of clayey soil on the bearing ratio of piled rafts was studied considering the coefficient of variation (COV) as the most affecting statistical parameter. The contribution of the raft and the piles in bearing capacity were calculated employing the log-normally distributed undrained shear strength and Monte Carlo simulation; the most important findings of this study are as follows:

1. The COV of shear strength and its scale of fluctuation were revealed to be the most influential parameters in the stochastic analyses.

2. Increasing the COV leads to a minimal decrease in the mean cap bearing ratio, in a constant scale of fluctuation.

3. Generally, increasing the scale of fluctuation results in marginal increase of the raft's bearing ratio.

4. Regarding the effect of geometrical position of the piles on their load bearing contribution, it can be concluded that the corner and central piles have the most and the least role in bearing capacity, respectively, mainly due to the axi-symmetrical nature of the model. Moreover, the piles located in the same position indicated the same bearing ratio, implying that the heterogeneous soils have the same effect of homogeneous soils regarding the position of the piles. This seems to be caused because the mean shear strength random field was kept constant, although it was spatially variant through the deposit.

5. Importantly, it was observed that taking into account the soil heterogeneity generally results in more contribution of the raft in bearing capacity than that of the homogenous soils obtained by experimental relationships, although the Fleming method seems to have more consistency to heterogeneous soil. This implies the significance of carrying out stochastic analyses where the soil properties are intensively variable.

6. Variation of the soil properties generally showed no remarkable impact on the results by varying neither the COV nor the fluctuation scale of the soil properties in a heterogeneous deposit. This is because a constant mean for the soil properties under study is considered throughout analyses, in spite of being varied through the whole random field. Therefore, a single deterministic analysis with equivalent mean strength and stiffness properties is expected to render the bearing contribution of different elements of the system.

## REFERENCES

- Adler, R.J. (2010). "The geometry of random fields", Society for Industrial and Applied Mathematics, SIAM edition.
- Ahmed, A. and Soubra, A.H. (2012). "Probabilistic analysis of strip footings resting on a spatially random soil using subset simulation approach", *Georisk*, 6(3), 188–201.
- Albusoda, B.S. and Salem L.A.K. (2016). "The effect of interaction on pile-raft system settlement subjected to earthquake excitation", *Applied Research Journal*, 2(4), 205-214.
- Bajad, S. and Sahu R. (2008). "An experimental study on the behavior of vertically loaded piled raft on soft clay", *The 12<sup>th</sup> International Conference of International Association for Computer Methods and Advances in Geomechanics (IACMAG)*, Goa, India.
- Baziar, M., Ghorbani A. and Katzenbach R. (2009). "Small-scale model test and three-dimensional analysis of pile-raft foundation on medium-dense sand", *International Journal of Civil Engineering*, 7(3), 170-175.
- Bourgeois, E., De Buhan, P. and Hassen, G. (2012). "Settlement analysis of piled-raft foundations by means of a multiphase model accounting for soil-pile interactions", *Computers and Geotechnics*, 46, 26-38.
- Ching, J. and Phoon, K.K. (2013). "Probability distribution for mobilized shear strengths of spatially variable soils under uniform stress states", *Georisk*, 7(3), 209–224.
- Cho, J., Lee, J.H., Jeong, S. and Lee, J. (2012). "The settlement behavior of piled raft in clay soils", *Ocean Engineering*, 53, 153-163.
- Dasaka, S.M. and Zhang, L.M. (2012). "Spatial variability of in situ weathered soil", *Géotechnique*, 62(5), 375-384.

- DeGroot, D.J. (1996). "Analyzing spatial variability of in situ soil properties", *Uncertainty in the Geologic Environment: From Theory to Practice*, ASCE.
- Elahi, A. (2011). "Evaluation of piled-raft bearing capacity located on heterogeneous soils using random field theory", M.Sc. Dissertation, University of Guilan, Rasht, Iran, (in Persian).
- Fan, H., Huang, Q. and Liang, R. (2014). "Reliability analysis of piles in spatially varying soils considering multiple failure modes", *Computers and Geotechnics*, 57(Apr), 97-104.
- Fleming, K., Weltman, A., Randolph, M. and Elson, K. (2008). *Piling engineering*, CRC press.
- Griffiths, D. and Fenton G.A. (2001). "Bearing capacity of spatially random soil: The undrained clay prandtl problem revisited", *Geotechnique*, 51(4), 351-360.
- Haldar, S. and Babu G.S. (2008a). "Reliability measures for pile foundations based on cone penetration test data", *Canadian Geotechnical Journal*, 45(12), 1699-1714.
- Haldar, S. and Babu, G.S. (2008b). "Effect of soil spatial variability on the response of laterally loaded pile in undrained clay", *Computers and Geotechnics*, 35(4), 537-547.
- Husain, A. (2016). "Probabilistic study for single pile in cohesionless soil Using Monte Carlo simulation technique", *International Journal of Scientific and Engineering Research*, 7(2). 628-633.
- Itasca, F.D. (2009). "Fast Lagrangian analysis of continua in 3 dimensions, Version 4.0", Minneapolis, Minnesota, Itasca Consulting Group 438.
- Jamshidi Chenari, R. and Mahigir, A. (2014). "The effect of spatial variability and anisotropy of soils on bearing capacity of shallow foundations", *Civil Engineering Infrastructures Journal*, 47(2), 199-213.
- Jamshidi Chenari, R., and Alaie, R. (2015). "Effects of anisotropy in correlation structure on the stability of an undrained clay slope", *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards*, 9(2), 109-123.
- Jamshidi Chenari, R. and Behfar, B. (2017). "Stochastic Analysis of Seepage through Natural Alluvial Deposits Considering Mechanical Anisotropy", *Civil Engineering Infrastructures Journal*, 50(2), 233-253.
- Jiang, S.H., Li, D.Q., Cao, Z.J., Zhou, C.B. and Phoon, K.K. (2014). "Efficient system reliability analysis of slope stability in spatially variable soils using Monte Carlo simulation", *Journal of Geotechnical and Geoenvironmental Engineering*, 141(2): 04014096.
- Kalos, M.H. and Whitlock, P.A. (2008). *Monte Carlo methods*, John Wiley & Sons.
- Kenarsari, E., Oloomi, R., Jamshidi Chenari, R. and Eslami, A. (2011). "Effect of vertical heterogeneity in soil strength on pile bearing capacity prediction from CPT data", *Proceedings of the 36<sup>th</sup> Annual Conference on Deep Foundations*, Boston, MA.
- Lee, J., Kim, Y. and Jeong, S. (2010). "Three-dimensional analysis of bearing behavior of piled raft on soft clay", *Computers and Geotechnics*, 37(1), 103-114.
- Lee, J., Park, D. and Choi, K. (2014). "Analysis of load sharing behavior for piled rafts using normalized load response model", *Computers and Geotechnics*, 57, 65-74.
- Lee, J., Park, D. and Park, K. (2015). "Estimation of load-sharing ratios for piled rafts in sands that includes interaction effects", *Computers and Geotechnics*, 63, 306-314.
- Li, D.Q., Qi, X.H., Cao, Z.J., Tang, X.S., Phoon, K.K. and Zhou, C.B. (2016). "Evaluating slope stability uncertainty using coupled Markov chain", *Computers and Geotechnics*, 73: 72-82.
- Li, M., Lu, X., Lu, X. and Ye, L. (2014). "Influence of soil-structure interaction on seismic collapse resistance of super-tall buildings", *Journal of Rock Mechanics and Geotechnical Engineering*, 6(5), 477-485.
- Lloret-Cabot, M., Fenton, G.A. and Hicks, M.A. (2014). "On the estimation of scale of fluctuation in geostatistics", *Georisk*, 8(2), 129-140.
- Matsuo, M. and Kuroda, K. (1974). "Probabilistic approach to design of embankments", *Soils and Foundations*, 14(2), 1-17.
- Morse, R. (1971). "Importance of proper soil units for statistical analysis", *Proceedings of the 1<sup>st</sup> International Conference on Applications of Statistics and Probability to Soil and Structural Engineering*, Hong Kong.
- Nguyen, D.D.C., Jo, S.B. and Kim, D.S. (2013). "Design method of piled-raft foundations under vertical load considering interaction effects", *Computers and Geotechnics*, 47, 16-27.
- Niandou, H. and Breysse, D. (2007). "Reliability analysis of a piled raft accounting for soil horizontal variability", *Computers and Geotechnics*, 34(2), 71-80.
- Papadopoulos, C.E. and Yeung, H. (2001). "Uncertainty estimation and Monte Carlo simulation method", *Flow Measurement and Instrumentation*, 12(4), 291-298.
- Park, D., Park, D. and Lee, J. (2016). "Analyzing load response and load sharing behavior of piled rafts installed with driven piles in sands", *Computers and Geotechnics*, 78, 62-71.
- Patil, J.D., Vasanwala, S.A. and Solanki, C.H. (2014). "An experimental investigation on behavior of

- piled raft foundation", *International Journal of Geomatics and Geosciences*, 5(2), 300.
- Phoon, K.K. and Kulhawy, F.H. (1999). "Characterization of geotechnical variability", *Canadian Geotechnical Journal*, 36(4), 612-624.
- Popescu, R., Deodatis, G. and Nobahar, A. (2005). "Effects of random heterogeneity of soil properties on bearing capacity", *Probabilistic Engineering Mechanics*, 20(4), 324-341.
- Poulos, H. (2002). "Simplified design procedure for piled raft foundations", *Deep Foundations 2002: An International Perspective on Theory, Design, Construction and Performance*, pp: 441-458.
- Poulos, H.G. and Davids, A.J. (2005). "Foundation design for the emirates twin towers, Dubai." *Canadian Geotechnical Journal*, 42(3), 716-730.
- Randolph, M. (1992). "Design methods for pile groups and piled rafts", *XIII ICSMFE*, 61-82.
- Reul, O. and Randolph M. (2003). "Piled rafts in overconsolidated clay: comparison of in situ measurements and numerical analyses", *Geotechnique*, 53(3), 301-315.
- Reul, O. and Randolph, M.F. (2004). "Design strategies for piled rafts subjected to nonuniform vertical loading", *Journal of Geotechnical and Geoenvironmental Engineering*, 130(1), 1-13.
- Saeedi Azizkandi, A. and Fakher, A. (2014). "A simple algorithm for analyzing a piled raft by considering stress distribution", *Civil Engineering Infrastructures Journal*, 47(2), 215-227.
- Salgado, R., and Kim, D. (2014). "Reliability analysis of load and resistance factor design of slopes", *Journal of Geotechnical and Geoenvironmental Engineering*, 140(1), 57-73.
- Tung, Y.K. and Yen, B.C. (2005). *Hydrosystems engineering uncertainty analysis*, ASCE, McGraw Hill.
- Vanmarcke, E. (2010). *Random fields: Analysis and synthesis*, World Scientific.
- Zhang, J., Zhang, L. and Tang, W.H. (2011). "New methods for system reliability analysis of soil slopes", *Canadian Geotechnical Journal*, 48(7), 1138-1148.
- Zhang, L., Goh, S.H. and Yi, J. (2016). "A centrifuge study of the seismic response of pile-raft systems embedded in soft clay", *Géotechnique*, 67(6), 479-490.