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Reliability Analysis of Corroded Reinforced Concrete Beams Using Enhanced HL-RF Method

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Abstract: Steel corrosion of bars in concrete structures is a complex process which leads to the reduction of the cross-section bars and decreasing the resistance of the concrete and steel materials. In this study, reliability analysis of a reinforced concrete beam with corrosion defects under the distributed load was investigated using the enhanced Hasofer-Lind and Rackwitz-Fiessler (EHL-RF) method based on relaxed approach. Robustness of the EHL-RF algorithm was compared with the HL-RF using a complicated example. It was seen that the EHL-RF algorithm is more robust than the HL-RF method. Finally, the effects of corrosion time were investigated using the EHL-RF algorithm for a reinforced concrete beam based on flexural strength in the pitting and general corrosion. The model uncertainties were considered in the resistance and load terms of flexural strength limit state function. The results illustrated that increasing the corrosion time-period leads to increase in the failure probability of the corroded concrete beam.

Keywords: Corrosion, Enhanced HL-RF method, Failure probability, Reliability analysis, Reinforced concrete.

INTRODUCTION

Preventing structural and non-structural damages are the aims of resisting deterioration that can lead to the reduction of the safety factor of the concrete structures against the applied external loads by corrosion defects. These failures include the reduction of cross-section bars and the changes in the steel mechanical behavior (Stewart, 2009). Therefore, the resistance of reinforced concrete structures i.e. ultimate stress and cross-section are reduced in corrosion environments. The approximation of structural failure probability can be used for estimating the performance and life-time of corroded concrete structures.

Stewart and Rosowsky (1998) and Stewart (2000) suggested an experimental model which can estimate the reduction of bars using two variables of initiation time and propagation time of corrosion. Rondringuez et al. (1996) studied the corrosion rate of bar diameter in general and pitting corrosion. Tarighat and Jalalifar (2014) suggested a mathematical model for the corrosion rate based on the results of Rondringuez et al. (1996). The corroded steel cross-section with pitting was presented by Stewart (2004). This corroded model was also applied by other researchers (Stewart and Al-Harthy, 2008; Stewart, 2009; Stewart et al., 1994- 2009). The failure probability of corroded reinforced concrete beam was investigated using HL-RF method by Stewart and

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Rosowsky (1998), Stewart (2004), Stewart and Al-Harthy (2008), and Stewart (2009). The first order reliability method is widely used due to simplicity and efficiency (Naess et al., 2009; Keshtegar and Miri, 2014a). The iterative FORM was established by Hasofer and Lind (1974), and Rackwitz and Fiessler (1978) extended the Hasofer and Lind approach to include the distribution information of random variables (HL-RF method). The failure probability is approximated based on reliability index (β) in FORM as (Santosh et al., 2006):

$$P_f = 1 - \int_{-\infty}^{\beta} \frac{1}{\sqrt{2\pi}} \exp[-\frac{1}{2}(z)^2]dz = 1 - \Phi(\beta) = \Phi(-\beta) \tag{1}$$

where Φ is standard normal cumulative distribution function while z is standard normal variable. The HL-RF contains the numerical instability such as periodic, bifurcation and chaos in nonlinear performance functions (Yang, 2010). Liu and Kiureghian (1991) introduced a merit function which monitors the convergence of HL-RF iterations. Santosh et al. (2006) improved the modified HL-RF method by selecting an appropriate step size based on Armijo rules. Yang (2010) suggested stability transformation method to control iterative instability of FORM approach. Keshtegar and Miri (2013) developed the Enhanced HL–RF without line search rule to achieve stabilization of FORM. Recently, Keshtegar and Miri (2014a) introduced the non-linear conjugate line search to overcome the difficulties of the HL-RF method. The conjugate HL-RF method is more robust than the HL-RF, but extensive computational iterative formula was developed to achieve stability solution with Wolfe conditions (Keshtegar and Miri, 2014a). The line search rules are important factor to achieve stabilization in FORM algorithms. Consequently, the Enhanced HL-RF can be used to search the MPP without line search such as merit function (Liu and Kiureghian, 1991), Armijo (Santosh et al., 2006), or Wolfe conditions (Keshtegar and Miri, 2014a), efficiently and simply.

In this paper, the EHL–RF is described to estimate the failure probability of corroded reinforcing concrete beam with pitting and general corrosions. By comparing the HL–RF and EHL-RF algorithms in a complex and non-Normal dynamic problem, it shows that the EHL-RF approach is more robust than the HL-RF method. Then, a corroded model of reinforced concrete beams is presented based on various uncertainties including model, resistance, load, and yield resistance corroded bars with normal and non-normal random variables. The effect of time-period in general and pitting corrosion is investigated.

RELIABILITY ANALYSIS METHOD

The main effort of FORM searches the most probable point (MPP), which corresponds to the maximum likelihood of failure occurrence (Santosh et al., 2006). Generally, the HL-RF iterative formula is used for the MPP search.

The HL-RF Algorithm

The iterative formula of HL-RF method in FORM is defined as:

$$U_{k+1}^{HL} = \frac{\nabla^T G(U_k)\, U_k - G(U_k)}{\nabla^T G(U_k)\, \nabla G(U_k)} \nabla G(U_k) \tag{2}$$

where $\nabla G(U_k)$: is the gradient vector of the limit state function. *k*: is the iteration number and *U*: is the standard normal variable which can be written as follows (Santosh et al., 2006):

$$U = \frac{(X - \mu_X^e)}{\sigma_X^e} \tag{3}$$

where μ_x^e and σ_x^e: are the equivalent mean and the standard deviation respectively, which are given as (Keshtegar and Miri, 2013):

$$\sigma_x^e = \frac{1}{f_X(x)}\phi[\Phi^{-1}\{F_X(x\ \ )\}] \quad (4)$$

$$\mu_x^e = x - \sigma_x^e \Phi^{-1}[F_X(x)] \quad (5)$$

in which ϕ: is normal probability density function and $F_X(x)$ and $f_X(x)$: are the cumulative distribution and probability density function of the random variable *X*, respectively.

The EHL-RF Algorithm

Recently, Keshtegar and Miri (2013) proposed an enhanced HL-RF algorithm based on relaxed approach, which is defined based on second-order fitting between 0 and 1 using the information from the new and previous iterations. The iterative formula of enhanced HL-RF algorithm is given by the following relations (Keshtegar and Miri, 2013):

$$U_{k+1}^{EHL} = U_k^{EHL} + \alpha_k [U_{k+1}^{HL} - U_k^{EHL}] \quad (6)$$

where α_k: is known as the relaxed coefficient at the k^{th} iteration; It is a real and positive number given as:

$$\alpha_k = \frac{d_{k-1}^T.d_k}{2[f(U_{k+1}^{HL}) - f(U_k^{HL}) + d_{k-1}^T.d_k]} \quad (7)$$

where d_k: is the search direction vector computed as:

$$d_k = U_{k+1}^{HL} - U_k^{EHL} \quad (8)$$

$f(U_k^{HL})$ and $f(U_{k+1}^{HL})$ are computed as:

$$f(U_k^{HL}) = \frac{\left|U_k^{HL}\right|^2}{2} - \frac{\nabla^T G(U_k^{HL}).\ U_k^{HL}}{\left|\nabla G(U_k^{HL})\right|^2} G(U_k^{HL}) \quad (9)$$

$$f(U_{k+1}^{HL}) = \frac{\left|U_{k+1}^{HL}\right|^2}{2} - \frac{\nabla^T G(U_k^{HL}).\ U_{k+1}^{HL}}{\left|\nabla G(U_k^{HL})\right|^2} G(U_{k+1}^{HL}) \quad (10)$$

in which U_{k+1}^{HL}: is the new point from the HL-RF algorithm it is also assumed that $0 \le \alpha_k \le 1$. Keshtegar and Miri (2013) showed that the EHL-RF is more robust than the HL-RF method and it has acceptable convergence in FORM.

Validation of the EHL–RF Algorithm

A complicated structural limit state function with non-normal variables is selected from Keshtegar and Miri (2014a) to compare the robustness of EHL–RF algorithm with HL-RF. A two degree of freedom primary-secondary dynamic system was applied in this example. Its dynamic characteristics were defined by lumped mass as M_s and M_p, spring stiffness of K_s and K_p, and damping coefficient of ξ_s and ξ_p for the primary (subscript p) and secondary (subscript s) oscillators, respectively (Figure 1).

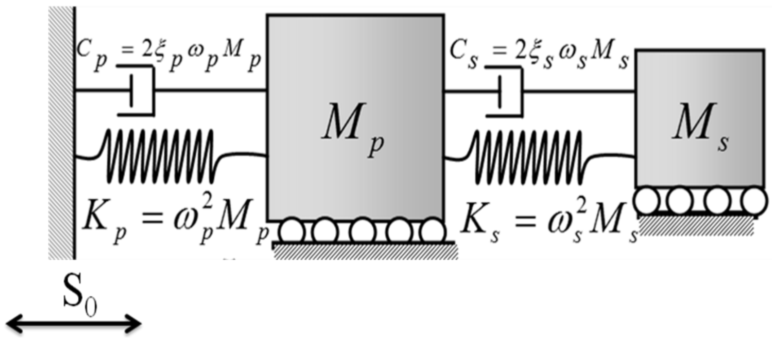

Fig. 1. Two-degree of freedom dynamic system

The force capacity of the secondary spring is considered for limit state function as (Keshtegar and Miri, 2014a):

$$G = Fs - Ks \times P(E[x_s^2])^{1/2} \qquad (11)$$

where Fs: denotes force capacity. P: is the peak factor that is considered as a deterministic constant with the value of 3 and $E[x_s^2]$: is mean-square relative displacement response:

$$E[x_s^2] = \frac{\pi S_0}{4\xi_s \omega_s}[\frac{\xi_a \xi_s}{\xi_p \xi_s (4\xi_a^2 + \theta^2) + \gamma \xi_a^2} \times \frac{(\xi_p \omega_p^3 + \xi_s \omega_s^3)\omega_p}{4\xi_a \omega_a^4}] \qquad (12)$$

where $\gamma = \frac{M_s}{M_p}$: is mass ratio, $\omega_a = \frac{\omega_p + \omega_s}{2}$ and $\xi_a = \frac{\xi_p + \xi_s}{2}$: are average frequency and damping ratio of the two systems, $\theta = \frac{\omega_p - \omega_s}{\omega_a}$: is a tuning parameter and S_0: is intensity of the white-noise base excitation. This example included eight independent basic random variables with lognormal distributions, their means and standard deviations are listed in Table 1.

According to the results extracted from Kiureghian and Stefano (1991), reliability index was equal to 2.01. This example was recently analyzed by Keshtegar and Miri (2014a), in which the converged results based on conjugate HL-RF method attained a safety index of 2.016446 after 24 iterations. The HL–RF method is yielded to unstable solutions as periodic-2 of the safety index i.e. {1.04958, 1.15364}. The probability of failure is obtained based on the Mont Carlo simulation using 2.5 $\times 10^5$ samples as $P_f = 0.004793$ (β = 2.59042). The converged reliability index and the MPP point were attained at β = 2.013352 and X^*= [ξ_p =0.0280, ξ_s =0.0121, F_s =13.7367, S_0 =103.7133, M_p = 1.0019, M_s = 0.0101, K_p =1.1020, K_s =0.0112] after 22 iterations by the EHL–RF algorithm, respectively. It can be seen that the EHL–RF algorithm is converged to excellent and stable results and it is more robust than the HL-RF.

MATHEMATICAL MODEL FOR CORRODED CONCRETE BEAM

A reinforced concrete beam under distributed load with rectangular cross-section is shown Figure 2. The limit state function is defined based on the maximum binding moment.

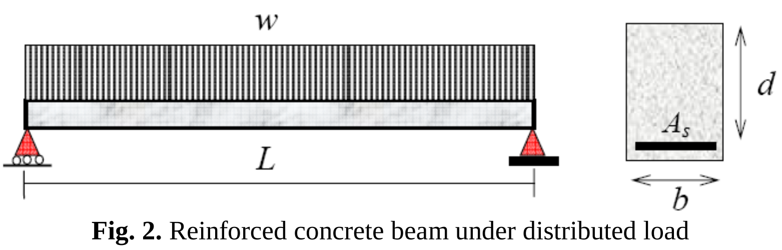

Fig. 2. Reinforced concrete beam under distributed load

Table 1. Statistical characteristics of basic random variables for dynamic system in Figure 1

Variables	Mean	standard deviation	Variables	Mean	standard deviation
ξ_p	0.05	0.4	M_p	1	0.1
ξ_s	0.02	0.5	M_s	0.01	0.1
F_s	15	0.1	K_p	1	0.2
S_0	100	0.1	K_s	0.01	0.2

The flexural limit state function of this beam is defined by the following equation (Stewart, 2009):

$$G(M) = \eta A(t) f_y(t)(d - K \frac{A_s(t).f_y(t)}{b.f_c}) - \lambda M_u$$
$$f_y(t) = (1 - \alpha \frac{A_s - A_s(t)}{A_s}) f_y \tag{13}$$

In the limit state (13), empirical coefficient's η and α: are the model uncertainty (Nowak and Collins, 2000) and tension resistance bars (Stewart, 2009), respectively. According to the experimental results from Cairns et al. (2005), the α value was reported from 0.017 to 0.06 for pitting corrosion. Du et al. (2005) suggested $\alpha = 0.005$, this α was applied by Stewart (2009). The statistical characteristics of the resistance variable K and the load uncertainty variable λ, (Nowak and Collins, 2000; Stewart, 2009; Vu and Stewart, 2000) are listed in Table 2.

Vu and Stewart (2000) and Vu et al. (2005) applied normal probability function for modeling the concrete compressive in the range of 25 to 40 *MPa* with coefficient of variation (CoV) of 0.15 to 0.18. The normal distribution function with CoV of about 0.1 to 0.12 was considered for concrete cover distribution function by Rodriguez et al. (1996) and Vu et al. (2005). The Lognormal probability distribution function with CoV of 0.11 was applied to define the statistical properties of steel yield tension variable in reliability analysis by Bhargava et al. (2011). The statistical properties of beam dimensions were applied based on the assumption by Bhargava et al. (2011) in this study. The statistical characteristics of material (concrete and steel) strength, applied load and beam dimensions (Figure 3) are shown in Table 3.

The corroded cross-sectional steel bar is calculated as:

$$A_s(t) = A_s - A_P(t) \tag{14}$$

$A_s(t)$ depends on the physical, mechanical and geometrical of the concrete beam. Thus, the two forms of modeled corrosion can be defined as pitting and general corrosion.

General Corrosion

The cross-sectional area of the bars at time, t (years) in general corrosion is suggested by the following relation (Stewart, 2004; Darmawan, 2010):

$$A_s(t) = \frac{\pi(D_0 - 2P_{av})^2}{4} \tag{15}$$

in which,

$$P_{av} = 0.0116.i_{corr}(t).T \tag{16}$$

and D_0: is the nominal bar diameter (mm), P_{av}: is the average pit depth (mm/year), T: is the time since corrosion initiation (year) and $i_{corr}(t)$: is the corrosion rate ($\mu A/cm^2$) which can be computed as (Vu and Stewart, 2000; Tarighat and Jalalifar, 2014):

$$i_{corr}(t) = 0.85\ i_{corr}(1).T^{-0.29} \tag{17}$$

Table 2. Statistical characteristics of the model, resistance and load uncertainties

Variables	Variable description	(CoV)	Mean (μ)	Probability function
K	Resistance ratio	0.05	0.6	Normal
η	Model coefficient	0.1	1	Normal
λ	Load coefficient	0.1	1.05	Normal
α	Yield empirical coefficient	0.12	0.005	Lognormal

Table 3. Statistical characteristics of resistance, beam dimensions and load variables

Variables	Variable description	(CoV)	Mean (μ)	Probability function
f_c	Concrete compressive (*MPa*)	0.18	30	Normal
f_y	Steel yield tension (*MPa*)	0.11	400	Lognormal
M_n	Applied moment (*kN-m*)	0.12	120	Gumbel
b	Section width (*mm)*	0.07	350	Normal
d	Effective height (*mm)*	0.07	500	Lognormal
C	Concrete cover (*mm)*	0.12	50	Normal

where $i_{corr}(1)$ is the corrosion rate ($\mu A/cm^2$) at the start of corrosion propagation ($\mu A/cm^2$) which is calculated as (Vu and Stewart, 2000; Tarighat and Jalalifar, 2014):

$$i_{corr}(1) = \frac{37.8(1-wc)^{-1.64}}{C} \tag{18}$$

and C: is the concrete cover (*cm*). Water - cement ratio (*wc*) of the concrete was calculated using the compressive strength of the concrete from Bolomey's formula as (Vu and Stewart, 2000):

$$wc = \frac{27}{f'_{cy}+13.5} \quad , f'_{cy} = f'_c + 7.4 \;\; MPa \tag{19}$$

These relations can be effective for cities in countries such as Asia, Europe, America and Australia whose humidity ratio is around 70% (Vu et al., 2005).

Pitting Corrosion

Loss cross- sectional bars subjected to the pitting corrosion can be calculated using the following equations (Stewart and Al-Harthy, 2008; Stewart, 2009; Darmawan, 2010):

$$A_P(t) = \begin{cases} A_1 + A_2 & P(t) \le \dfrac{D_0}{\sqrt{2}} \\ A_s - A_1 + A_2 & \dfrac{D_0}{\sqrt{2}} < P(t) \le D_0 \\ A_s & P(t) > D_0 \end{cases} \tag{20}$$

in which

$$P(t) = 0.0116.i_{corr}(t).R.T \tag{21}$$

$$\begin{aligned} & A_s = \frac{\pi.D_0^{\,2}}{4} \\ & A_1 = 0.5[\theta_1(\frac{D_0}{2})^2 - b\left|\frac{D_0}{2} - \frac{P(t)^2}{D_0}\right|] \\ & A_2 = 0.5[\theta_2 P(t)^2 - b\frac{P(t)^2}{D_0}] \\ & b = 2P(t)\sqrt{1-(\frac{P(t)}{D_0})^2} \;,\; \theta_1 = 2\arcsin(\frac{b}{D_0}) \\ & ,\theta_2 = 2\arcsin(\frac{b}{2P(t)}) \end{aligned} \tag{22}$$

where $P(t)$: is the maximum penetration of pitting and R: is pitting factor (rate between pitting and average depth $R = P(t)/P_{av}$) which varied from 4 to 8 (Stewart, 2009). Therefore, the random variables for corroded cross-sectional steel bars are listed in Table 4.

NUMERICAL RESULTS OF RELIABILITY ANALYSIS

The failure probability of beam is estimated by using enhanced HL-RF method based on relaxed approach. The results of failure probability based on the general and pitting corrosion are plotted in Figure 3. It is clear that the failure probabilities to the corrosion times in pitting corrosion are more than the general corrosion in a time more than 20 years.

Compression of the failure probability to external load moment is plotted in Figure 4 for several times since corrosion initiation (T). It can be seen that the failure probabilities of this beam are increased with respect to increase time since corrosion initiation. For $T<15$ years, the rate of change for the failure probabilities is insignificantly increased, but is significantly increased for $T>50$ years. Also, differences of the failure probability in pitting corrosion obtained more changes in the time since corrosion initiation of $T<15$ while, they showed significant differences of failure probabilities for general corrosion in this domain of T. Therefore, the corrosion defects can reduce the performance of the beam in long time corrosion.

Table 4. Statistical characteristics of the basic random variables of corroded cross-sectional steel bars

Variables	Variable description	(CoV)	Mean (μ)	Probability function
D_0	Bar diameter (*mm*)	0.05	6Φ18	Normal
R	The ratio of maximum to average corrosion	0.2	6	Gumbel
T	Corrosion initiation time (years)	0.35	variable	Lognormal

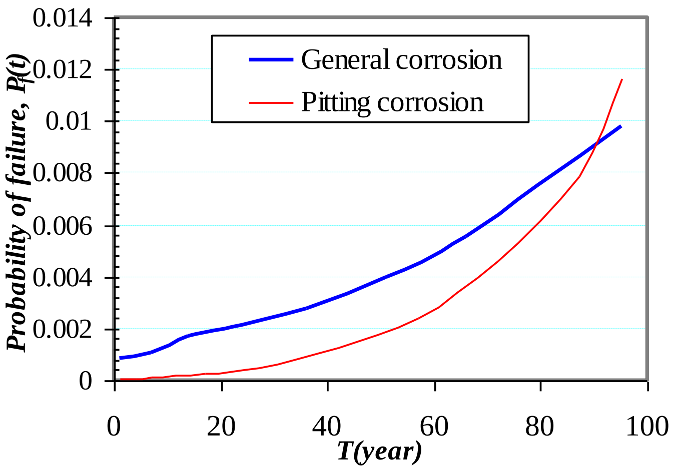

Fig. 3. Comparing the failure probability under the general and pitting corrosion

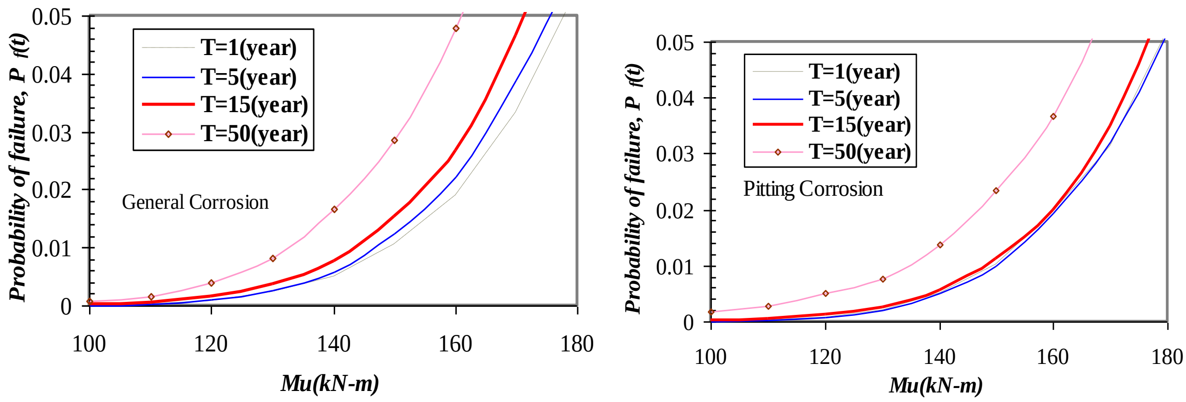

Fig. 4. Compression of failure probability to external moment for initial time since corrosion

CONCLUSIONS

The enhanced HL-RF (EHL-RF) method without line search rules was applied for the reliability analysis of concrete beam with general and pitting corrosion. The EHL-RF approach based on the dynamical step size is more robust than the HL-RF. Based on the reliability analysis of corroded beam, it is seen that increasing the corrosion time period leads to increasing the failure probability of the concrete beam. The maximum rate of failure probabilities were concluded to increase the time since corrosion initiation (*T>15* years) in the pitting corrosion of reinforced concrete beam.

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