Civil Engineering Infrastructures Journal, 50(2): 233 – 253, December 2017
Print ISSN: 2322-2093; Online ISSN: 2423-6691
DOI: 10.7508/ceij.2017.02.003

Stochastic Analysis of Seepage through Natural Alluvial Deposits Considering Mechanical Anisotropy

Jamshidi Chenari, R.[1*] and Behfar, B.[2]

[1] Associate Professor, University of Guilan, Guilan, Iran.
[2] M.Sc., University of Guilan, Guilan, Iran.

Received: 18 Oct. 2016; Revised: 31 Jul. 2017; Accepted: 01 Aug. 2017

ABSTRACT: The soil is a heterogeneous and anisotropic medium. Hydraulic conductivity, an intrinsic property of natural alluvial deposits varies both deterministically and randomly in space and has different values in various directions. In the present study, the permeability of natural deposits and its influence on the seepage flow through a natural alluvial deposit is studied. The 2D Finite Difference code, FLAC 5.0, is used for modeling permeability as a random variable with lognormal distribution and correlated structure. Effect of spatially varying permeability on the seepage flow through deposit is investigated for both isotropic and anisotropic conditions. Results show that in isotropic condition, the mean discharge flow rate calculated from stochastic analyses is less than the equivalent deterministic value and this reduction depends on the coefficient of variation, COV of permeability and the correlation length. The directionality of permeability introduced as mechanical anisotropy was also studied along with the heterogeneity. It was found that increasing the anisotropy ratio of permeability leads to the formation of horizontal flow canals and increasing the seepage flow consequently at a constant vertical permeability. Variation of permeability coefficient was found to have almost no impact on mean discharge flow rate for anisotropic fields in comparison to the isotropic condition.

Keywords: Anisotropy, Heterogeneity, Permeability, Random Field Theory, Seepage.

INTRODUCTION

In geotechnical problems, the soil is assumed to be isotropic and homogeneous in most classic analytical methods as well as advanced numerical methods, and average values for the properties are adopted for calculation and analysis purposes. Having accepted anisotropy and heterogeneity as undisputed properties of natural soil deposits, it is necessary to consider these effects in geotechnical calculations.

Since the first use of Random Finite Element Method (RFEM) in steady state seepage problem by Griffiths and Fenton (1993), several recent studies on seepage flow analysis have focused on the spatial fluctuations of soil permeability by using random field theory (Fenton and Griffiths, 1995, 1996, 1997; Srivastava et al., 2010).

Although most of the seepage analyses in geotechnical engineering practice assume homogeneous permeability field (Salmasi et al., 2015), several researchers have attempted to investigate the influence of spatial variability of permeability on seepage flow

* Corresponding author E-mail: jamshidi_reza@yahoo.com

through earth dams. Griffiths and Fenton (1993) studied the effect of stochastic soil permeability on confined seepage occurring beneath water retaining structures. They analyzed the case of a dam with two cut-off walls. Parametric studies were conducted to gauge the effect of the standard deviation and correlation structure of the permeability on the output statistics relating to seepage quantities, exit gradients, and uplift pressures. Fenton and Griffiths (1996) assumed the soil permeability in an earth dam of typical geometry as a spatially random field following a lognormal distribution with prescribed mean, variance, and spatial correlation structure. They computed the statistics of flow and free surface drawdown through the dam using Monte Carlo simulations, and the statistics of the flow rate and downstream exit-point were finally established. Fenton and Griffiths (1997) investigated the effect of the stochastic nature of the permeability field on the hydraulic gradient distribution in earth dam in comparison to the deterministic case. They also modeled steady seepage beneath a single sheet-pile wall embedded in a natural alluvial deposit in three-dimensional condition (Griffiths and Fenton, 1997). Hydraulic conductivity was assumed as a spatially random property with specified mean, variance, and spatial correlation length. The influence of three-dimensionality was investigated and was compared with 2D analysis results. It was found that the "randomness" and thence the variance of flow rate reduce in a 3D case over those values observed in the 2D case with the same statistics. Srivastava et al. (2010) studied the influence of spatial variation of soil permeability properties on the steady state seepage flow analysis and showed that with increase in coefficient of variation of permeability, there is a decrease in estimated mean seepage discharge, and at the same time there is an increase in the coefficient of variation of the estimated mean seepage discharge value. Seyed Noori (2014) and Jamshidi Chenari and Seyed Noori (2016) studied the effect of permeability and volume compressibility variation on 1D consolidation of natural alluvial deposits. Permeability was assumed both deterministically and stochastically varying with depth. In all studies mentioned before, analyses are based on isotropic condition. However, Ahmed (2012) studied confined flow under hydraulic structures considering anisotropy in the correlation structure of the hydraulic conductivity which is so-called "anisotropy of heterogeneity". Some other researchers have considered the effect of anisotropy of heterogeneity on the stability of geostructures. Jamshidi Chenari and Alaie (2015) focused on slope stability problem considering different correlation properties in vertical and horizontal directions. There are two types of anisotropy namely, heterogeneity and mechanical (hydraulic) anisotropy. Jamshidi Chenari and Mahigir (2014) considered anisotropy of un-drained and drained shear strength of natural alluvial deposits by generalizing the conventional isotropic Mohr-Coulomb failure criterion to the anisotropic one in FLAC software. The study showed that disregarding the anisotropic nature of undrained shear strength might lead to the un-conservative prediction of bearing capacity of shallow foundations. They simultaneously considered heterogeneity and mechanical anisotropy of shear strength in short and long term bearing capacity predictions.

Both types of anisotropy exhibited in terms of the directionality in hydraulic conductivity are intrinsic attributes of porous media which are not avertable. The second type of anisotropy is called "anisotropy of hydraulic conductivity" hereinafter. Natural deposits have unequal permeability in different directions. In order to properly model the hydraulic property of natural deposits, it is essential to choose a proper distribution and select the parameters related to the distribution based on in situ test data. Since there is layering present in the natural deposits, the anisotropy of conductivity is also considered as an unavoidable property of these soils in addition to heterogeneity. It is possible for the coefficient of permeability to be one order of magnitude greater in the horizontal direction than in the vertical direction (Budhu, 2011). Hence, in addition to heterogeneity, proper values for the anisotropy of conductivity ratio should be taken into account. More recently, some researchers have focused on mechanical anisotropy of the permeability and its effect on water flow through rock and other materials (Zhang et al., 2011; Cho, 2012; Le et al., 2012; Rafiezadeh and Ataie Ashtiani, 2013; and Yu et al., 2013). Estimation of water seepage flow beneath dams is a vital task in flow analysis of waterfront structures. The effects of deterministic and random heterogeneity and anisotropy of hydraulic conductivity of natural soil deposits are studied. Indeed, this study is presented in two parts: the first section considers the heterogeneity issue and the effect of permeability variability on discharge flow rate through the heterogeneous stratum and the second part evaluate the effect of mechanical anisotropy on the discharge behavior of heterogeneous stratum. The anisotropy is different from what considered by Ahmed (2012). The random field is assumed isotropic statistically. However, the hydraulic conductivity is assumed directional and different values are adopted for vertical and horizontal directions. Therefore, assuming anisotropy of hydraulic conductivity (not anisotropy of heterogeneity) and investigation of its effect on water seepage through natural sedimentary deposits in dam foundations is considered the novelty of current research. It should, however, be noted that gravity dam considered in current study is not normally founded on permeable deposits. It is instead constructed on rock foundations. The assumption of gravity dam herein is only to assume a water barrier structure, and the foundation as a naturally occurred alluvial deposit is considered a random medium and investigated accordingly.

UNCERTAINTY IN GEOTECHNICAL PARAMETERS

Uncertainty in geotechnical problems falls into two categories. The first category is the inherent uncertainty of the soil, which shows the random natural variations of a variable, and the second is the uncertainty which occurs as a result of the lack of knowledge about the studied variable, which includes the uncertainty of measurements, statistical uncertainties, and uncertainty in correlated models. Unlike the first group of uncertainties, in the second group uncertainties can be reduced by collecting more data, improving the measuring accuracy, and utilizing better calculation methods. In the current study, the first category of uncertainty will be studied which is an inherent property of soil and affects the permeability of natural deposits.

Almost all natural deposits possess physical and mechanical parameters which are heterogeneous. Heterogeneity in soil results from natural geological processes in which soil is formed as a geomechanical material and undergoes changes. Heterogeneity in natural deposits can be categorized into two groups of lithological and inherent categories. Lithological heterogeneity is characterized by a loose or dense soil layer within a denser or looser soil. Another type of heterogeneity is due to inherent variations in the soil properties such as variations of physical and mechanical properties of the soil from one point to another. Inherent heterogeneity can take two forms:

1- The deterministic process, which is related to the variations in the constitutive parameters of the soil due to variation in overburden pressure or effective stress.
2- A random process, which is related to the random variations of the constitutive parameters of the soil about its mean value in the problem.

Figure 1 depicts different components of the inherent heterogeneity of the coefficient of permeability including deterministic and random variations. As can be seen from the Figure, profile of variations of the coefficient of permeability can be separated into two parts using a single regression analysis, with a deterministic part which can also be nonlinear, and a random part with constant variations from a fixed mean value.

Like deformability and strength parameters, it is predicted that the coefficient of permeability in the vertical direction is a function of effective stress level and pre-consolidation ratio. On the basis of soil mechanics principles and considering the weight of the soil, as we proceed to the deeper areas of the soil stratum, the overburden pressure increases and as a result, the soil is more compact and less porous. Considering the relation between porosity of the soil and its permeability, the coefficient of permeability in a single homogeneous soil layer can be considered to decrease with depth. In the current study, this decreasing trend is represented linearly with the slope, λ (Figure 1).

RANDOM FIELD THEORY

Since constitutive parameters of soil have random variability in the domain under study, stochastic modeling with the help of a random field theory can be considered as a useful tool. In recent years, along with the progress in fast computation technology and development of advanced computation algorithms, scientists are paying more attention to the modeling of heterogeneous materials using random field theory. The investigated parameters are considered as a random variable with a specific probability distribution function and the related statistical parameters.

The most important parameters used in probability distributions are: mean value and the standard deviation which shows data variability around the mean value. It is mainly in the form of the coefficient of variation *CoV*, which is dimensionless. Correlation distance is another important parameter which shows the distance from which the data dependence becomes negligible. Therefore, mean value μ_k, standard deviation σ_k, and scale of fluctuation θ_k, along with probability distribution function of the data, can be mentioned as the most important statistical parameters. In the current study, μ_k was assumed to be equivalent to the surface intercept value of the deterministic permeability trend k_0 (Figure 1), and σ_k was assumed according to *CoV* definition.

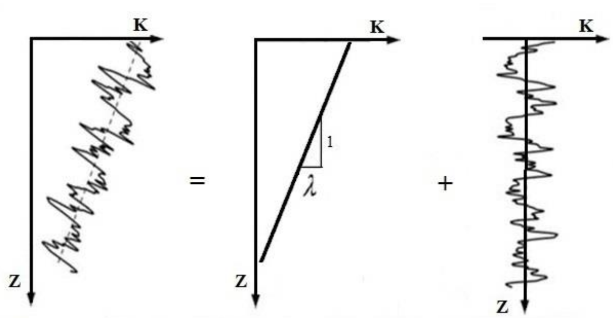


Fig. 1. Schematic representation of components of the inherent variability of the coefficient of permeability of the soil

Normal or log normal distributions are commonly used in generic engineering applications. However, log normal distribution seems proper for the coefficient of permeability due to being strictly nonnegative. Results of statistical analysis of field data for the coefficient of permeability, confirms the assumption of log normal distribution for this parameter (Lumb, 1974, Benson, 1993). A log normal distribution can be expressed in Equation 1.

$$k(\tilde{x}) = exp\{\mu_{\ln k} + \sigma_{\ln k} G(\tilde{x})\} \quad (1)$$

in which, $\tilde{x}$: is location of the points that the coefficient of permeability is being investigated, and $G(\tilde{x})$: is a Gaussian correlated random field with zero mean value and unit standard deviation. The values of μ_{lnk} and σ_{lnk}, can be calculated by using the log normal transformation, provided in Eqs. (2) and (3).

$$\sigma_{\ln k}^2 = \ln\left(1 + \frac{\sigma_k^2}{\mu_k^2}\right) = \ln(1 + \mathrm{COV}_k^2) \quad (2)$$

$$\mu_{\ln k} = \ln(\mu_k) - \frac{1}{2}\sigma_{\ln k}^2 \quad (3)$$

Griffiths and Fenton (1993) showed that the correlation, $\rho_k(\tau)$ between the logarithms of the coefficient of permeability $lnk(x)$ of two points of x and $x+\tau$ conforms to the Gauss–Markov model which is a decreasing exponential function. In the current study, the correlation with a decreasing exponential function is considered as introduced in Eq. (4):

$$\rho_k(\tau) = exp\left(-\frac{2\tau}{\theta_k}\right) \quad (4)$$

in which, $\tau = |\tilde{X}_1 - \tilde{X}_2|$ is the absolute distance between two points and θ_k: is the scale of fluctuation.

The distance τ, which is used in the generation of the correlation matrix (Equation4), is, in fact, the distance between the center to center of the elements, or consecutive cells in the generated mesh. Correlation calculation process in the Finite Difference mesh can be explained from Figure 2. For instance, if the distance between the centers of elements number 1 and 2 is defined as dx, the corresponding correlation between two elements can be obtained by substituting in $\tau=dx$ in Eq. (4). Using the same process, correlation of element number 1 with elements number 3, 4, and 5 can be obtained by substituting τ with the distance values of $2dx$, $3dx$, and $4dx$ respectively. Likewise, the correlation distance between element number 1 and elements number 61, 62, and 63 is dy, $\sqrt{dx^2 + \mathrm{dy}^2}$ and $\sqrt{(2dx)^2 + \mathrm{dy}^2}$ respectively; and the same process is continued for the other elements.

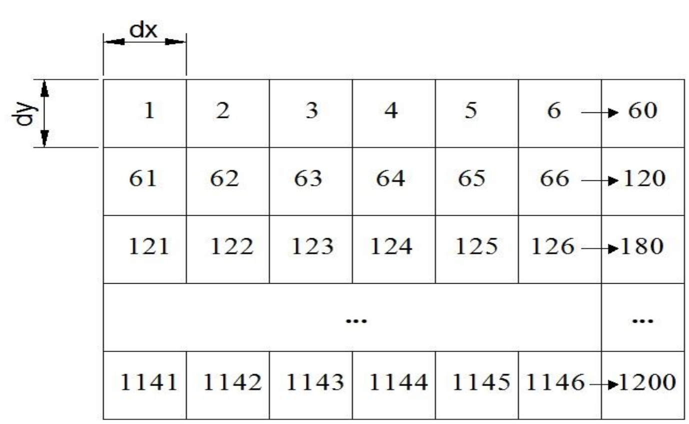


Fig. 2. Discretization of Finite Difference grid in the random field modeling

Therefore, the values in the first row of the correlation matrix are the correlation coefficients between element number 1 and other elements, which lead to a row of 1200 arrays as a result of 20×60 Finite Difference cells in the current investigation. Hence, the correlation matrix will be 1200×1200 considering all the cells. L matrix is extracted after obtaining the correlation matrix and its decomposition into multiplication of a lower triangular matrix and its transpose using Cholesky decomposition technique (Eq. (5)).

$$\rho_k = L.L^T \tag{5}$$

The normal correlated random field can be written after obtaining the matrix L:

$$G_i = \sum_{j=1}^{i} L_{ij} Z_j \qquad i = 1,2,3,\dots,n \tag{6}$$

in which Z_j: is an uncorrelated random variable sequence.

Different studies in the past have been conducted on the variability of geotechnical parameters, and common ranges of variability for different parameters used in random analyses are presented in the technical literature. Lumb (1974) has reported a range of 200% to 300% for the coefficient of variability of permeability coefficient based on his observations. Benson (1993) reported values of 50% to 200% as the result of his field tests, but he has mentioned that in some soils, the *CoV* reaches up to 500% or even 700%. Duncan (2000) suggested a range of 68% to 90% for the coefficient of variation based on the tests he performed and based on this, Srivastava et al. (2010) used a range of 60% to 90% for the coefficient of variation. Griffiths and Fenton (1993) used a range of 12.5% to 1600% for the coefficient of variation of permeability in investigation of water seepage under water containing structures in heterogeneous medium; and later, theses researchers applied a value of 50% as the coefficient of variability for the coefficient of permeability in analysis of critical hydraulic gradient in the body of earth dams. In all the studies mentioned above, the deterministic variability of the coefficient of permeability was neglected, and a constant mean value was considered instead. Table 1 shows different values adopted by various researchers for the coefficient of variation of the permeability coefficient.

Table 1. Suggested values for the coefficient of variation of the permeability coefficient

Reference	CoV_k (%)
Lumb (1974)	200-300
Benson (1993)	50-200
Griffiths and Fenton (1993)	12.5-1600
Duncan (2000)	240 (80% saturation) 90 (100% saturation)
Srivastava et al. (2010)	60-90
Ahmed (2012)	12.5-800

Different ranges of the scale of fluctuation for the permeability of soils have been used by the previous researchers as shown in Table 2.

Table 2. Utilized values for the correlation lengths for the coefficient of permeability

Reference	Scale of (m) Fluctuation, θ_k
Griffith and Fenton (1993)	0, 1, 2, 4, 8
Fenton and Griffith (1995)	0.1, 0.5, 1, 2, 8
Fenton and Griffith (1996)	0.1, 0.5, 1, 2, 4, 8, 16
Fenton and Griffith (1997)	1
Srivastava et al. (2010)	0.5-15
Ahmed (2012)	1-24

PERMEABILITY ANISOTROPY

Anisotropy is the property of being directionally dependent, as opposed to isotropy, which implies identical properties in all directions. Two types of anisotropy are called inherent and induced anisotropy based on the causes of this property. Inherent anisotropy is a property of granular soils which is the result of sedimentation and arrangement of the particles. In the sedimentation process, coarser aggregates are deposited in such a way that their main axis is perpendicular to the direction of sedimentation. Another type of anisotropy called induced anisotropy is identified as the change in arrangement and orientation of soil particles as a result of strains induced by unequal stresses on the soil structure. When under load, the internal structure of the soil changes in a way that the stronger orientation becomes parallel with maximum principal stress (Oda, 1972). In most cases, natural deposits undergo the sedimentation process, dissimilar gravitational stresses, aging chemical bonds and biological activities; hence most soils contain a combination of both inherent and induced anisotropy which is called initial anisotropy. Natural deposits mainly tend to deposit in the vertical direction and experience the same horizontal stresses. In other words, natural deposits are symmetrical about the vertical axis. Therefore, a common method of describing anisotropy of natural deposits is by using cross anisotropy assumption. In a naturally layered soil, permeability is maximum for a current in the direction parallel with the layering and minimum in the direction perpendicular to the layering. It might even be possible for the horizontal permeability to be one order of magnitude greater than its vertical direction. Anisotropy ratio A_R is defined as the ratio of horizontal to the vertical permeability coefficient. Clennell et al. (1999) believe that within any uniform layer, levels of permeability anisotropy are modest and does not reach the high levels predicted by simple models of clay particle reorientation. The discrepancy arises from particle clustering and irregularities in particle packing. Although somewhat higher levels of anisotropy may exist as a consequence of lamination within individual beds, values > 10 that are known to exist on the formation scale are produced by strong contrasts between the permeability of interlayered beds. They emphasize that, as argillaceous sediments have permeability ranges of many orders of magnitude, apparently subtle lithological layering in a shale unit may lead to a highly anisotropic flow behavior. An overview of research on permeability anisotropy was given by Scholes et al. (2007), and an expression for predicting anisotropy as a function of void ratio was offered. They described hydraulic permeability for lignite as a function of flow path tortuosity only. The lignite permeability was shown to be up to eight times greater in the direction perpendicular to compression.

An overview of research on permeability anisotropy was given by Scholes et al. (2007), and an expression for predicting anisotropy as a function of void ratio was offered. They described hydraulic permeability for lignite as a function of flow path tortuosity only. The lignite permeability was shown to be up to eight times greater in the direction perpendicular to compression.

MONTE CARLO SIMULATION

In the realization process, it can be seen that after each realization, as permeability changes in the elements, the results of the analyses which are the discharge rate of water under dam foundation also undergo variation. Hence, results from a single analysis cannot be utilized by its own, and Monte Carlo simulation has been used to reach more realistic and representative results. Monte Carlo simulation is a powerful method which can be used for all linear and nonlinear problems, but reaching a trustworthy distribution of responses needs a lot of realizations. Applying this method is easy, and the result will be more realistic if the realizations are done completely. Figure 3 demonstrates a detailed flowchart of the aforesaid calculation process.

NUMERICAL SIMULATIONS

Numerical simulation of seepage problem is done for a concrete gravity dam sitting on a 10-meter-thick alluvial deposit with Finite Difference method and using the two-dimensional software FLAC 5.0. The selected model consists of 1200 elements of 0.5×0.5 meters. Boundary conditions are chosen in a way that all the lateral boundaries and the lower boundary are impermeable and the upstream and downstream have constant hydraulic heads. The geometry of the model is shown in Figure 4 for the isotropic hydraulic condition. The selected dimensions were adopted through sets of sensitivity analyses intended to eliminate boundary effects. However, for the anisotropic cases, the horizontal width extends to higher values to render consistent results.

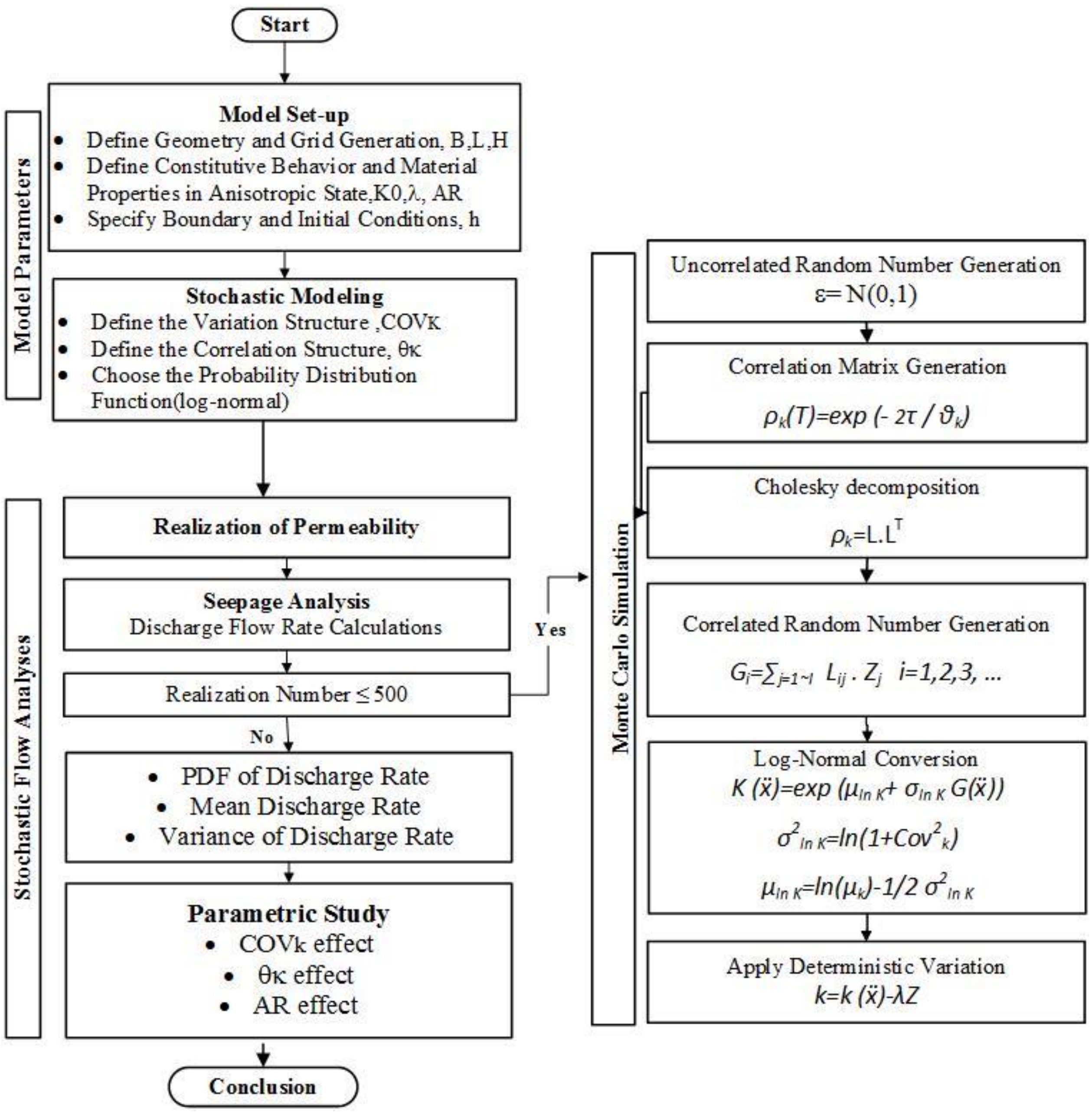


Fig. 3. Flowchart for Monte Carlo simulation process adopted in the current study

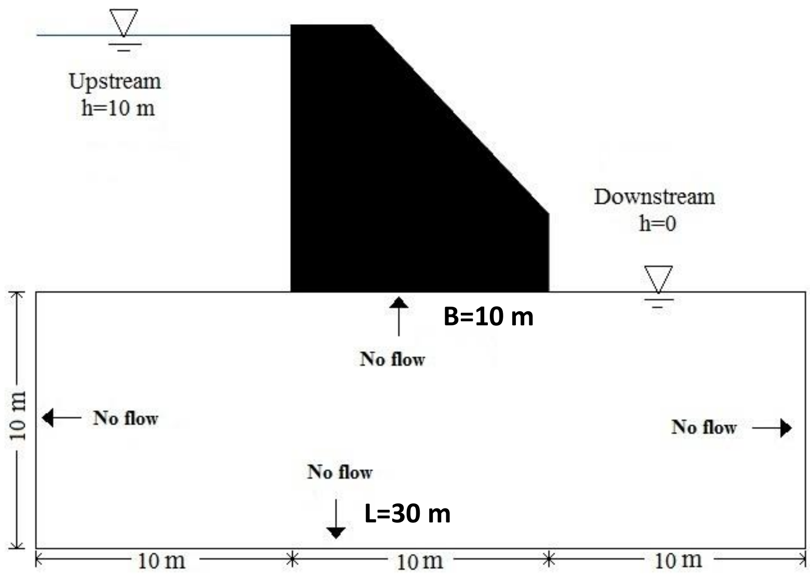


Fig. 4. Problem geometry and boundary conditions

Total dimensions of the model are 10×30 m. A coefficient of permeability of 10^{-4} cm/s is assigned to the permeable soil in average sense $\mu_k=k_0$ (Figure 1), and different coefficients of variation of 20, 40, 60, 80, and 100% are applied for stochastic analyses. Moreover, according to the dimension of the model and elements size, different scales of fluctuation of 2, 4, 10, 20, 30 and 60 m are used.

Finite Difference formulation combined with the random field theory in Monte Carlo simulation framework rendered calculation of the seepage flow through the dam foundation. To assure the adequacy of the number of realizations, the mean discharge flow rate is calculated against the number of realizations for different values of the coefficients of variation of permeability CoV_k in an uncorrelated scheme as presented in Figure 5. As can be seen from Figure 5, mean discharge rates converge after 100 realizations for different CoV_k values; however, 500 numbers of realizations are considered for more confidence to be appropriate for all sets of analysis as depicted subsequently.

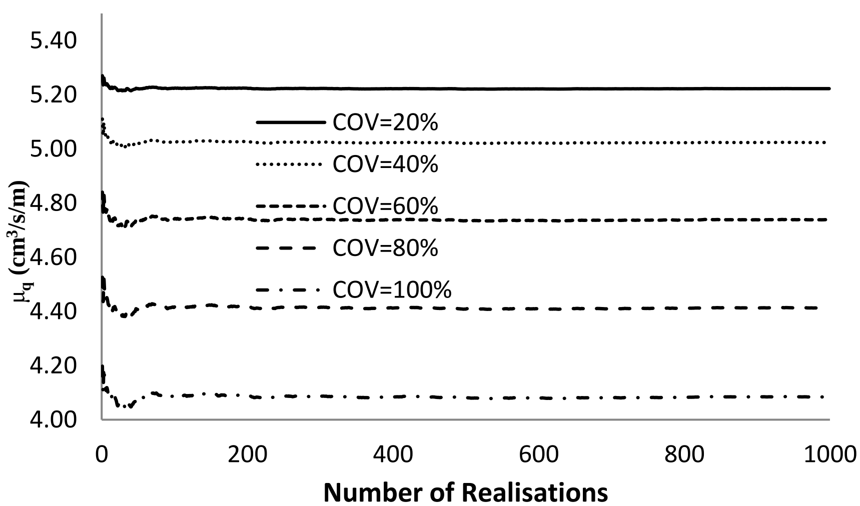
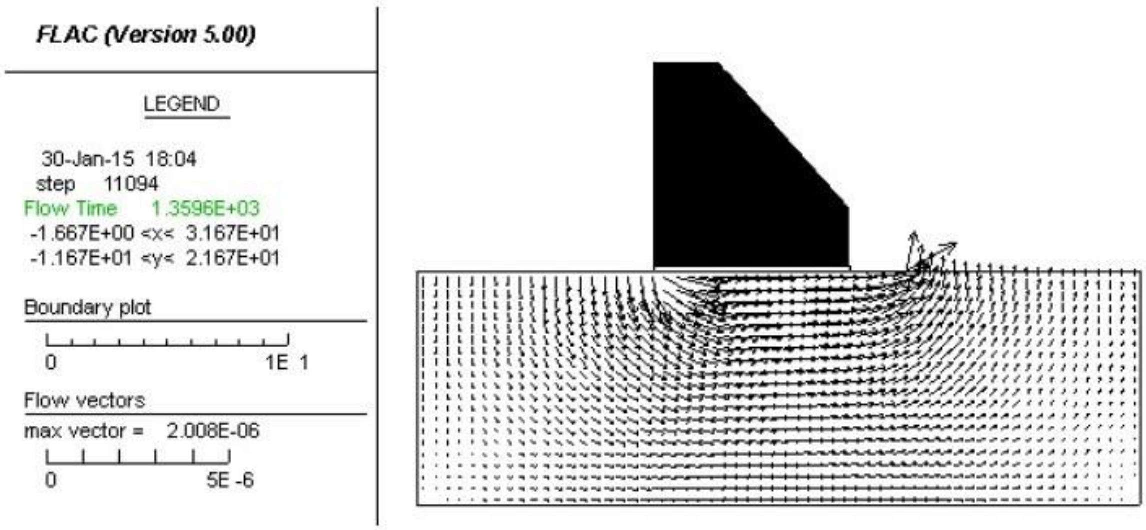


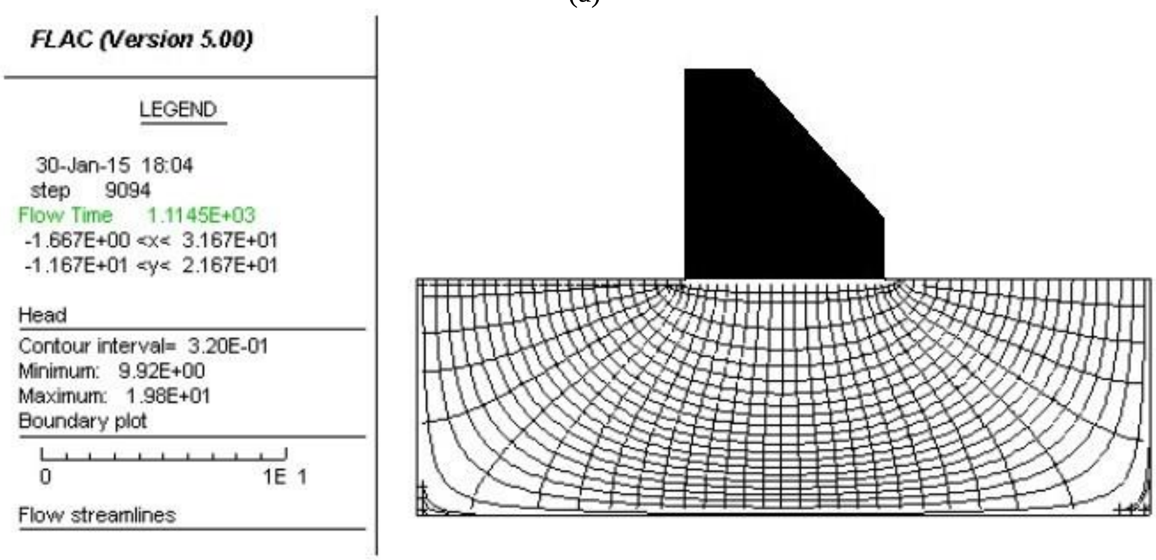
Fig. 5. The mean discharge flow rate against the number of realizations for different values of CoV_k

EFFECT OF HETEROGENEITY

Initially, a mass of homogeneous and isotropic soil with a constant permeability of 10^{-4} cm/s was modeled and analyzed. The results show a flow rate of 5.295 cm^3/s/m for the homogeneous and isotropic case. This value is used as a basis for comparison with heterogeneous analyses afterward. Before proceeding to more sophisticated and advanced flow analyses, the homogeneous and isotropic analysis was calibrated and verified with a simple, classic calculation of discharge rate beneath water front structures. It was shown that the result of flow analysis is accurate enough and simulation of the anisotropy and heterogeneity problem was then continued. Figure 6 shows the stream and equipotential lines along with the velocity vectors of the flow inside the homogeneous and isotropic soil.



(a)



(b)

Fig. 6. Simulation of homogeneous and isotropic conditions for k= 10^{-4} cm/s; a) flow vectors and b) flow net

As mentioned earlier, the coefficient of permeability decreases with depth, and the decreasing rate depends on various factors such as soil type, porosity, and gradation. One order of magnitude reduction of the coefficient of permeability through 10 meters in depth is considered in this investigation. Considering linear trend of variations for the coefficient of permeability with depth, this parameter is varied through depth z using Eq. (7).

$$k_z = k_0 - \lambda z \qquad (7)$$

in which, λ: is the rate at which the coefficient of permeability varies through the deterministic process. The value of the surface coefficient of permeability k_0 was assumed equivalent to the homogeneous and isotropic case, and the analyses were done assuming $\lambda=9\times10^{-8}$ s^{-1} and $k_0=10^{-4}$ cm/s. A sensitivity analysis as illustrated in Figure 7 shows that no dramatic changes occur by adopting a different variation rate in case the mean value is constant.

Figure 8 shows the linear distribution of the coefficient of permeability with depth in the soil mass. Discharge flow rate of the soil is calculated with deterministic inherent heterogeneity and is equal to 3.595 cm^3/s. It is also clear that as the coefficient of permeability decreases with depth, discharge flow rate decreases in comparison to the homogenous case, where the discharge flow rate value was computed to be 5.295 cm^3/s.

Stochastic numerical analyses of seepage flow underneath the gravity dam under study were performed by generating a random log-normally distributed coefficient of permeability field using the FISH programming feature of FLAC 5.0 software. Realization of the coefficient of permeability in two-dimensional space of the problem is performed by adopting the mean value trend and the coefficient of variation of the permeability which are basic stochastic parameters along with the respective scale of fluctuation. Deterministic and stochastic parts of the heterogeneity are taken into account simultaneously and superimposed as demonstrated in Figure 9.

	K_t	K_b	K_m	λ (s^{-1})	Q (cm^3/s)
1	10^{-04} cm/s	10^{-04} cm/s	10^{-04} cm/s	0	5.30
2	1.01×10^{-04} cm/s	0.99×10^{-04} cm/s	10^{-04} cm/s	0.2×10^{-08}	5.35
3	1.02×10^{-04} cm/s	0.98×10^{-04} cm/s	10^{-04} cm/s	0.4×10^{-08}	5.40
4	1.05×10^{-04} cm/s	0.95×10^{-04} cm/s	10^{-04} cm/s	10^{-08}	5.56

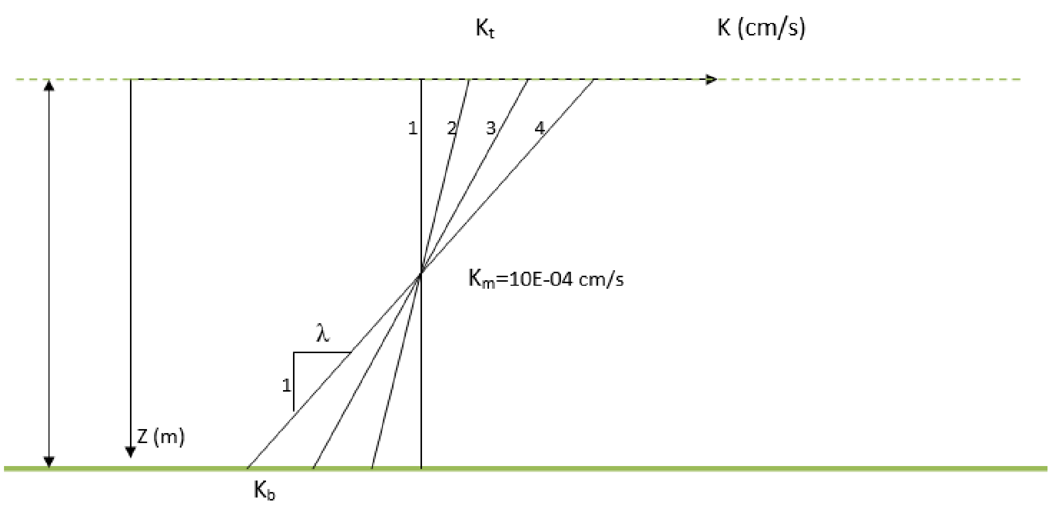


Fig. 7. Effect of variation trend on discharge rate assuming constant mean value

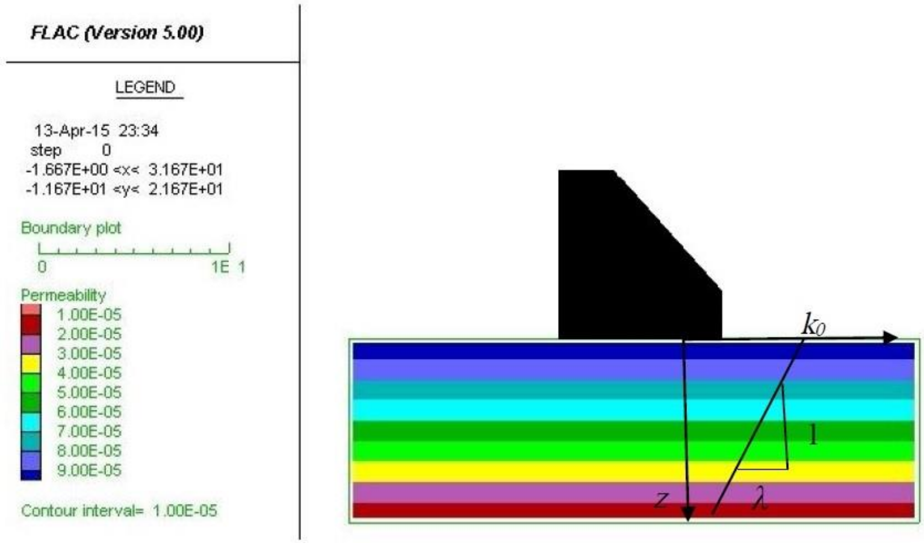
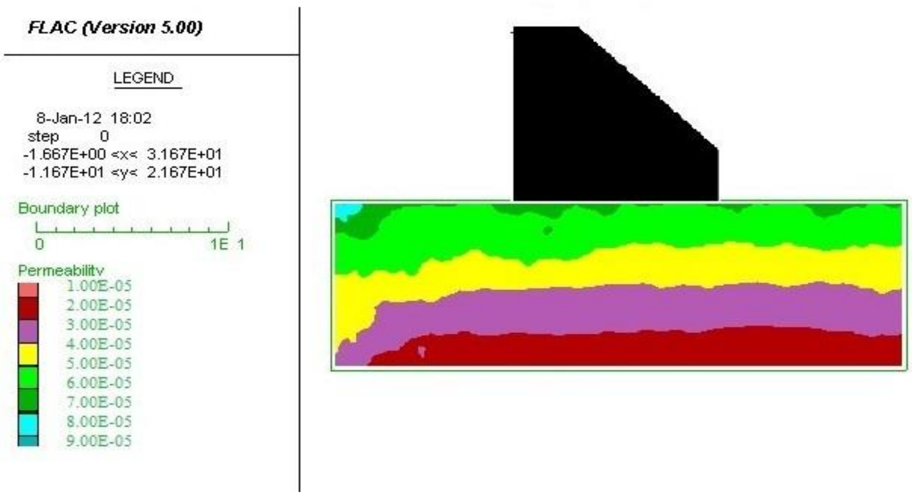
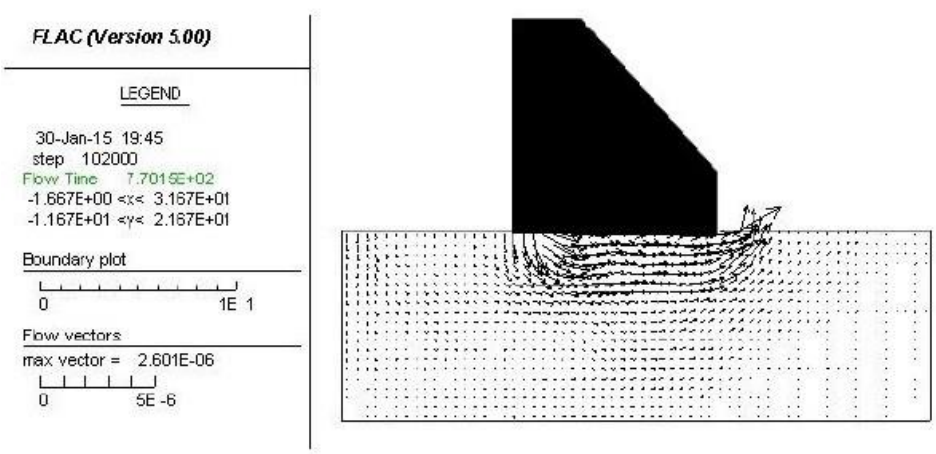


Fig. 8. Deterministic variation of the coefficient of permeability with depth



(a)



(b)

Fig. 9. Sample realization of the coefficient of permeability for k_0=10^{-4} cm/s, λ=9×10^{-8}s^{-1}; CoV_k=100% and θ_k=20 m; a) permeability realization and b) flow vectors

Flow analysis is repeated numerous times using Monte Carlo simulations. Each simulation of the Monte Carlo process involves the same mean trend, standard deviation and spatial correlation length of the permeability; however, the spatial distribution of properties varies from one simulation to next. Following a "sufficient" number of simulations, output quantities of interest such as the discharge flow rate beneath the gravity dam can be simulated and statistically analyzed to produce estimates of important statistical parameters. A sample realization of the inherent variability of the permeability consisting simultaneous consideration of deterministic and stochastic variation of the permeability coefficient is shown in Figure 9 along with the velocity vectors. The effect of random variation of the permeability is obvious when considering velocity vectors. Tortuosity and flow interlocking occur due to the sharp variation of permeability within adjacent elements. The overall implication is that the equivalent conductivity of the medium decreases due to long flow paths and flow interlocking caused by tortuous stream lines.

Table 3 briefly represents the adopted input parameters for modeling the proposed problem. For each set of analysis, *500* realizations were made, and the moving average discharge flow rate is depicted in Figure 10 for different coefficients of variation and scales of fluctuation.

Table 3. Input parameters varied in the study

Parameter	Values Considered
Geometry of the model	L=30m, H=10m, B=10m
CoV_k (%)	20,40,60,80,100
θ_k (m)	2,4,10,20,30,60
Anisotropy Ratio, A_R	1,10,100
λ (s^{-1})	9×10^{-8}
k_0 (cm/s)	10^{-4}
Hydraulic head	h=10m

It is observed that for a specific scale of fluctuation, discharge flow rate of water beneath the gravity dam decreases with COV_k. Another translation of Figure 10 is provided in Figure 11 demonstrating the variation of the mean discharge flow rate against the coefficient of variation of the permeability for different scales of fluctuation. The input parameters relating to the mean, standard deviation, and spatial correlation length are assumed to be defined at the "point" level. While statistics at this resolution are obviously impossible to measure in practice, they represent a fundamental baseline of the inherent soil variability which cannot be avoided when Cholskey decomposition technique is utilized to take account of correlation. At the point scale, one could just as easily be inside a void (very high permeability) or inside a "chunk" of granite (very low permeability). The introduction of impervious elements or elements with very low permeability beneath dam foundation which is probable when high variability is assumed is implied as flow barriers or "blocked" pipe which induces tortuosity owing to reduced discharge flow rate (Figure 11).

Another important observation from Figure 11 is the effect of the correlation length on the mean discharge flow rate which can be found in Figure 12 more clearly. For "ragged" random fields with lower spatial correlation length, a sharp reduction in discharge flow rate is seen with variation in COV_k due to the formation of so-called "chunks of granite" inside dam foundation. It is well known that as the scale of fluctuation becomes negligible, the effective permeability approaches the geometric mean $K_G = \mu_k \exp(-\frac{1}{2}\sigma_{lnk}^2)$ (Dagan, 1989) which for fixed μ_k illustrates the reduction in flow rate with variation of the permeability due to the decreased equivalent permeability K_G. However, for smooth random fields with higher correlation length, more discharge flow rate is calculated due to uniform formation of permeable elements. In limit as $\theta_k \to \infty$, the permeability field coincides with the deterministic distribution everywhere as observed from Figure 12. In this case, the flow statistics are expected to approach these obtained by using a single deterministic variable k to model the field $k(\tilde{x}) = k_z$ introduced in Eq. (7). In such case, it can be shown that $\mu_{lnq} = ln(q_{\mu k}) - \frac{1}{2}\sigma_{lnk}^2$ and $\sigma_{lnq}^2 = \sigma_{lnk}^2$, so from lognormal transformation laws, the mean flow rate μ_q is concluded to approach $q_{\mu k}$ independent of CoV_K as illustrated by the θ_k=60 case in Figures 11 and 12.

The coefficient of variability of the discharge flow rate for different levels of variability of permeability is illustrated in Figure 13 for different values of the scale of fluctuation. It is clear that the dispersity of discharge flow results is in linear proportion to the variability of permeability coefficient.

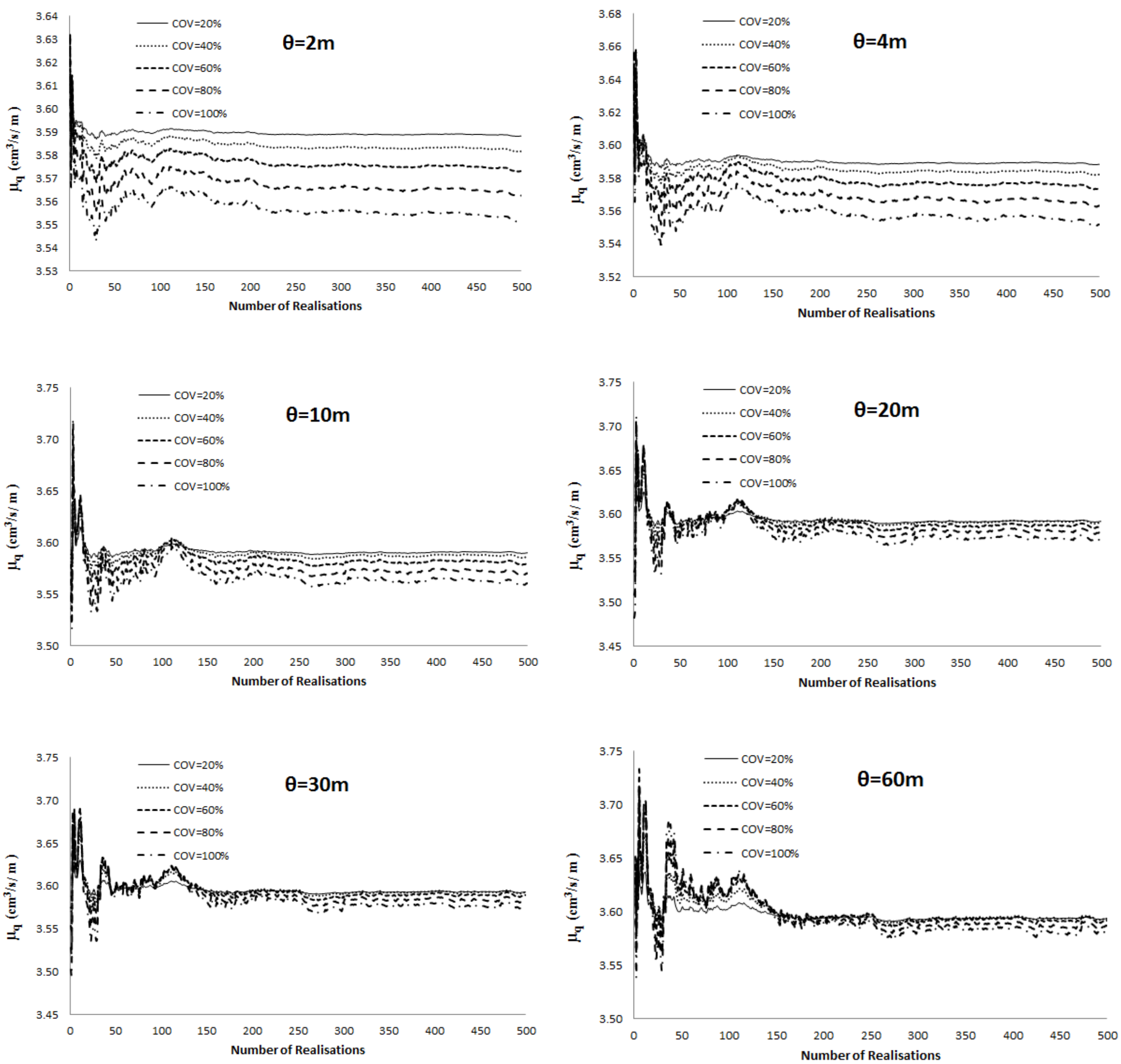


Fig. 10. Moving average discharge flow rate variation with the number of realizations

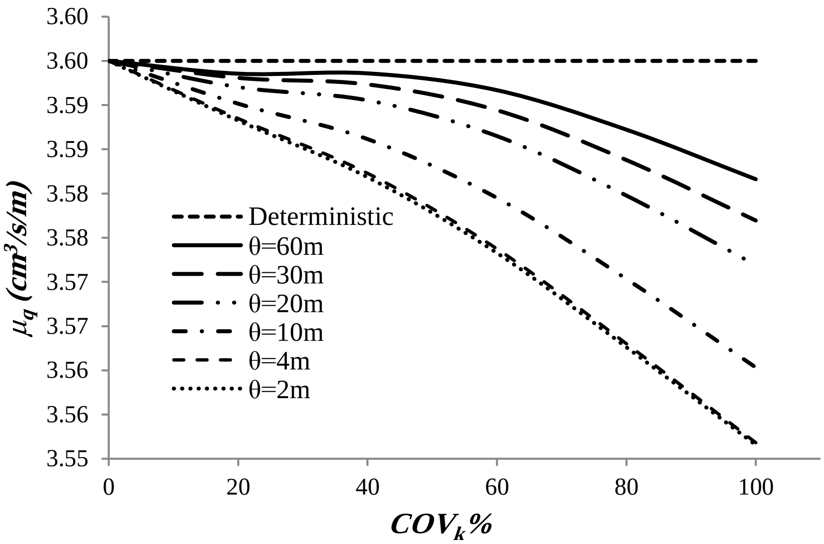


Fig. 11. The mean discharge flow rate variation with respect to COV_k in different scales of fluctuation

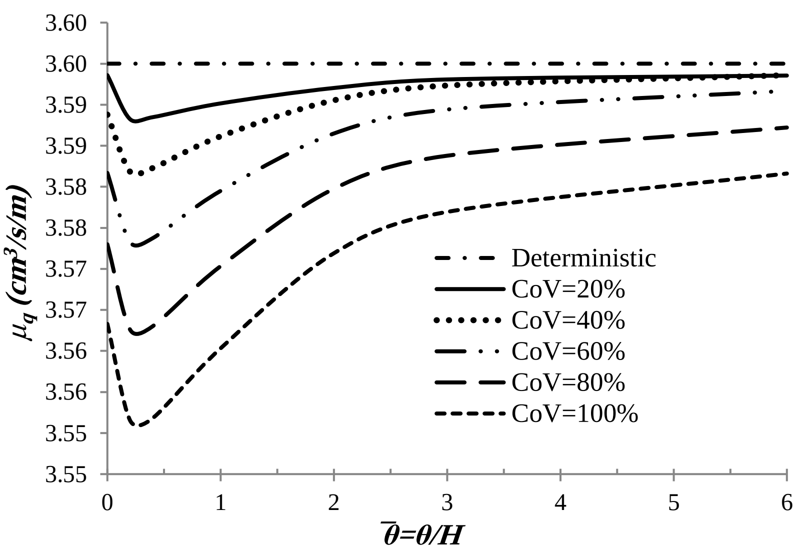


Fig. 12. The mean discharge flow rate variation with the scale of fluctuation for different CoV_k

This means that for a specific correlation distance, the spatial variability of the coefficient of permeability increases the variability of discharge flow rate. Correlation distance is also affecting the variance of discharge flow rate. For very short sales of fluctuations, the variance of flow rate is very small, as evidenced by trivial CoV_q values in Figure 13, however, it increases as the scale of fluctuation increases. In the limit as $\theta_k \rightarrow \infty$, it can be shown that $\sigma^2_{lnq} = \sigma^2_{lnk}$ as shown in Figure 13.

EFFECT OF MECHANICAL ANISOTROPY

Mechanical anisotropy implies directionality in hydraulic conductivity. This means that different coefficients of permeability are expected for horizontal and vertical directions. Heterogeneity anisotropy is regarded to the stochastic variation of permeability in two different directions. If the correlation length of the permeability, representing a random field is different in two directions, it is called a stochastic anisotropic field.

The case with which fluids flow through natural alluvial deposits has long been identified as one of the most important parameters controlling reservoir performance. If permeability were the same at all places and in all directions (homogeneous and isotropic) then measuring the flow through a sample of soil would reveal its value. However, sediment type and grain size distribution may vary through a reservoir leading to variation in permeability. Therefore, permeability measured at the same point in the horizontal direction k_h, may be different from permeability measured in vertical direction k_v. Although anisotropy strictly refers to the directional dependency of a measurement; the ratio k_h/k_v is often used to quantify permeability anisotropy or anisotropy ratio, A_R. Values of 1, 10 and 100 are adopted to investigate the effect of anisotropy in seepage problem. Discharge flow rates resulting from the deterministic analysis for each of anisotropic cases are shown in Table 4. It is evident from Table 4 that as the anisotropy ratio increases, horizontal permeability of the soil increases and discharge flow rate increases significantly as horizontal flow channels are generated as shown in Figure 14. These results can show the importance of vertical drains in liquefaction controlling systems underneath important structures.

Figure 14 shows the flow net beneath gravity dam for two different anisotropy ratios of 10 and 100. It is evident from the picture that an increase in the anisotropy ratio gives rise to the formation of horizontal flow channels and the flow practically occurs in horizontal channels when the anisotropy ratio is large enough.

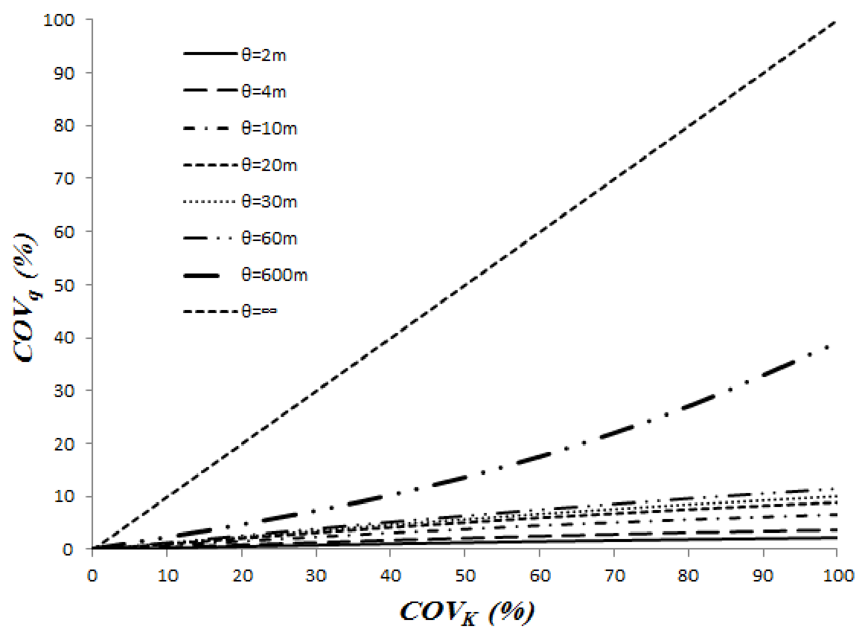
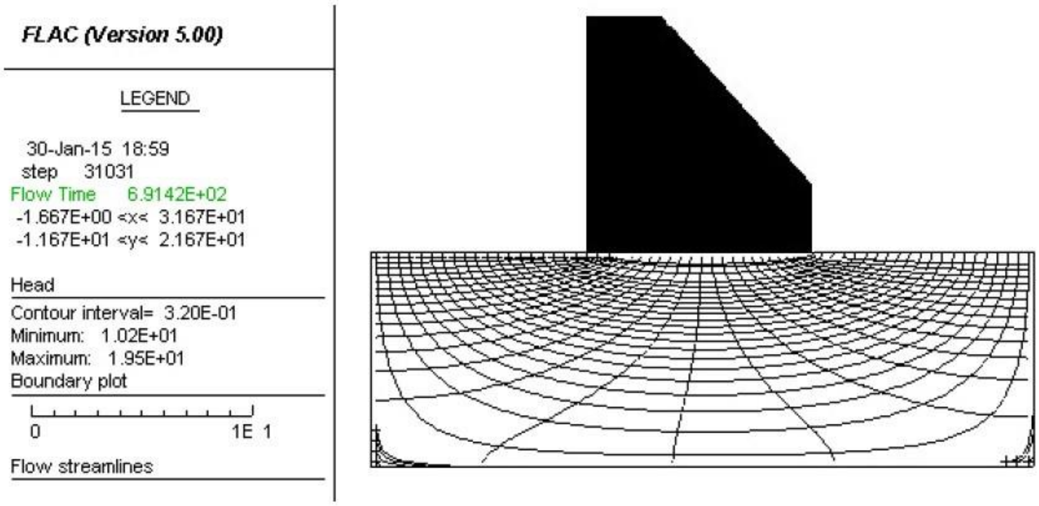


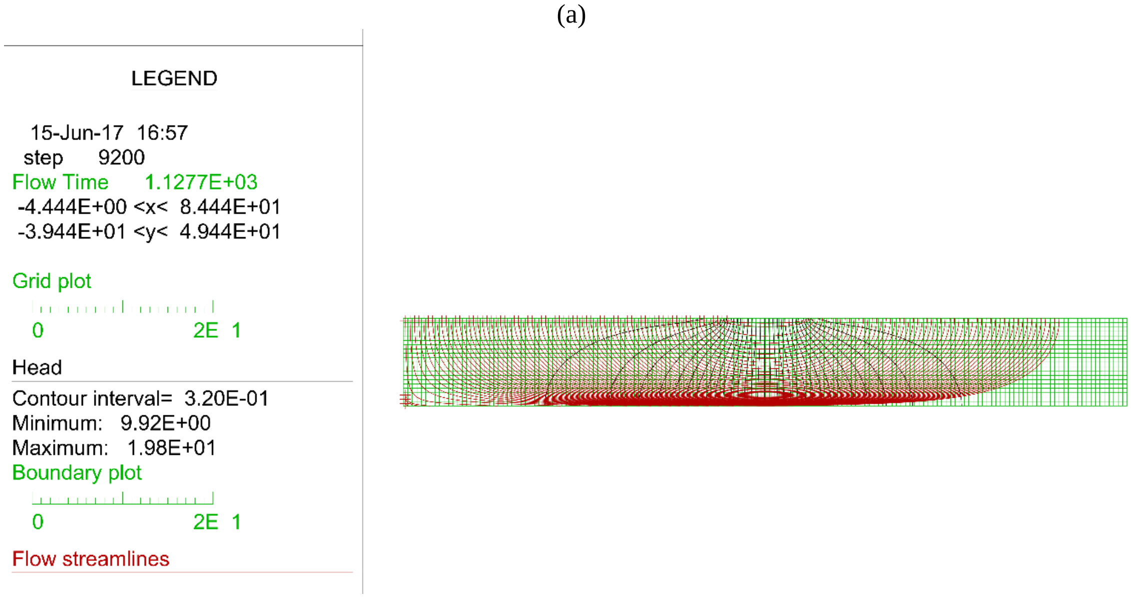
Fig. 13. Coefficient of variation of discharge flow rate for different values of CoV_k and scale of fluctuation

Table 4. Results of deterministic anisotropic flow analyses

Anisotropy Ratio, A_R	**q (cm³/s/m)**
1	3.59
10	24.30
100	91.70



(a)



(b)

Fig. 14. Flow net in anisotropic soil mass with; a) $A_R = 10$, b) $A_R = 100$

Figures 15 and 16 displays how the average discharge flow rate varies with the anisotropy ratio for different values of CoV_k and two different scales of fluctuation of 2 m and 60 m respectively. Two observations are made when considering these Figures. It is first noted that as the anisotropy ratio increases, the mean discharge flow rate increases dramatically at a constant vertical permeability. It is seen that introduction of anisotropy ratios of 10 and 100 will lead to 580% and 2450% increase in discharge flow rate. The second observation is that CoV_k effect is not practically noticeable when anisotropic deposits are encountered. This effect even vanishes when the scale of fluctuation is large enough.

Intuitively, one can think of the increase in mean discharge flow rate by first considering one-dimensional flow down a 'pipe' as confirmed by considering Figure 13, the total flow rate of which is heavily dependent on the minimum permeability encountered along the way. In the case of the small scale of fluctuation, (Figure 15), the chance of getting a small permeability or 'blocked pipe' also increases as the CoV_k increases, resulting in a potential decrease of the mean flow. This effect, however, is vanished as even if one flow pipe gets blocked, other parallel pipes direct the flow to the downstream. This is the reason why CoV_k does not have a major effect on discharge flow rate through anisotropic deposits even if when the scale of fluctuation is too low or a "ragged" random field is adopted. In limits, when the scale of fluctuation bears very high values in comparison to the element size, the estimated discharge flow rate converges to the deterministic values as provided in Table 4. Figure 16 illustrates this effect efficiently. In general, it is observed that mechanical anisotropy fades the heterogeneity effect and horizontal channel formation causes the seepage flow to find its way through parallel channels irrespective to the degree of heterogeneity observed in the stratum.

MECHANICAL VS. HETEROGENEITY ANISOTROPY

A discussion is made herewith when comparing the results of normalized discharge flow rate of the current study with those of Ahmed (2012) as illustrated in Figure 17. It is seen that for both types of anisotropy, discharge flow rate increases when the anisotropy ratio is increased.

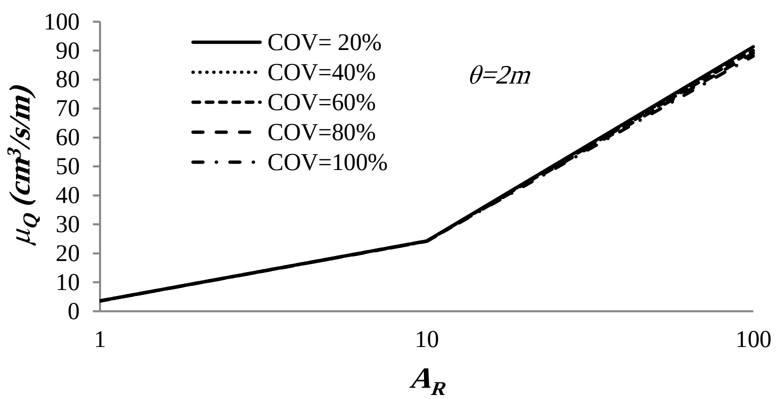


Fig. 15. Variation of the average discharge flow rate with anisotropy ratio for different COV_k values and low correlation assumption

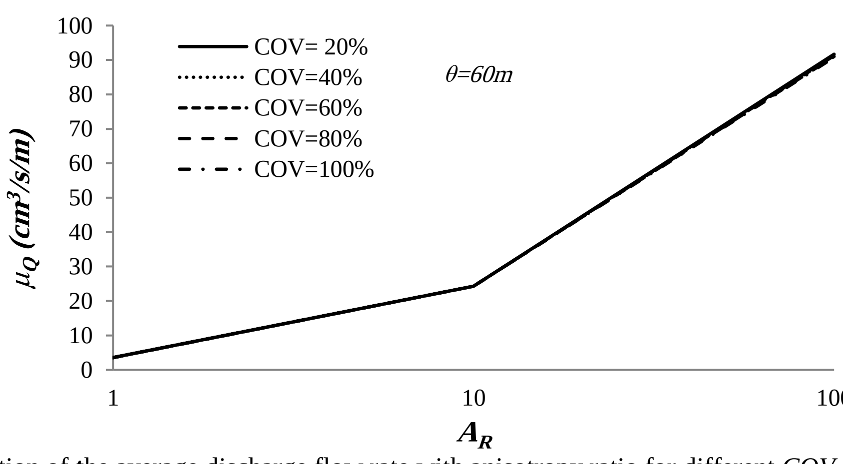


Fig. 16. Variation of the average discharge flow rate with anisotropy ratio for different COV_k values and high correlation assumption

However, the difference between increasing rate in two effects is noticeable. Ahmed (2012) explains that for larger values of anisotropy ratio, each row of cells underneath the water barrier structure is strongly correlated and may be conceived to act as an autonomous flow pipe. Hence, the deposit under the structure can be thought of as a group of flow pipes, even though each has its own hydraulic conductivity. He also emphasizes that the chance of encountering low hydraulic conductivity cells along each row of elements is small due to the strong correlation of the cells in the horizontal direction. The same explanation applies when justifying the trend of variation against the mechanical anisotropy. Even though the trends of variation in two types of anisotropy are similar, the approaching points at limits are totally different. As the effect of mechanical anisotropy is concerned, it is observed that the values of normalized discharge flow rate approach to the deterministic values corresponding to each anisotropy ratios as illustrated in Table 4. For the mechanical anisotropy effect, the starting point is the deterministic solution of the isotropic case, and the values move far beyond the isotropic deterministic solution, whereas for the heterogeneity anisotropy effect the isotropic deterministic solution of the problem studies by Ahmed (2012) will be the approaching point (not the starting point). The small difference between the isotropic deterministic solutions of two cases lies behind the fact that the seeping volumes in two problems are different. Indeed, if we divide the normalized discharge flow rate by the seeping volume, the isotropic deterministic solutions will almost coincide. This comparison reveals that the effect of mechanical anisotropy is more pronounced in comparison to the heterogeneity anisotropy investigated by Ahmed (2012). It means that mechanical anisotropy negligence will lead to very unsafe prediction of discharge flow rate if one compromises on an isotropic deterministic solution.

CONCLUSIONS

The need for considering heterogeneity and anisotropy of natural alluvial deposits is emergent n common engineering practice when solving various geotechnical problems.

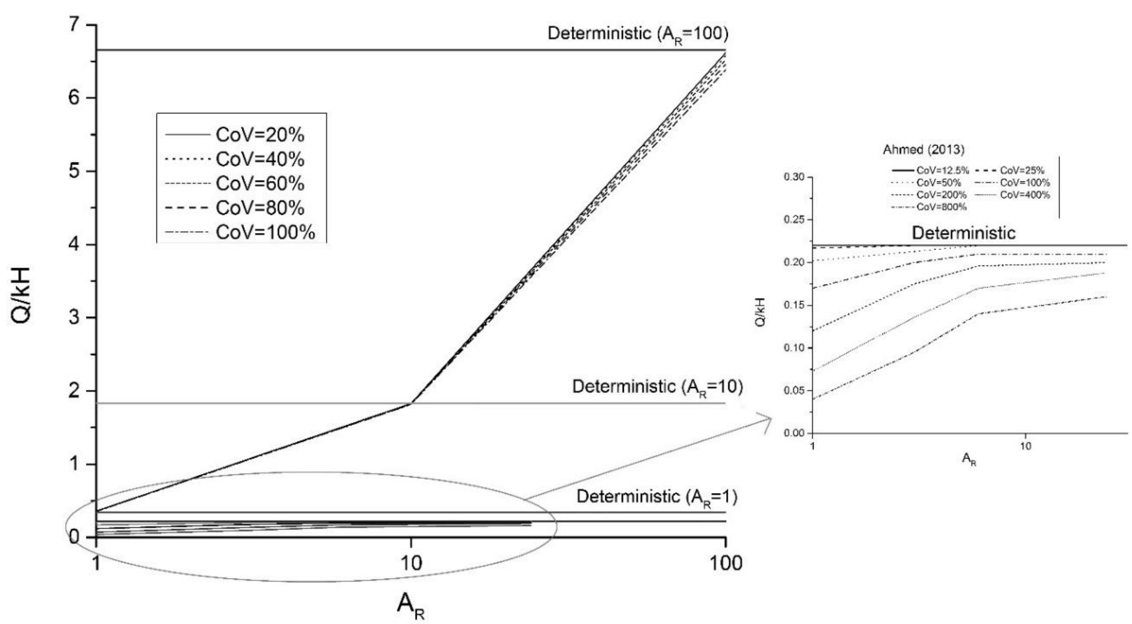


Fig. 17. Comparison between mechanical anisotropy of current study and anisotropy of heterogeneity (Ahmed, 2012)

Numerous experimental works prove that in-situ soil properties of soil and rock foundations deviate from homogeneous and isotropic assumptions as discussed earlier. Inherent variability along with the anisotropy of hydraulic conductivity were studied into a seepage problem by considering a deterministic and stochastic variation of permeability coefficient in a natural alluvial deposit and mechanical anisotropic of permeability field simultaneously. Linear variation was assumed for the deterministic part, and the stochastic component was assumed log-normally distributed stationary and correlated random field. The coefficient of variation and the scale of fluctuation of the permeability coefficient were varied through different sets of analyses, and their effects were studied on the statistics of the discharge flow rate through a natural alluvium for both isotropic and anisotropic conditions. Mean and variance of the discharge flow rate were investigated, and following results were concluded:

1. High coefficient of variation of the permeability coefficient leads to the formation of impermeable or low-permeability zones in dam foundation owing to decreased discharge flow rate.
2. Low scale of fluctuation acts similar to high variation due to "ragged" permeability field generation leading to reduced discharge flow rate through the alluvial deposit.
3. In the extreme, where the scale of fluctuation is too high, the mean discharge flow rate converges to the deterministic solution obtained by a single deterministic flow analysis.
4. Variability in permeability coefficient induces variation in discharge flow rate calculation results. A linear variation between CoV_k and CoV_q was confirmed by flow analysis results.
5. In the limit, when a smooth random field is generated through the adoption of a very high scale of fluctuation, maximum variation in discharge flow results is expected.
6. Anisotropy of hydraulic conductivity will result in dramatic increase in discharge flow rate at a constant vertical permeability in alluvial deposit through the formation of long and horizontal flow channels inside the stratum.
7. For highly anisotropic alluviums, variation in permeability coefficient will lead to the formation of impermeable zones in some horizontal flow channels although other parallel channels control the performance. This implies that when the stratum is mechanically and hydraulically anisotropic, the variability of permeability is not dominant for seepage discharge flow rate estimation and indeed a single deterministic analysis was proven sufficient.

The final remark is that for isotropic permeability fields, disregarding the effect of random heterogeneity of permeability in natural alluvial deposits and relying solely upon a single deterministic seepage analysis will lead to overestimation of the discharge flow. This implies to remain on the safe side. However, disregarding mechanical anisotropy was shown leading to a very unsafe solution. It is believed that anisotropy in hydraulic conductivity is an unavoidable property of alluvial deposits which are cross anisotropic in nature. It was shown that for anisotropic permeability filed, the degree of variability of permeability field is not that important due to the formation of highly permeable parallel flow channels which fades the effect of impervious overlying or underlying parallel channels. This means that for highly anisotropic stratum, a single anisotropic deterministic seepage analysis would suffice to render discharge flow rate.

REFERENCES

Ahmed, A.A. (2012). "Stochastic analysis of seepage under hydraulic structures resting on anisotropic heterogeneous soils", *Journal of Geotechnical and Geoenvironmental Engineering*, 139(6), 1001-1004.

Budhu, M. (2011). Soil Mechanics and Foundations (3rd Edition), John Wiley & Sons.

Benson, C. H. (1993). "Probability Distributions for Hydraulic Conductivity of Compacted Soil Liners", *Journal of Geotechnical and Geoenvironmental Engineering*, 119(3), 471–486.

Cho, S.E. (2012). "Probabilistic analysis of seepage that considers the spatial variability of permeability for an embankment on soil foundation", *Engineering Geology*, 133-134, 30–39.

Clennell, M.B., Dewhurst, D.N., Brown, K.M. and Westbrook, G.K. (1999). Permeability anisotropy of consolidated clays, Geological Society, London, *Special Publications*, 158(1), 79-96.

Dagan, G. (1989). *Flow and Transport in Porous Formations*, Springer-Verlag, New York, NY.

Duncan, J.M. (2000). "Factors of safety and reliability in geotechnical engineering", *Journal of Geotechnical and Geoenvironmental Engineering*, 126(4), 307-316.

Fenton, G. A. and Griffiths, D.V. (1995). "Flow through earth dams with spatially random permeability", Proceedings of the 10th ASCE Engineering Mechanics Conference, Boulder, Colorado.

Fenton, G.A. and Griffiths, D.V. (1996). "Statistics of free surface flow through stochastic earth dam", *Journal of Geotechnical Engineering*, 122(6), 427-436.

Fenton, G.A. and Griffiths, D.V. (1997). "Extreme hydraulic gradient statistics in stochastic earth dam", *Journal of Geotechnical and Geoenvironmental Engineering*, 123(11), 995-1000.

Griffiths, D.V. and Fenton, G.A. (1993). "Seepage beneath water retaining structures founded on spatially random soil", *Geotechnique*, 43(4), 577-87.

Griffiths, D.V. and Fenton, G.A. (1997), "Three-dimensional seepage through spatially random soil", *Journal of Geotechnical and Geoenvironmental Engineering*, 123(2), 153-160.

Jamshidi Chenari, R., and Seyed Noori, N. (2016). "Uncoupled consolidation analysis of clay deposits with linearly varying characteristics with depth", *Iranian Journal of Science and Technology, Transactions of Civil Engineering*, 41(1), 49-53.

Jamshidi Chenari, R. and Alaie, R. (2015). "Effects of anisotropy in correlation structure on the stability of an undrained clay slope", *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards*, 9(2), 109-123.

Jamshidi Chenari, R. and Mahigir, A. (2014). "The effect of spatial variability and anisotropy of soils on bearing capacity of shallow foundations", *Civil Engineering Infrastructures Journal*, 47(2), 199-213.

Le, T.M.H., Gallipoli, D., Sanchez, M. and Wheeler, S.J. (2012). "Stochastic analysis of unsaturated seepage through randomly heterogeneous earth embankments", *International Journal for Numerical and Analytical Methods in Geomechanics*, 36(8), 1056-1076.

Lumb, P. (1974). *Application of statistics in soil mechanics*, Soil Mechanics New Horizons, IK Lee, (ed.).

Oda, M. (1972). "Initial fabrics and their relations to mechanical properties of granular material", *Soils and Foundations*, 12(1), 17-36.

Rafiezadeh, K. and Ataie-Ashtiani, B. (2013). "Three dimensional flow in anisotropic zoned porous media using boundary element method", *Engineering Analysis with Boundary Elements*, 36 (5), 812-824.

Salmasi, F., Mansuri, B., and Raufi, A. (2015). "Use of numerical simulation to measure the effect of relief wells for decreasing uplift in a homogeneous earth dam", *Civil Engineering Infrastructures Journal*, 48(1), 35-45.

Seyed Noori, N. (2014). "Analytical investigation on consolidation of heterogeneous natural alluvial deposits", M.Sc. Thesis, Faculty of Pardis, The University of Guilan, Guilan, Iran.

Scholes, O.N., Clayton, S.A., Hoadley, A.F. and Tiu, C. (2007). "Permeability anisotropy due to consolidation of compressible porous media", *Transport in Porous Media*, 68(3), 365-387.

Srivastava, A., Babu, G.S. and Haldar, S. (2010). "Influence of spatial variability of permeability property on steady state seepage flow and slope stability analysis", *Engineering Geology*, 110(3), 93-101.

Yu, C., Deng, S.C., Li, H.B., Li, J.C. and Xia, X. (2013). "The anisotropic seepage analysis of water-sealed underground oil storage caverns", *Tunneling and Underground Space Technology*, 38 (September), 26-37.

Zhang, F., Cosson, B., Comas-Cardona, S. and Binetruy C. (2011). "Efficient stochastic simulation approach for RTM process with random fibrous permeability", *Composites Science and Technology*, 71 (12), 1478.