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Particle Swarm Optimization for Hydraulic Analysis of Water Distribution Systems

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Abstract: The analysis of flow in water-distribution networks with several pumps by the Content Model may be turned into a non-convex optimization uncertain problem with multiple solutions. Newton-based methods such as GGA are not able to capture a global optimum in these situations. On the other hand, evolutionary methods designed to use the population of individuals may find a global solution even for such an uncertain problem. In the present paper, the Content Model is minimized using the particle-swarm optimization (PSO) technique. This is a population-based iterative evolutionary algorithm, applied for non-linear and non-convex optimization problems. The penalty-function method is used to convert the constrained problem into an unconstrained one. Both the PSO and GGA algorithms are applied to analyse two sample examples. It is revealed that while GGA demonstrates better performance in convex problems, PSO is more successful in non-convex networks. By increasing the penalty-function coefficient the accuracy of the solution may be improved considerably.

Keywords: Content Model, Global Gradient Algorithm, Hydraulic Analysis, Particle-Swarm Optimization, Water Distribution Systems

INTRODUCTION

In a water distribution network (WDN) flow rates and nodal heads are computed by solving a mixture system of linear (continuity) and nonlinear (energy) equations. Since 1936 various methods have been devised for WDN analysis that directly solve the system equations, e.g., the Hardy Cross method (Cross, 1936), the Newton-Raphson method (Martin and Peters, 1963; Shamir and Howard, 1968), and the linear theory method (Wood and Charles, 1972). However, Collins et al. (1978) proposed a mathematical optimization technique, the so-called Content Model, that minimized a nonlinear convex objective function subject to a set of linear equality constraints. The convexity of the objective function guaranteed the existence and uniqueness of the solution. However, the nonlinear programming methods used for the solution of the Content Model were time-consuming, reducing the practical application of the model in large complex networks. Todini and Pilati (1988) developed a Newton-based global gradient algorithm (GGA), originally based on the minimization of a slightly modified Content Model. Basically, GGA involves two iterative steps, where the heads and flows are obtained, respectively (Elhay and Simpson, 2011). Simpson (2010) compared Q-equations and GGA

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formulation in analyzing water distribution systems. Todini and Rossman (2013) introduced a unified framework for deriving simultaneous equation algorithms for WDNs. Moosavian and Jaefarzadeh (2014) applied an efficient higher-order method and reduced the number of iterations of the Hardy Cross algorithm. Recently, some innovative techniques have been developed for simplifying the topological representation of pipe networks while preserving the accuracy of the analysis (see for example Giustolisi, 2010; Giustolisi et al., 2012).

In a pipe network problem quite a few pumps may be provided externally to supply water from reservoirs, or internally within the network as booster pumps to augment pressure and discharge accordingly at certain locations within the system. At a constant rotational speed, there is a unique relationship between the delivered head h_p and supplied discharge Q, known as the pump head-discharge curve. The head-discharge curves for various kinds of pump are different. Usually in screw pumps these curves are stable and strictly monotonically decreasing, i.e., the head decreases as the flow rate increases. Thus, for a given head there is only one value for the flow rate. However, for some centrifugal and half-axial pumps, the characteristic head-discharge curve is unstable or not continuously decreasing as the flow rate increases. In other words, for the same head, two or three discharges may exist (Bhave and Gupta, 2006). The analysis of a distribution network with several pumps with unstable or in some cases even stable head-discharge curves is a non-convex uncertain problem with multiple solutions as operating points. Generally, the convergence characteristics of the Newton-type methods such as GGA are highly sensitive to the initial guesses of the solution. Specifically in non-convex problems, these methods will fail if the initial guesses are not sufficiently close to the global minimum (Luenberger and Yinyu, 2008). In other words, they may trap into a local minimum, leading to the wrong solution.

On the other hand, evolutionary methods are intrinsically designed to find a global solution even in uncertain problems. Over the last two decades many evolutionary techniques have been successfully applied to minimize the cost function of pipes. This is a non-convex function with discrete decision variables. The most important techniques include genetic algorithms (Murphy and Simpson,1992; Simpson et al., 1994; Dandy et al.,1996; Savic and Walters, 1997); simulated annealing (Cunha and Sousa, 2001); harmony search (Geem, 2006); the shuffled frog-leaping algorithm (Eusuff and Lansey, 2003); ant-colony optimization (Maier et al., 2003); particle-swarm optimization (Suribabu and Neelakantan, 2006); cross entropy (Perelman and Ostfeld, 2007); scatter search (Lin et al., 2007); differential evolution (Suribabu, 2010) and Vasan and Simonovic, 2010); self-adaptive differential evolution (Zheng et al., 2013) and the soccer-league competition algorithm (Moosavian and Kasaee, 2014). Moosavian and Jaefarzadeh (2014) applied a shuffled complex-evolution technique (SCE) in a head-based optimization model (Co-Content Model) for the hydraulic analysis of pipe networks. This methodology was able to accurately simulate pressure-driven demand and leakage in pipe networks.

In this article the Content Model is optimized using an evolutionary-type algorithm called particle swarm optimization (PSO). In this methodology, there is no need to solve a system of linear or non-linear equations where a proper initial solution vector is crucial to the convergence of non-convex problems.

CONTENT MODEL APPROACH

As mentioned, the solution of the content-optimization model by Collins et al. (1978) yielded the network analysis. A simplified version of this model is presented herein by applying it to the one-loop network shown in Figure 1, where nodes 1 and 2 are source nodes with known head values H_1, and H_2, and nodes 3, 4 and 5 are known demand nodes q_3, q_4 and q_5. Let pipes 1 to 5 have the known resistances R_1 to R_5, respectively. The Content Model aims to minimize the energy function $C(Q)$ (Bhave and Gupta, 2006):

$$C(Q) = R_1 |Q_1|^{n+1}/(n+1) + R_2 |Q_2|^{n+1}/(n+1) + R_3 |Q_3|^{n+1}/(n+1) + R_4 |Q_4|^{n+1}/(n+1) + R_5 |Q_5|^{n+1}/(n+1) - Q_1 H_1 - Q_2 H_2 \tag{1}$$

This is subject to the following node-flow continuity constraints:

$$\begin{aligned} &Q_1 - Q_5 - Q_4 - q_3 = 0 \\ &Q_5 + Q_3 - q_4 = 0 \\ &Q_4 + Q_2 - Q_3 - q_5 = 0 \end{aligned} \tag{2}$$

The objective function (1) contains integral functions of the head loss terms in pipes 1 to 5 and potential energy terms in source nodes 1 and 2, respectively. Note that the node-flow continuity constraints should be written for all nodes of the network with unknown heads. Since all node-flow continuity constraints and loop-head loss equations have to be satisfied, the solution of the optimization model gives the discharges in the pipes, hence the network analysis. If there is a pump, for example in pipe (2), with a head-discharge curve approximated by a quadratic equation $h_p = AQ^2+BQ+C$, the energy function model may be modified accordingly:

$$\begin{aligned} C(Q) = {} & R_1 |Q_1|^{n+1}/(n+1) + R_2 |Q_2|^{n+1}/(n+1) + \\ & R_3 |Q_3|^{n+1}/(n+1) + R_4 |Q_4|^{n+1}/(n+1) \\ & + R_5 |Q_5|^{n+1}/(n+1) - Q_1 H_1 - Q_2 H_2 - \\ & (A|Q_2|^3/3 + B|Q_2|^2/2 + C|Q_2|) \end{aligned} \tag{3}$$

where A, B and C: are constant coefficients. The Content Model in a general form may be expressed as:

$$\text{Minimize}: C(Q) = \sum_j R_j |Q_j|^{n+1}/(n+1) - \sum_k Q_k H_k - \sum_l \left(A|Q_l|^3/3 + B|Q_l|^2/2 + C|Q_l|\right) \tag{4.1}$$

$j \in pipe, k \in source\ node, l \in pump$

Subject to:

$$\sum_j Q_j - q_i = 0,\ for\ all\ i \tag{4.2}$$

Note that the second summation in the objective function is only for source nodes with known heads, while the constraints are written for nodes with unknown heads.

In the global gradient algorithm presented by Todini and Pilati (1987), the optimization model of (4.1) and (4.2) is unconstrained by a number of Lagrange Multipliers, and the resulting equations are solved by the Newton-Raphson method. Thereby, the estimates for Q and H are updated at each iteration directly. The convergence rate of this algorithm may be of the second order, provided that initial guesses are sufficiently close to the final solution (Luenberger, Yinyu, 2008). At present, GGA is applied for water distribution network analysis in commercial and industrial software such as WaterGEMS and EPANET (Rossman, 2002).

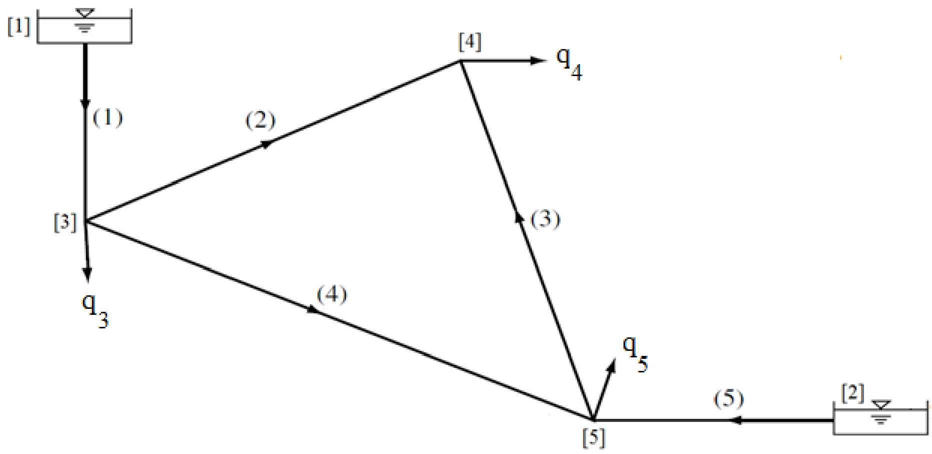

Fig. 1. Schematic representation of the looped pipe network with 5 pipes.

On The Convexity Property of the Content Model

If an optimization model is convex, Newton-based methods can easily minimize it with any arbitrary initial guess, and the existence and uniqueness of the solution are guaranteed. However, for non-convex functions with several optima, these methods may not necessarily capture a global optimum. Instead, they are likely to be trapped in a local optimum, depending on initial guesses at the beginning of the solution process.

On the other hand, meta-heuristic algorithms examine the whole domain of the problem as much as possible and seek a global optimum independent of the initial guesses. To illustrate the convexity behaviour of the Content Model in the presence or absence of a pump, consider the network shown in Figure 2, including one reservoir, two nodes and three pipes. The Content Model may be written as:

$$\begin{aligned}&\text{Minimize}: C(Q) = R_1 Q_1^{\,n+1}/(n+1) + \\ &R_2 Q_2^{\,n+1}/(n+1) + R_3 Q_3^{\,n+1}/(n+1) - \\ &Q_1 H_1 - Q_2 H_1\end{aligned} \tag{5.1}$$

$$\begin{aligned}&\text{Subject to}: \\ &Q_1 - Q_3 - q_2 = 0 \\ &Q_2 + Q_3 - q_1 = 0\end{aligned} \tag{5.2}$$

In Eq. (5.1) the absolute values are removed, presuming the flow in the pipes is selected in proper directions. Substituting for Q_2 and Q_3 from constraints (5.2) into (5.1), the Content Model may be modified as:

$$\begin{aligned}&\text{Minimize}: C(Q) = R_1 Q_1^{\,n+1}/(n+1) + \\ &R_2 (Q_1 - q_2 - q_1)^{n+1}/(n+1) + \\ &R_3 (Q_1 - q_2)^{n+1}/(n+1) - \\ &Q_1 H_1 - (Q_1 - q_2 - q_1) H_1\end{aligned} \tag{6}$$

The second derivative of Equation (6) yields a positive function:

$$\begin{aligned}&\frac{\partial^2 C}{\partial Q_1^2} = nR_1 Q_1^{\,n-1} + nR_2 (Q_1 - q_2 - q_1)^{n-1} + \\ &nR_3 (Q_1 - q_2)^{n-1} = nR_1 (Q_1)^{n-1} + \\ &nR_2 (Q_2)^{n-1} + nR_3 (Q_3)^{n-1} \geq 0\end{aligned} \tag{7}$$

The positiveness of Eq. (7) assures the convexity of the Content Model and GGA is therefore able to find the proper solution. When two pumps are placed in pipes 1 and 2, the Content Model is modified as:

$$\text{Minimize}: C(Q) = R_1 Q_1^{n+1} / (n+1) + R_2 (Q_1 - q_2 - q_1)^{n+1} / (n+1) + R_3 (Q_1 - q_2)^{n+1} / (n+1)$$
$$- \left(A_1 Q_1^3 / 3 + B_1 Q_1^2 / 2 + C_1 Q_1 \right) - \left(A_2 (Q_1 - q_2 - q_1)^3 / 3 + B_2 (Q_1 - q_2 - q_1)^2 / 2 \right.$$
$$\left. + C_2 (Q_1 - q_2 - q_1) \right) - Q_1 H_1 - (Q_1 - q_2 - q_1) H_1 \tag{8}$$

The second derivative of Eq. (8) gives:

$$\frac{\partial^2 C}{\partial Q^2} = nR_1 Q_1^{n-1} + nR_2 (Q_1 - q_2 - q_1)^{n-1} + nR_3 (Q_1 - q_2)^{n-1}$$
$$- (2A_1 Q_1 + B_1) - (2A_2 (Q_1 - q_2 - q_1) + B_2) =$$
$$nR_1 (Q_1)^{n-1} + nR_2 (Q_2)^{n-1} + nR_3 (Q_3)^{n-1} - (2A_1 Q_1 + B_1) - (2A_2 Q_2 + B_2) \tag{9}$$

In this case, Eq. (9) may not be positive for some values of *A* and *B*; even for a strictly monotonically decreasing head-discharge curve, the Content Model is non-convex. As an example, for typical values of constant coefficients, consider the sample energy curve in Figure 3 for the one-looped pipe network shown in Figure 2. The Content Model has two minima; however, GGA is able to find only one local solution depending on the selection of initial guesses. This solution may not necessarily be a global optimum. Evolutionary methods are to be used to resolve this problem and to find the global minimum. In the following, a powerful evolutionary approach for minimizing the Content Model will be introduced.

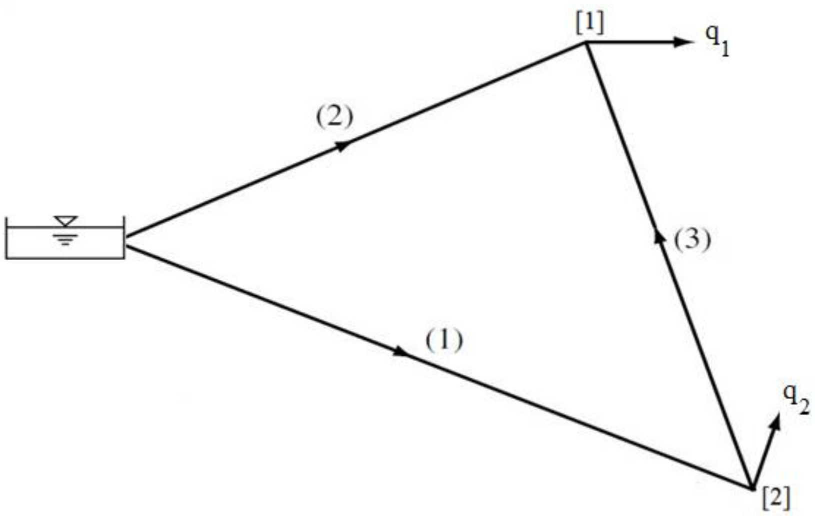

Fig. 2. Schematic representation of the looped pipe network with three pipes.

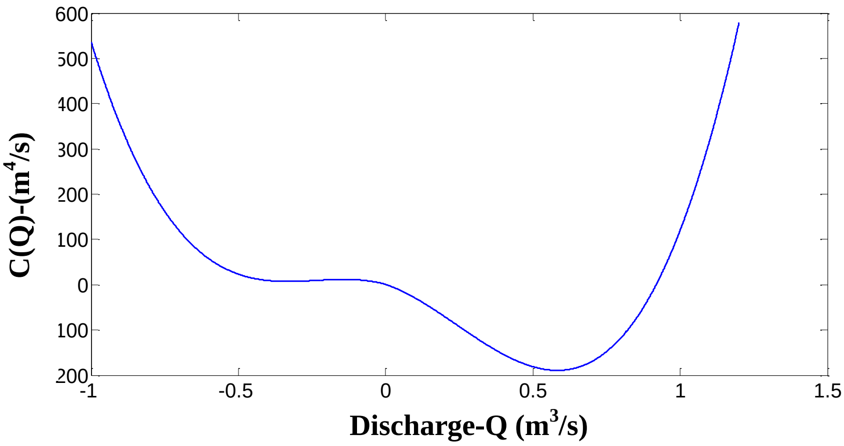

Fig. 3. System energy curve for one- looped pipe network.

PARTICLE SWARM OPTIMIZATION

Particle swarm optimization (PSO) is an adaptive evolutionary algorithm based on population search (Kennedy and Eberhart, 1995). It is a simple and fast converging technique simulating the social behaviour of birds flocking and fish schooling. Consider a swarm of flying birds looking for a piece of food (global optimum) in an area. No bird in the group knows the exact location of the food, but they may identify the one closest to the food, and obviously their best choice is to follow that bird (Zhu et al., 2011). In PSO each solution is taken as a bird in the group or a particle in the swarm, having a certain position and velocity, but with zero mass. The location and velocity of the particles are updated in consecutive iterations. The various steps in this algorithm may be classified as follows.

Step 1: Define the optimization problem and constraints

The optimization problem together with its constraints (if any) are specified as:

$$\begin{aligned}&\text{Minimize} \quad : C(\boldsymbol{X}_i)\\ &\text{Subject to} : \boldsymbol{h}(\boldsymbol{X}_i) = 0\end{aligned} \tag{10}$$

where $C(\boldsymbol{X}_i)$: is an objective function, $\boldsymbol{X}_i$: is the set of decision variables for particle i and $\boldsymbol{h}(\boldsymbol{X}_i)$: is the set of constraints. PSO is basically an unconstrained algorithm but may be used for constrained problems as well with a penalty function, as will be explained later.

Step 2: Generate samples

Each particle i is associated with three vectors:

1. The particle's current position

$$\boldsymbol{X}_i = \left(x_{i1}, x_{i2}, \ldots, x_{iN}\right) \tag{11}$$

where N: is the number of decision variables;

2. The best location it has reached so far

$$\boldsymbol{Pbest}_i = \left(pbest_{i1}, pbest_{i2}, \ldots, pbest_{iN}\right) \tag{12}$$

3. Its current velocity, which enables it to evolve to a new position

$$\boldsymbol{V}_i = \left(v_{i1}, v_{i2}, \ldots, v_{iN}\right) \tag{13}$$

The initial population of the PSO may be created arbitrarily by

$$\boldsymbol{X}_i = \boldsymbol{X}_{imin} + \boldsymbol{\tau}_i\left(\boldsymbol{X}_{imin} - \boldsymbol{X}_{imax}\right) \tag{14}$$

where τ_i: denotes a uniformly distributed random vector within the range of [0,1], and $\boldsymbol{X}_{imin}$ and $\boldsymbol{X}_{imax}$: are the maximum and minimum bounds of particle i. Then, the objective functions ($C_i(\boldsymbol{X}_i)$, $i = 1, \ldots, nPop$) of all the individuals in the population are calculated. The position matrix of the population of generation G may be represented as:

$$P^{(G)} = \left[C_1(\boldsymbol{X}_1), C_2(\boldsymbol{X}_2), \cdots C_{nPop}\left(\boldsymbol{X}_{nPop}\right)\right] \qquad (15)$$

where *nPop*: is the number of populations.

Step 3: Start the iterative process

In this step, the best position that every particle has reached so far (***Pbest***) is found and the best ***Pbest*** is set as the best global position (***Gbest***). Then, two main steps of PSO are performed sequentially to create the new solution vectors.

Step 3.1: Update velocities

The new velocity, V_i, may be obtained as follows:

$$\boldsymbol{V}_i \leftarrow \omega \boldsymbol{V}_i + c_1\, \alpha_i \left(\boldsymbol{P}best_i - \boldsymbol{X}_i\right) + c_2\, \beta_i \left(\boldsymbol{G}best - \boldsymbol{X}_i\right) \qquad (16)$$

where ***Gbest***: is the best position vector attained by any particle during the optimization process, c_1 and c_2: are the acceleration constants, representing the weights of the stochastic acceleration terms that respectively pull each particle simultaneously towards its best position and the best global position, α_i and β_i: are constant functions, generating uniform pseudo-random numbers between 0 and 1 (sometimes referred to as learning rates or factors), and ω: is an inertia term proposed by Shi and Eberhart (1998), which controls the impact of the velocity history into the new velocity and provides improved performance in a number of applications. This last parameter may be suitably adapted during the calculation process. The operation (16) allows a balance between the local and global search.

Step 3.2: Update particles' positions

The particles' positions are updated accordingly as follows:

$$\boldsymbol{X}_i \leftarrow \boldsymbol{V}_i + \boldsymbol{X}_i \qquad (17)$$

Step 4: Check the stopping criterion

The steps 3.1 and 3.2 are repeated until a termination criterion is satisfied. This condition may be stated either in terms of a maximum number of iterations or a certain value for the objective function.

Penalty Function

The PSO algorithm is designed for unconstrained optimization models. The penalty function method may be used for changing constrained models into unconstrained ones (Luenberger and Yinyu, 2008). A penalty function is defined as the summation of square constraints:

$$P(\boldsymbol{X}_i) = \sum_{j=1}^{m} \boldsymbol{h}_j(\boldsymbol{X}_i)^2 \qquad (18)$$

where m: is the number of constraints. An unconstrained model is then obtained by adding the penalty function to the objective function:

$$\begin{aligned} &\text{Minimize} : C(\boldsymbol{X}_i) + \mu P(\boldsymbol{X}_i) \\ &\text{i} = 1, \text{nPop} \end{aligned} \qquad (19)$$

where μ: is a large positive constant coefficient.

Implementation of PSO in Pipe Network Analysis

Pipe-network analysis in Content Model, i.e., as in Eqs. (4.1) and (4.2), is a constrained optimization problem. To eliminate the constraints this model may be treated by a penalty function similar to Eq. (19). The PSO algorithm may then be used to minimize Eq. (19), where decision variables are flow-rate in pipes with a maximum bound set to the sum of all demands and a minimum bound of zero.

Both the PSO and GGA algorithms are applied to analyse two network problems in this section. The residuals of node - balance summed up over all nodes and the residuals of loop energy balance summed up over the loops are calculated to examine the accuracy of solutions. All of the computations were executed in a MATLAB environment with an Intel(R) Core(TM) 2Duo CPU P8700 with 2.53GHz and 4.00 GB RAM.

NUMERICAL EXAMPLE 1

Consider a three-loop network with 5 nodes and 7 pipes, where the node-, pipe- and loop-numbers are shown in Figure 4 as reported by Todini (2006). The pipes' resistances are R_1=1.5625, R_2=50, R_3=100, R_4=12.5, R_5=75, R_6=200, and R_7=100, respectively. The algorithms of PSO, minimizing the Content Model plus penalty functions and GGA by applying Newton-Raphson iterative method are applied to analyse this network. Each of these algorithms was tested 100 times with different initial guesses. The selected parameters for the implementation of the PSO algorithm included: number of decision variables = 7; population = 700 (100 times the number of decision variables); and number of iterations = 300. Based on the authors' experience, if the population is selected as 100 times the decision variables, the algorithm performance will be efficient in all runs. The decision variables were set between a lower bound of 0 and an upper bound of 1 m^3/s, equal to the sum of demands in all nodes. The convergence properties of objective function evaluation and the total mass and energy balances against the number of iterations for the PSO method are shown in Figures 5 and 6 for the two penalty-function coefficients $\mu=10^6$ and $\mu=10^9$, respectively. The accuracy of mass balance increases the larger the value of μ. The objective function was evaluated for 100 runs of the GGA and PSO methods. In Table 1, the best, worst, average and standard deviation of the solutions obtained from the different runs indicate that both methods generate similar results. However, in Table 2 the mass balance at nodes and the energy balance around the loops reveal that GGA produces more accurate results than PSO. This is due to the convexity of the Content Model in the absence of any pump in the system, where the Newton-based models are highly successful in detecting the global optimum solution.

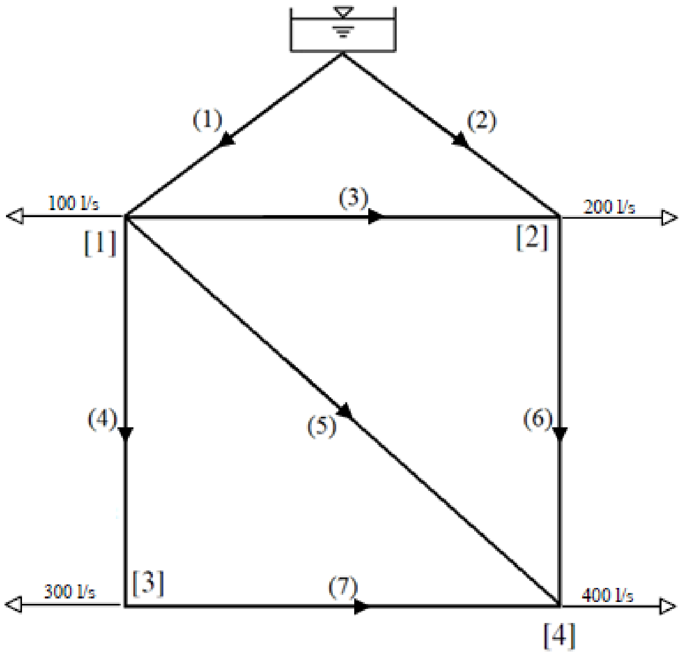

Fig. 4. Schematic representation of the three-looped pipe network (Todini, 2006).

Table 1. Evaluation of objective function in PSO and GGA for example 1($\mu=10^9$).

	Best	Worst	Mean	Std
PSO	-99	-99	-99	1.57e-014
GGA	-99	-99	-99	7.03E-13

Table 2. Evaluation of mass and energy balance in PSO and GGA for Example 1 ($\mu=10^9$).

Mass Balance (Node)	PSO	GGA	Energy Balance (Loop)	PSO	GGA
1	4.95E-8	-3.47E-15	123	-3.42E-08	2.22E-16
2	4.90E-8	1.72E-15	356	7.56E-07	4.44E-16
3	4.85E-8	8.88E-16	457	-6.16E-07	-1.11E-16
4	4.80E-8	2.83E-15			

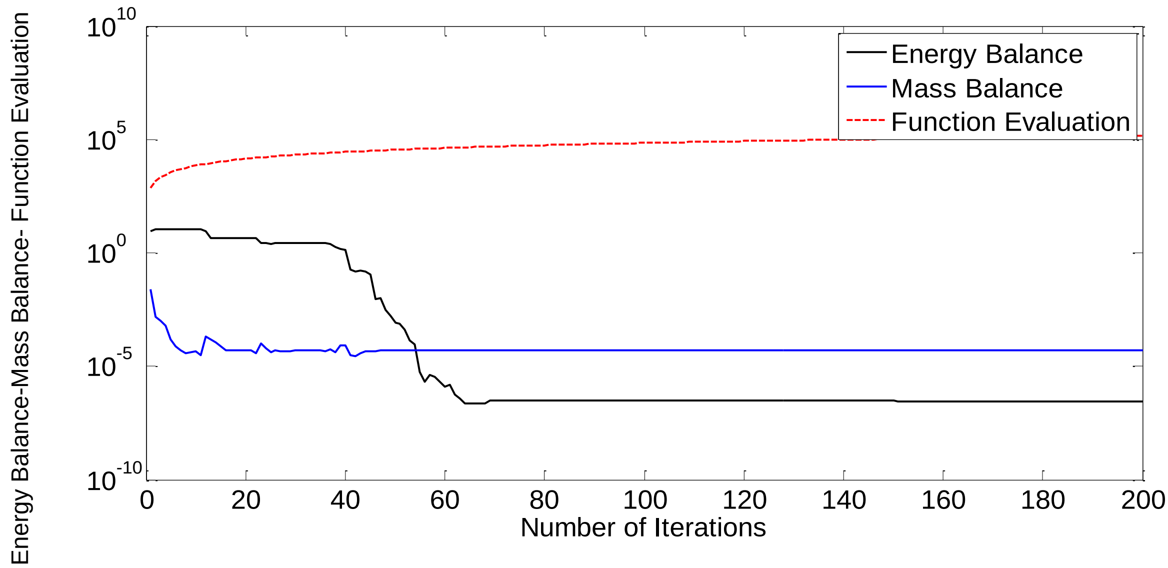

Fig. 5. Mass and energy balances and function evaluation versus the number of iterations for the PSO algorithm with $\mu=10^6$, Example 1.

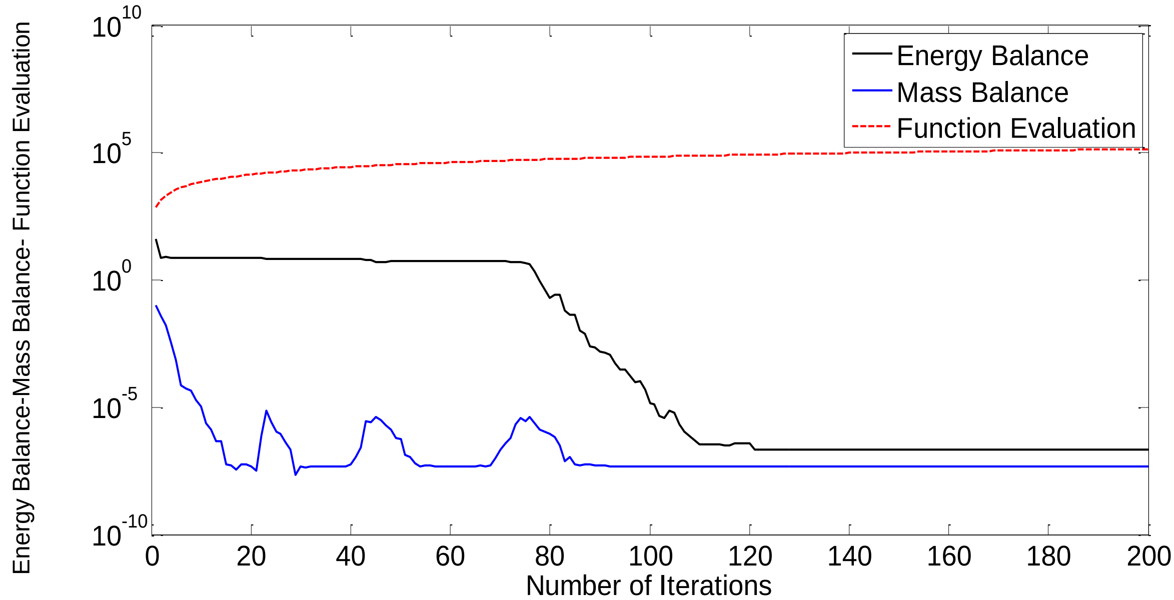

Fig. 6. Mass and energy balances and function evaluation versus the number of iterations for the PSO algorithm with $\mu=10^9$, Example 1.

NUMERICAL EXAMPLE 2

Consider the eight-pipe network shown in Figure 7, including two reservoirs, a source pump that supplies some of the system demand, and a booster pump placed in pipe 1. There are two globe valves, which have a loss coefficient of 10, in pipes 7 and 8, and two of the meter in pipe 3 (Larock et al., 2000). The pipes' resistances are R_1=1160, R_2=613, R_3=1160, R_4=690, R_5=1292, R_6=1115, R_7=322 and R_8=239, respectively, and the pumps' characteristic curves are approximated by $h_{p1} = -2220Q^2+44.4Q+12.28$ and $h_{p2} = -55.6Q^2+1.667Q+4.1$. In a similar manner to Example 1, this network was analysed 100 times with different initial guesses using the two methods of GGA and PSO. The parameters selected for the PSO model included: number of decision variables = 8; population = 800 (100 times the number of decision variables); and number of iterations = 500. The range of initial guesses was between 0.0 and 0.24 m^3/s. In Table 3, the best, worst, average and standard deviation of the objective function, obtained from 100 runs with different initial guesses, demonstrate the advantage of PSO over GGA. In fact, PSO captured the global minimum of -44.5989 in all runs, with a standard deviation of zero. Depending on the initial guesses, GGA found a local minimum of 10.8166 in some runs and a global minimum of -44.4839 in others. Figure 8 shows the performance of GGA in different runs in finding either the local or global minimum as the final solution of the network. Table 4 shows that although the total mass balance at nodes for both methods is satisfactory, GGA is not able to satisfy energy balances around the loops for either local or global minima as accurately as PSO. The convergence trend of the PSO method is shown in Figure 9 and 10 for $\mu=10^6$ and $\mu=10^9$ against the number of iterations, for objective function, and total mass and energy balance. Generally, mass balances converge quicker than energy balances. Larger values of penalty coefficient μ result in higher accuracy of mass balances.

Table 3. Evaluation of objective function in PSO and GGA for Example 2 ($\mu=10^9$).

	Best	**Worst**	**Mean**	**Std**
PSO	-44.5989	-44.5989	-44.5989	0
GGA	-44.4839	10.8166	-24.0951	26.8896

Table 4. Evaluation of mass and energy balance in PSO and GGA for example 2($\mu=10^9$).

Mass Balance (Node)	**PSO**	**GGA[a]**	**GGA[b]**
1	8.713E-8	2.77556E-17	-2.52576E-15
2	8.014E-8	5.13478E-16	9.14546E-15
3	8. 041E-8	-3.33067E-16	-5.27356E-16
4	9.131E-8	-1.19349E-15	-1.04083E-15
5	7.848E-8	-8.32667E-17	-5.50948E-15
Energy Balance (Loop)	**PSO**	**GGA[a]**	**GGA[b]**
123	7.134E-07	0	-1.77636E-15
356	-7.251E-07	0.001194775	8.187415863
457	-1.798E-06	0.006735264	-417.2911369

[a] Global optimum
[b] Local optimum

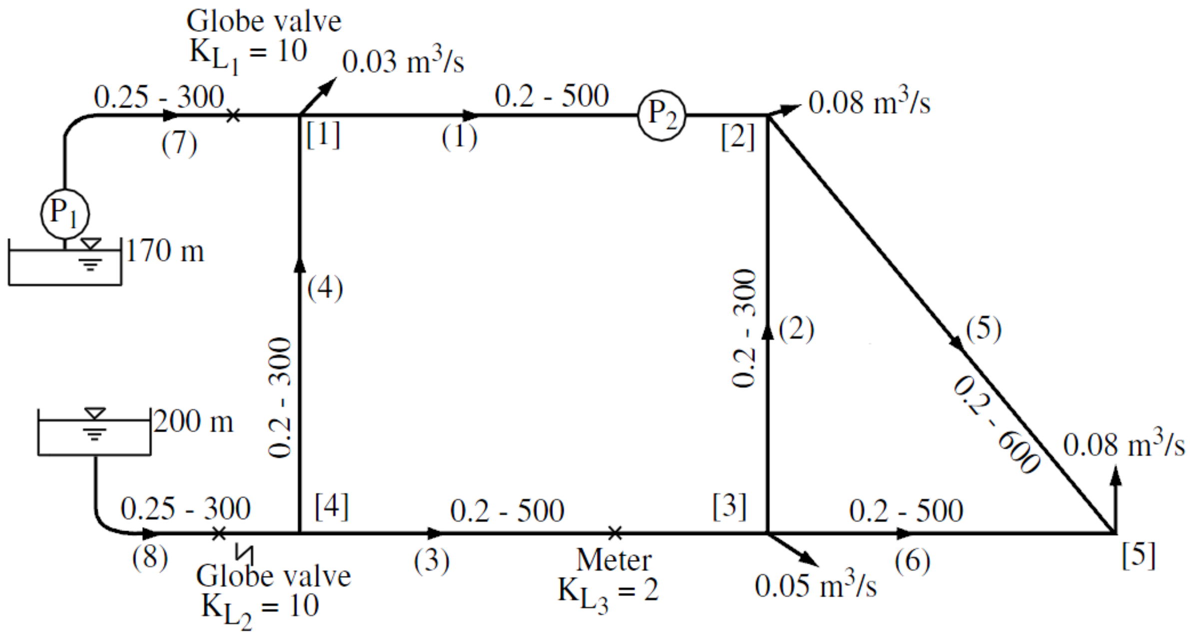

Fig. 7. Schematic representation of the looped pipe network for Example 2, (Larock et al., 2000).

Fig. 8. Performance of GGA in finding local or global minimum for 100 different runs.

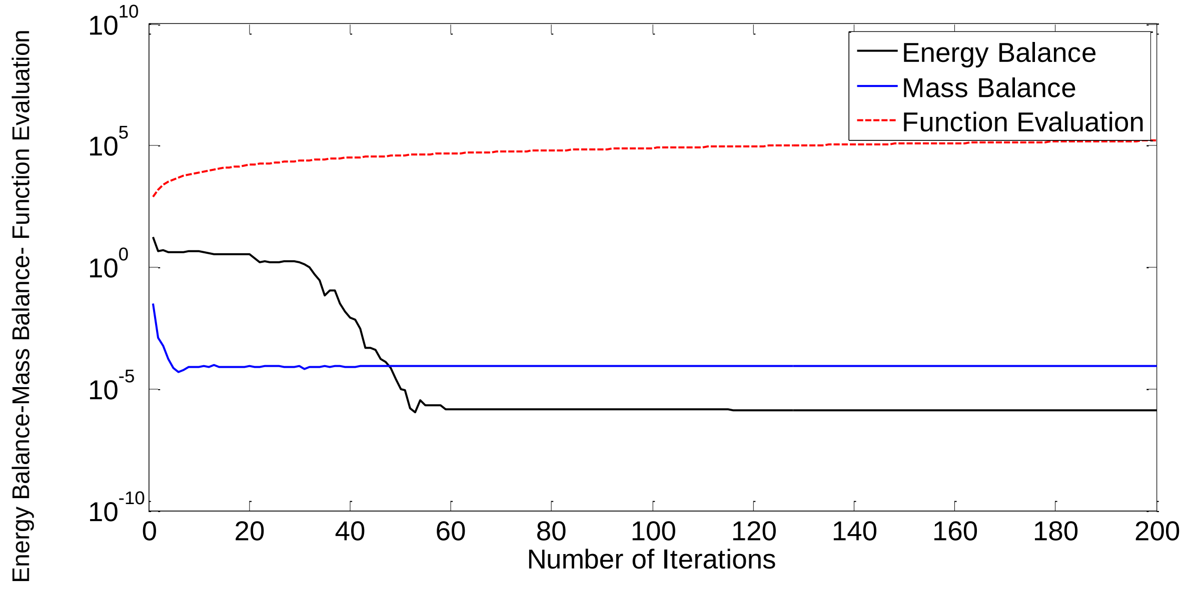

Fig. 9. Mass and Energy balances and function evaluation versus the number of iterations for the PSO algorithm with $\mu=10^6$, Example 2.

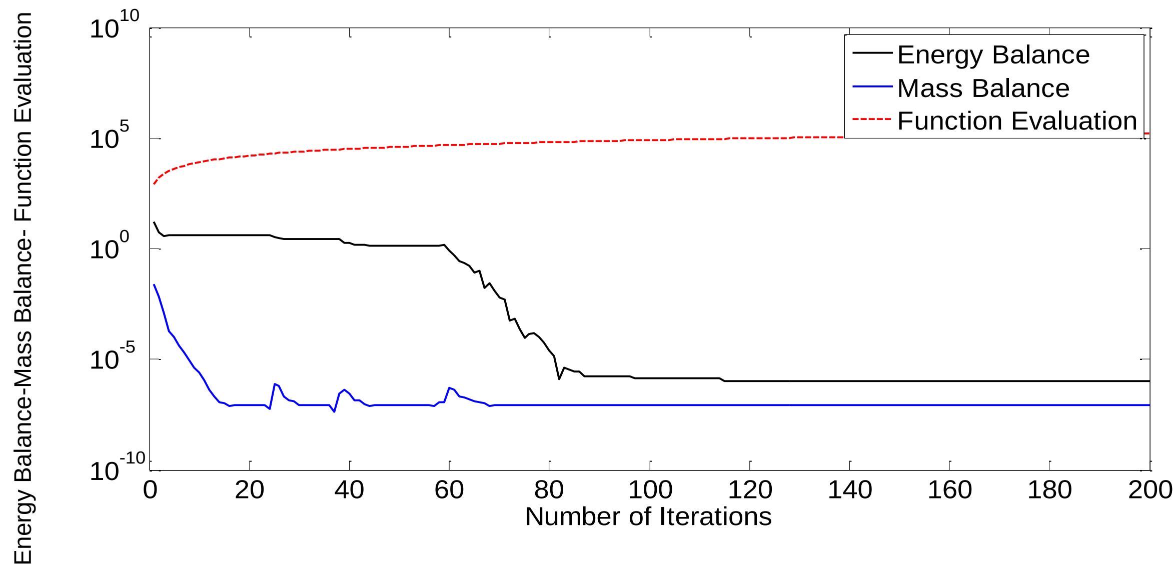

Fig. 10. Mass and Energy balances and function evaluation versus the number of iterations for the PSO algorithm with $\mu=10^9$, Example 2.

CONCLUSIONS

The Content model for the minimization of energy function is a suitable approach for pipe network analysis. For a network including several pumps, this model may become non-convex with multiple solutions. Newton-based methods such as GGA are not able to find the global optimum in this problem. Particle-swarm optimization (PSO) is an evolutionary phenomenon-mimicking algorithm for non-convex and unconstrained optimization problems that may be easily applied for the optimization of the Content Model by using a penalty function. It does not depend on any special initial solution vector, which is sometimes critical for the convergence of Newton-based methods. There is no need to solve linear systems of equations during the solution process and hence no need for huge memories. It has been shown that by enhancing the penalty function coefficient the accuracy of the solution may be improved considerably.

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