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Technical Note

Damage Detection of Axially Loaded Beam: A Frequency-Based Method

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ABSTRACT: The present study utilizes an analytical method to formulate the three lowest modal frequencies of axially-loaded notched beam through both crack location and load level in a specific format that can be used in existing frequency-based crack-identification methods. The proposed formula provides a basis to shift into two states, one with axial loading and the other without any loading whatsoever. When any two natural frequencies in simply-supported beam with an open crack, subjected to axial load, are measured, crack position and extent can be determined, using a characteristic equation, which is a function of crack location, sectional flexibility, and eigenvalue (natural frequency). Theoretical results show high accuracy for service axial loads. In this range, errors for crack location and extent are less than 12% and 10%, respectively.

Keywords: Axial Load, Characteristic Equation, Damage Detection, Eigen Frequency, Notched Beam.

INTRODUCTION

Deteriorating infrastructure, caused by aging, accidental events, overloading, etc., has caused the development of damage detection techniques for structural components during the last three decades. Structure health monitoring and damage detection at the possibly earliest stage is of utmost importance in civil, mechanical, and aerospace engineering communities, since damage accumulation may lead into a catastrophic structure failure.

Several analytical, numerical, and experimental methods have been proposed to investigate dynamic behavior of cracked structures (e.g. see Saavedra and Cuitino, 2001; Sinha and Friswell, 2002; Zheng and Kessissoglou, 2004; Kisa and Gurel, 2007; Orhan, 2007; Mazanoglu et al., 2009; Attar, 2012; Cademi and Calio, 2013; Gomes and Almeida, 2014). As a result of its experimentation simplicity, eigen Frequency changes have been widely investigated (e.g. see Zheng and Ji, 2012; Mazanoglu and Sabuncu, 2012; Bakhtiari-Nejad et al., 2014).

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Much of the work in this field deals with modeling a crack in beam, subject to various boundary conditions. For the case of multiple cracks, Caddemi and Calio (2009) presented exact closed-form solution expressions for the vibration modes of the Euler-Bernouli beam at the presence of multiple concentrated cracks. In their study, cracks are assumed to remain open during vibration and are modeled as a sequence of Dirac's delta generalized functions in the flexural stiffness. Zhiang and Ji (2012) provided an approximate method to determine the stiffness and the fundamental frequency of a cracked beam. Their method greatly simplified the calculation of cracked beam with complicated crack configurations. Jassim et al. (2013) conducted a research to analyze the vibration for occurrence of any damage in a cantilever beam. Their results showed that the change of natural frequency is a feasible and viable tool for damage detection. Labib et al. (2014) studied the problem of calculating the natural frequencies of beams with multiple cracks and frames with cracked beam, using Wattrick-Williams algorithm.

In these studies, effects of axial force were not taken into consideration, even though such impacts can be significant on frequency response of structural elements, such as building columns. Binici (2005) studied vibration of beams with multiple cracks, subjected to axial force. In his study, Cracks are assumed to introduce new local flexibility changes and are modeled as rotational springs. Meeting one set of boundary conditions as well as continuity and jump ones leads to mode shape functions and a second-order determinant that needs to be solved for its roots. The present paper adopts this method to solve the direct problem. Mei et al. (2006) proposed a concise and systemic approach to both free and forced vibration of complex-axially loaded Timoshenko beams with discontinuities such as cracks and sectional changes, based on wave vibration analysis. Viola et al. (2007) utilized a procedure in which dynamic stiffness matrix was combined, and thus introduced a line-spring element, letting them model the crack beam. Cicirello and Palmeri (2014) studied pre-damaged Euler-Bernouli beams with any number of cracks, subject to axial forces in combination with transverse loads. Dirac delta functions are utilized as switching variables to present cracks' opening and closing. Moradi and Jamshidi Moghadam (2014) investigated cracked post-buckled beams. Their results showed geometric imperfection and the impact of applied load on not only modal parameters but also the crack size and position.

Lele and Maiti (2002) derived the characteristic equation of simply-supported Timoshenko beam and used it for inverse problem by the variations of natural frequencies. Khiem (2006) developed the general frequency equation of damage beams for elastic end supports. Khiem and Toan (2014) proposed a novel method to detect an unknown number of multiple cracks from the measured natural frequencies. Their research also developed the so-called crack scanning method. The present study, however, offers a new approximate frequency-based method to identify the damages of simply-supported cracked beam, which is under the influence of axial load. It primarily focuses on the method, proposed by Binici (2005) to analyze the vibration of cracked beam at the presence of axial load in case of a direct problem, deriving new polynomial formulas that separate axial load effects on frequency domain. Approximate formulas generally depend on load level and crack location, not to mention the Eigen mode under consideration. The output of this procedure is a coefficient that translates the frequency of axially-loaded damaged beam to one with no axial force. As far as the authors know, there has not been any solution from the scholars to detect the damages of the beam, under axial load. This paper presents a frequency-based method to solve the existing inverse problem.

MATERIALS AND METHODS

Direct Method

Simply-supported beam with a single crack has been taken into consideration (Figure 1). Binici (2005) presented the determinant of a 2×2 matrix to be solved for eigenvalue, which results in natural frequencies of the cracked beam, subject to axial load. Crack is assumed to remain open. Crack breathing mechanism that needs time domain analysis, has not been considered in this study either.

Buckling load of damaged beam is determined by setting circular natural frequency (ω) to zero in order to solve the characteristic equation for P. Different levels of axial loads (P) are applied to beam due to the buckling load of each damage scenario. Results are extracted for the first three modes, more affected by axial load in comparison to the higher modes. Figures 2-7 represent variation of natural frequencies of damaged beam for different axial load levels and specified crack severity ($\xi = a/h = 0.4$). Material and geometry properties of the case study are elasticity modulus $E = 2.07\times10^{11}$ N/m^2, shear modulus $G = 79\times10^9$ N/m^2, Poisson ratio $\upsilon = 0.3$, mass per unit volume $\rho = 7860$ Kg/m^3, length $L = 400$ mm, and square cross section $B = H = 12.7$ mm.

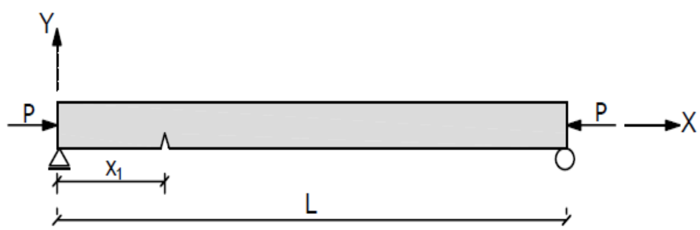

Fig. 1. Beam with a single crack subjected to axial force

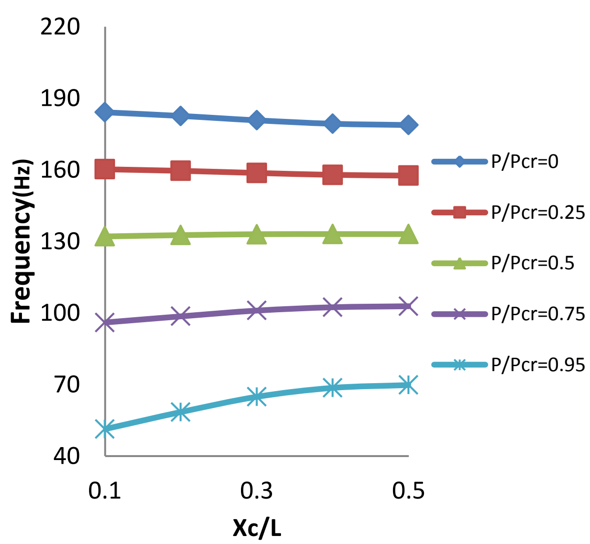

Fig. 2. First frequency of cracked beam, subject to compressive load

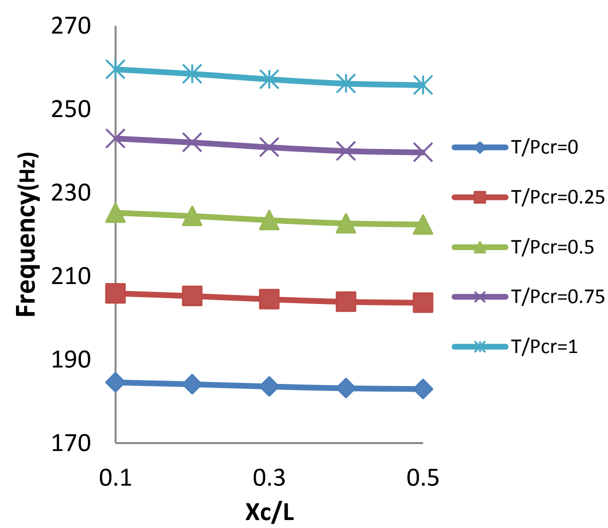

Fig. 3. First frequency of cracked beam, subject to tensile load

It can be observed that as the crack location approaches the center of the beam, the reduction in natural frequency of the beam, due to the presence of crack, increases. Non-dimensional formulas for fundamental frequency are derived below.

For compressive load (P):

$$f_{1dc}=f_{1d}\left[\lambda_{1c}(\beta)\sqrt{1-\left(\frac{P}{P_{cr(d)}}\right)^2}\right] \quad (1)$$

$$\lambda_{1c}(\beta)=a\beta^3+b\beta^2+c\beta+d \quad (2)$$

$$a=-1.552\left(\frac{P}{P_{cr(d)}}\right)^2+0.648\left(\frac{P}{P_{cr(d)}}\right)-0.142 \quad (3)$$

$$b=0.944\left(\frac{P}{P_{cr(d)}}\right)^2-0.376\left(\frac{P}{P_{cr(d)}}\right)-0.085 \quad (4)$$

$$c=0.176\left(\frac{P}{P_{cr(d)}}\right)^2-0.088\left(\frac{P}{P_{cr(d)}}\right)-0.016 \quad (5)$$

$$d=0.184\left(\frac{P}{P_{cr(d)}}\right)^2-0.43\left(\frac{P}{P_{cr(d)}}\right)+0.99 \quad (6)$$

For tensile load (T):

$$f_{1dt}=f_{1d}\left[\lambda_{1t}(\beta)\sqrt{1-\left(\frac{T}{P_{cr(d)}}\right)^2}\right] \quad (7)$$

$$\lambda_{1t}(\beta)=a\beta^3+b\beta^2+c\beta+d \quad (8)$$

$$a=0.108\left(\frac{T}{P_{cr(d)}}\right)^2+0.238\left(\frac{T}{P_{cr(d)}}\right)+0.004 \quad (9)$$

$$b=0.072\left(\frac{T}{P_{cr(d)}}\right)^2-0.156\left(\frac{T}{P_{cr(d)}}\right)-0.003 \quad (10)$$

$$c=0.004\left(\frac{T}{P_{cr(d)}}\right)^2-0.011\left(\frac{T}{P_{cr(d)}}\right)-0.001 \quad (11)$$

$$d=-0.276\left(\frac{T}{P_{cr(d)}}\right)^2+0.225\left(\frac{T}{P_{cr(d)}}\right)+1.04 \quad (12)$$

It is evident that even the small axial loads, which are actually realistic for service conditions of some structural elements, can result in shifts up to 15% in the first mode Eigen-frequencies.

The proposed formula for second mode at the presence of axial load is given as below:

For compressive load (P):

$$f_{2dc}=f_{2d}\times\lambda_{2c} \quad (13)$$

$$\lambda_{2c}=-0.129\left(\frac{P}{P_{cr(d)}}\right)+1.002 \quad (14)$$

For tensile load (T):

$$f_{2dt}=f_{2d}\times\lambda_{2t} \quad (15)$$

$$\lambda_{2t}=0.111\left(\frac{T}{P_{cr(d)}}\right)+1.001 \quad (16)$$

Figures 6 and 7 represent frequency variation for third mode at the presence of axial loads.

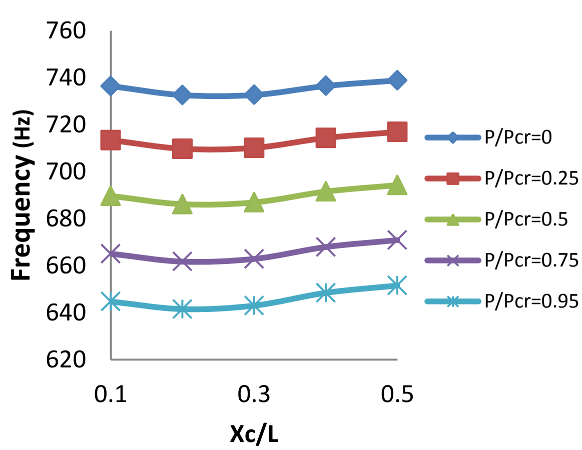

Fig. 4. Second frequency of cracked beam, subject to compressive loads

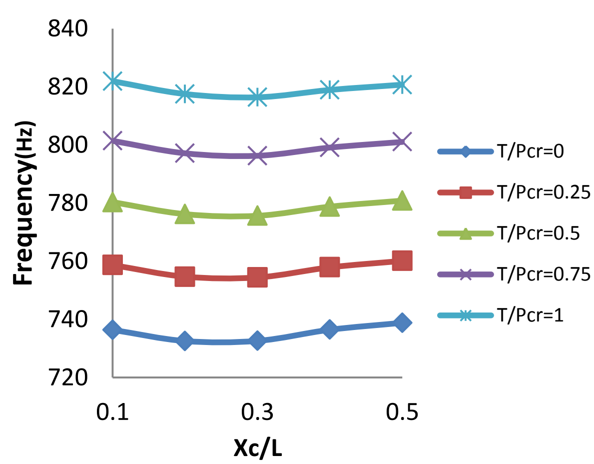

Fig. 5. Second frequency of cracked beam, subject to Tensile loads

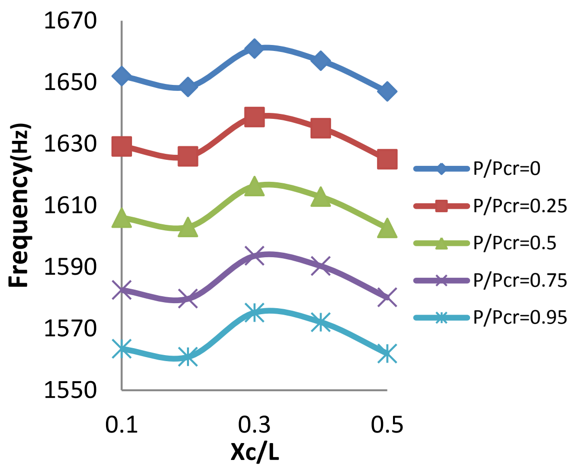

Fig. 6. Third frequency of cracked beam, subject to compressive load

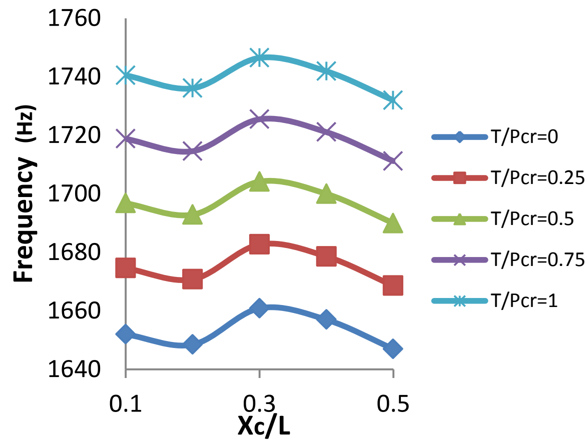

Fig. 7. Third frequency of cracked beam, subject to tensile load

As it can be seen, formulas for second and third modes are functions of axial load levels that simplify the damage-detection procedure for beam-like structures. Frequency changes for the third mode, at the presence of tensile load, can be formulated as:

For compressive load (*P*):

$$f_{3dc}=f_{3d}\times\lambda_{3c} \tag{17}$$

$$\lambda_{3c}=-0.055\left(\frac{P}{P_{cr(d)}}\right)+1.002 \tag{18}$$

For tensile load (*T*):

$$f_{3dt}=f_{3d}\times\lambda_{3t} \tag{19}$$

$$\lambda_{3t}=0.051\left(\frac{T}{P_{cr(d)}}\right)+1.000 \tag{20}$$

As it can be seen, formulas for second and third modes are functions of axial load levels that simplify the damage-detection procedure for beam-like structures. For the third frequency, *L*/6 is the location, resulting in the maximum decrease, compared to that of the undamaged beam with axial force.

Inverse Method

Applied method in this section represents a characteristic equation, using transfer matrix that can predict crack location as well as severity of a simply-supported beam when two natural frequencies of the damaged beam are available (Lin, 2004).

Since axial loads can significantly affect Eigen-frequencies, existing characteristic equation would have obvious deviation, resulting in great errors in inverse problem solution. The proposed formulas in the previous section translate the Eigen-frequencies of axially-loaded cracked beam to the case without any axial loading where an initial crack position has been assumed. Summary of revised algorithm, proposed by the authors, is presented in Figure 8.

RESULTS AND DISCUSSION

In order to investigate the practicality of the revised algorithm, two scenarios have been considered. Relatively high levels of axial loads for both compression and tension are applied to the case study, described in direct problem (Direct Method). The crack identification equation is a nonlinear one, capable of being solved by means of standard Newton-Raphson numerical iterations. The authors in turn programmed a code to solve nonlinear equation, using MATLAB 2011. It is noteworthy that for different frequency pairs, the obtained crack parameters are not unique.

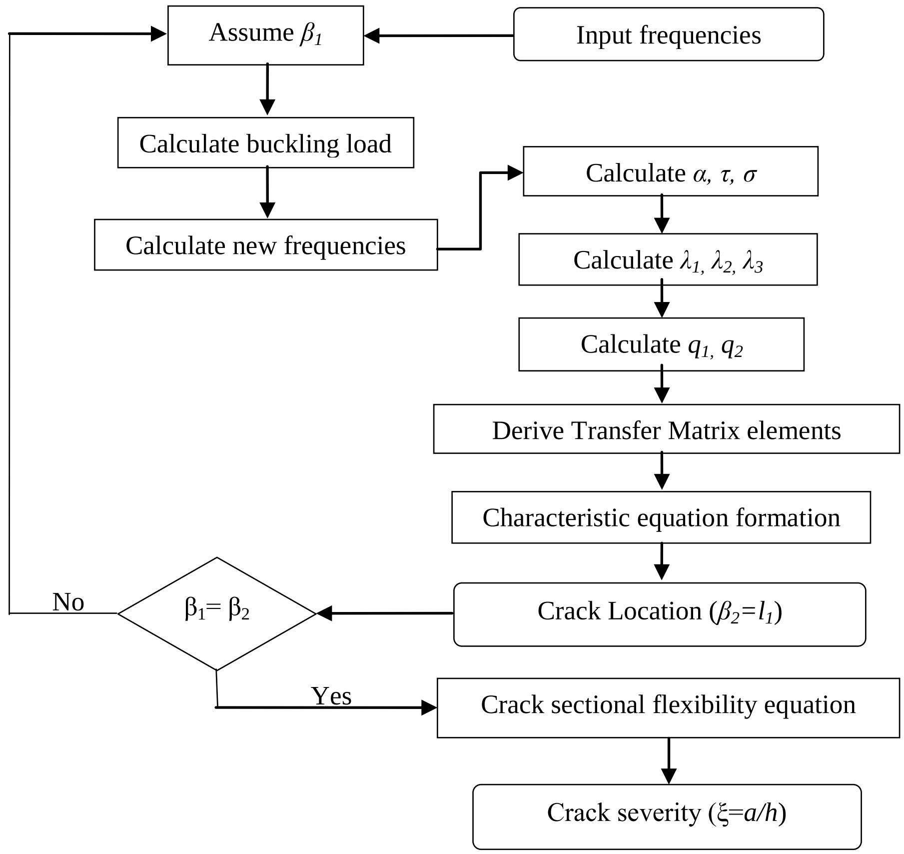

Fig. 8. Revised algorithm for damage detection of axially loaded beam with a crack

Cracked Beam, Influenced by Compressive Load

Compressive axial force P = 12775 N is applied to the model. Three lowest natural frequencies of cracked beam are reported in Table 1 with crack position and depth being $\beta = 0.3$ and $\xi = 0.4$ respectively.

Table 2 represents crack position and depth for each pair of Eigen-frequencies. As it can be seen, revised algorithm proves to be able to find out crack parameters.

Cracked Beam, Influenced by Tensile Load

Tensile axial force P = 19162 N is applied to the beam model with the same crack scenario, addressed in section "Cracked Beam, Influenced by Compressive Load". Table 3 represents natural frequencies of three lowest modes.

In Table 4 crack location and extent have been identified from any two of these three Eigen-frequencies.

It can be observed that the crack identification results, proposed in this article, are relatively acceptable for service axial loads ($0.5P_{cr} \geq P$). Higher axial load levels lead to an increase in the errors, belonging to calculated parameters. Results showed that crack position is strongly affected by axial load level.

Table 1. Frequencies of cracked beam subjected to compressive load

No. of Mode	Frequency (Hz)
1st Mode	132.5
2nd Mode	682.53
3rd Mode	1616.3

Table 2. Crack parameters obtained from proposed method (compressive load)

Eigen Frequencies of Cracked Beam	Crack Position (β = 0.3)		Crack Depth (ξ = 0.4)	
	Revised Method	Error (%)	Revised Method	Error (%)
ω_1 and ω_2	0.310	3.33	0.365	-8.75
ω_1 and ω_3	0.333	11.00	0.399	-0.25
ω_2 and ω_3	0.338	12.67	0.438	9.50

Table 3. Frequencies of cracked beam, subject to tensile load

No. of Mode	Frequency (Hz)
1st mode	238.6
2nd mode	789.38
3rd mode	1722.4

Table 4. Crack parameters obtained from proposed method (tensile load)

Eigen Frequencies of Cracked Beam	Crack Position (β = 0.3)		Crack Depth (ξ = 0.4)	
	Revised Method	Error (%)	Revised Method	Error (%)
ω_1 and ω_2	0.321	6.93	0.438	9.50
ω_1 and ω_3	0.328	9.33	0.379	-5.25
ω_2 and ω_3	0.344	14.67	0.368	-8.00

COCLUSIONS

The present study proposed a method to solve the inverse problem of a simply-supported beam with a crack, which subject to axial load. It revealed approximate formulas, which introduce frequency-translating coefficients in two states with and without axially loading. Crack identification is a frequency-based process that employs the proposed formulas to solve inverse problem. Results showed that obtained crack position and sectional flexibility for service axial loads are in good accord with the existing data. Other types of boundary conditions can also be considered through the similar procedure.

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REFERENCES

Attar, M. (2012). "A transfer matrix method for free vibration analysis and crack identification of stepped beams with multiple edge cracks and different boundary conditions", *International Journal of Mechanical Sciences*, 57(1), 19-33.

Bakhtiari-Nejad, F, Khorram, A. and Rezaeian, M. (2014). "Analytical estimation of natural frequencies and mode shapes of a beam having two cracks", *International Journal of Mechanical Sciences*, 78, 193-202.

Binici, B. (2005). "Vibration of beams with multiple open cracks subjected to axial force", *Journal of Sound and Vibration*, 287(1-2), 277-295.

Caddemi, S. and Calio, I. (2009). "Exact closed-form solution for the vibration modes of the Euler-Bernouli beam with multiple open cracks", *Journal of Sound and Vibration*, 327(3-5), 473-489.

Cicirello, A. and Palmeri, A. (2014). "Static analysis of Euler–Bernoulli beams with multiple unilateral cracks under combined axial and transverse loads", *International Journal of Solids and Structures*, 51(5), 1020-1029.

Gomes, H.M. and Almeida, F.J.J. (2014). "An analytical dynamic model for single-cracked beams including bending, axial stiffness, rotational inertia, shear deformation and coupling effects", *Applied Mathematical Modelling*, 38(3), 938-948.

Jassim, Z.A., Ali, N.N., Mustapha, F. and Abdul-Jalil, N.A. (2013). "A review on the vibration analysis for a damage occurrence of a cantilever beam", *Engineering Failure Analysis*, 31, 442-461.

Khiem, N.T. (2006). "Damage detection of beam by natural frequencies: General theory and procedure", *Vietnam Journal of Mechanics, VAST*, 28(2), 120-132.

Khiem, N.T. and Toan, L.K. (2014). "A novel method for crack detection in beam-like structures by measurements of natural frequencies", *Journal of Sound and Vibration*, 333(18), 4084-4103.

Kisa, M. and Gurel, M.A. (2007). "Free vibration analysis of uniform and stepped cracked beams with circular cross sections", *International Journal of Engineering Science*, 45(2-8), 364-380.

Labib, A., Kennedy, D. and Featherston, C. (2014). "Free vibration analysis of beams and frames with multiple cracks for damage detection", *Journal of Sound and Vibration*, 333(20), 4991-5003.

Lele, S.P. and Maiti, S.K. (2002). "Modeling of transverse vibration of short beams for crack detection and measurement of crack extension", *Journal of Sound and Vibration*, 257(3), 559-583.

Lin, H.P. (2004). "Direct and inverse methods on free vibration analysis of simply supported beams with a crack", *Engineering Structures*, 26(4), 427-436.

Mazanoglu, M. and Sabuncu, M. (2012). "A frequency based algorithm for identification of single and double cracked beams via a statistical approach used in experiment", *Mechanical Systems and Signal Processing*, 30, 168-185.

Mazanoglu, K., Yesilyurt, I. and Sabuncu, M. (2009). "Vibration analysis of multiple-cracked non-uniform beams", *Journal of Sound and Vibration*, 320(4-5), 977-989.

Mei, C., Karpenko, Y., Moody, S. and Allen, D. (2006). "Analytical approach to free and forced vibrations of axially loaded cracked Timoshenko beams", *Journal of Sound and Vibration*, 291(3-5), 1041-1060.

Moradi, S. and Jamshidi Moghadam, P. (2014). "Vibration analysis of cracked post-buckled beams", *Applied Mathematical Modelling*, 38(13), 3281-3294.

Orhan, S. (2007). "Analysis of free and forced vibration of a cracked cantilever beam", *NDT and E International*, 40(6), 443-450.

Rizos, P.F., Aspragathos, N. and Dimarogonas, A.D. (1990). "Identification of crack location and magnitude in a cantilever beam from the vibrating mode", *Journal of Sound and Vibration*, 138(3), 381-388.

Saavedra, P.N. and Cuitino, L.A. (2001). "Crack detection and vibration behavior of cracked beams", *Computers and Structures*, 79(16), 1451-1459.

Sinha, J.K. and Friswell, M.I. (2002). "Simulation of the dynamic response of a cracked beam", *Computers and structures*, 80(18-19), 1473-1476.

Viola, E., Ricci, P. and Aliabadi, M.H. (2007). "Free vibration analysis of axially loaded Timoshenko beam structures using the dynamic stiffness method", *Journal of Sound and Vibration*, 304(1-2), 124-153.

Zheng, D.Y. and Kessissoglou, N.J. (2004). "Free vibration analysis of a cracked beam by finite element method", *Journal of Sound and Vibration*, 273(3), 457-475.

Zheng, T. and Ji, T. (2012). "An approximate method for determining the static deflection and natural frequency of a cracked beam", *Journal of Sound and Vibration*, 331(11), 2654-2670.