

## Automatic Calibration of HEC-HMS Model Using Multi-Objective Fuzzy Optimal Models

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**ABSTRACT:** Estimation of parameters of a hydrologic model is undertaken using a procedure called “calibration” in order to obtain predictions as close as possible to observed values. This study aimed to use the particle swarm optimization (PSO) algorithm for automatic calibration of the HEC-HMS hydrologic model, which includes a library of different event-based models for simulating the rainfall-runoff process. Since a flood hydrograph has different characteristics such as time to peak, peak discharge and total runoff volume, the calibration process is addressed using a single-objective or multi-objective optimization model. In this context, the fuzzy set theory can be used to combine different objective functions and convert the multi-objective model to a single-objective one. In this research, the Tamar basin, a sub-basin of the Golestan-Dam Basin in north of Iran, was selected as the case study with four reliable measured flood events. The first three events were used for calibration and the fourth one for verification. As most of the models built in the HEC-HMS software were event-based, the concept of recalibration of parameters related to a basin initial condition was also introduced. The comparison of results obtained from the single and multi-objective scenarios showed the efficiency of the proposed HMS-PSO simulation-optimization approach in the multi-objective calibration of event-based hydrologic models.

**Keywords:** Automatic Calibration, HEC-HMS, Particle Swarm Optimization, Rainfall-Runoff Modeling.

### INTRODUCTION

Management of water resources in a basin requires the identification of the basin characteristics for modeling hydrologic processes such as the transformation of rainfall into runoff. Estimation of runoff from rainfall as the main driving force in the hydrologic cycle is important not only for

management and operational purposes but also for mitigating natural disasters such as floods and droughts. Mathematical hydrologic models include a set of parameters to represent different hydrologic processes, which need to be estimated in a procedure called calibration.

Manual calibration, which is undertaken by visual inspection and a trial-and-error

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procedure, can become tedious, time consuming and may require personal experience and experimental information especially in presence of a large number of parameters. In response to restrictions involved in manual calibration, automatic calibration (AC) relying on systematic search algorithms has been introduced since 1970. These algorithms are developed in a way to find the best parameter values based on predefined objective functions.

Different challenges in an AC exercise include 1) competence of optimization algorithms as the search engine to locate global optimum values of a parameter, 2) the type of objective functions, 3) non-uniqueness of parameter values, and 4) multi-objective nature of the calibration process. There are several works addressing one or two of the mentioned challenges. For example, Duan et al. (1994) introduced Shuffled Complex Evolution (SCE) as an efficient optimization algorithm for the AC.

Since there is no single-objective function that can consider all the aspects of a hydrograph such as peak flow, time to peak flow, and flood volume, the multi-objective calibration has been introduced (Yapo et al., 1998). Some of studies where multi-objective calibration models have been recently used can be named as Madson (2000), Bekele and Nicklow (2007), Moussa and Chahinian (2009) and Kamali et al. (2012).

It is worth noting that in most applications of multi-objective calibration, continuous hydrologic models have been dealt with and fewer studies have addressed the calibration of event-based models. Hence, this study is focused on the multi-objective calibration of the Hydrologic Engineering Center- Hydrologic Modeling Systems (HEC-HMS) model which is one of well-known, public-domain and practically

in-use event-based hydrological models available.

The task of multi-objective calibration is performed by testing different combinations of objective functions, considering three important characteristics of hydrographs including time to peak, peak volume and flood volume. A fuzzy optimal model (Cheng et al., 1993) specifying the preference structure of decision makers is also used to combine different objectives. The PSO algorithm is used as the optimization algorithm. The presented methodology is verified in the Tamar basin, one of the sub-basins of the Golestan Dam basin. Finally, the results of both single-objective and multi-objective calibrations are compared.

## **STUDY AREA**

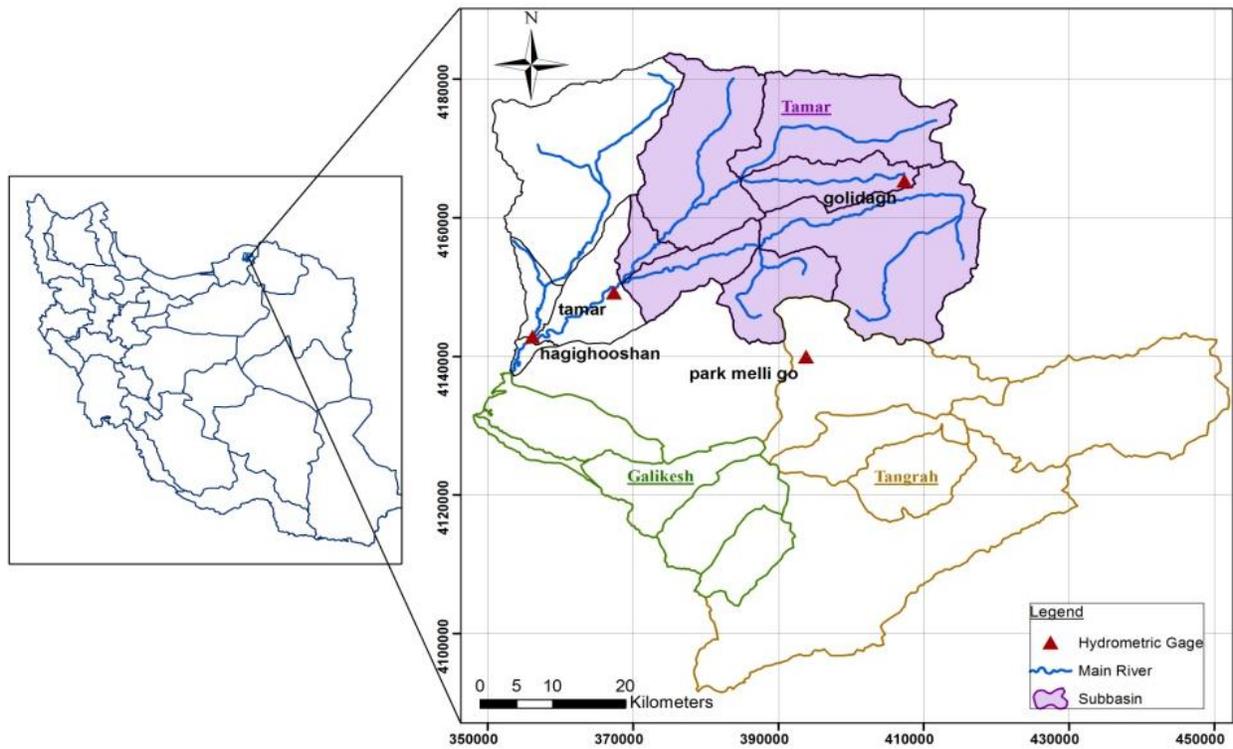
The study area is the Gorganroud River Basin in Iran, which is divided into three sub-basins namely, Tamar, Tangrah, and Galikesh with 11, 11, and 5 sub-basins, respectively. Figure 1 shows the schematic representation of the study area and its location on the country map.

Owing to flash floods and projected damages, there is an urgent need for a flood control management plan in this basin. Having a calibrated rainfall-runoff model for this basin due to the lack of measured flood data is a big challenge. The Tamar basin had the most reliable flood data and, hence, it was considered as the study area in this research (Figure 1).

Flood data were only available for four flood events, of which the first three events were used for calibration and the fourth one for verification. Table 1 presents the dates and the peak flows of the events. Figure 2 shows the corresponding hydrographs and the hyetographs.

**Table 1.** Characteristics of four flood events in Tamar Basin.

Event	Start Time	Peak Flow (cms)	Duration (hr)
1	19 Sep. 2004, 18:00	128	22
2	06 May 2005, 11:00	299	30
3	09 Aug. 2005, 20:00	783	19
4	08 Oct. 2005, 19:00	120	16



**Fig. 1.** Key map of Gorganroud River Basin and the study area.

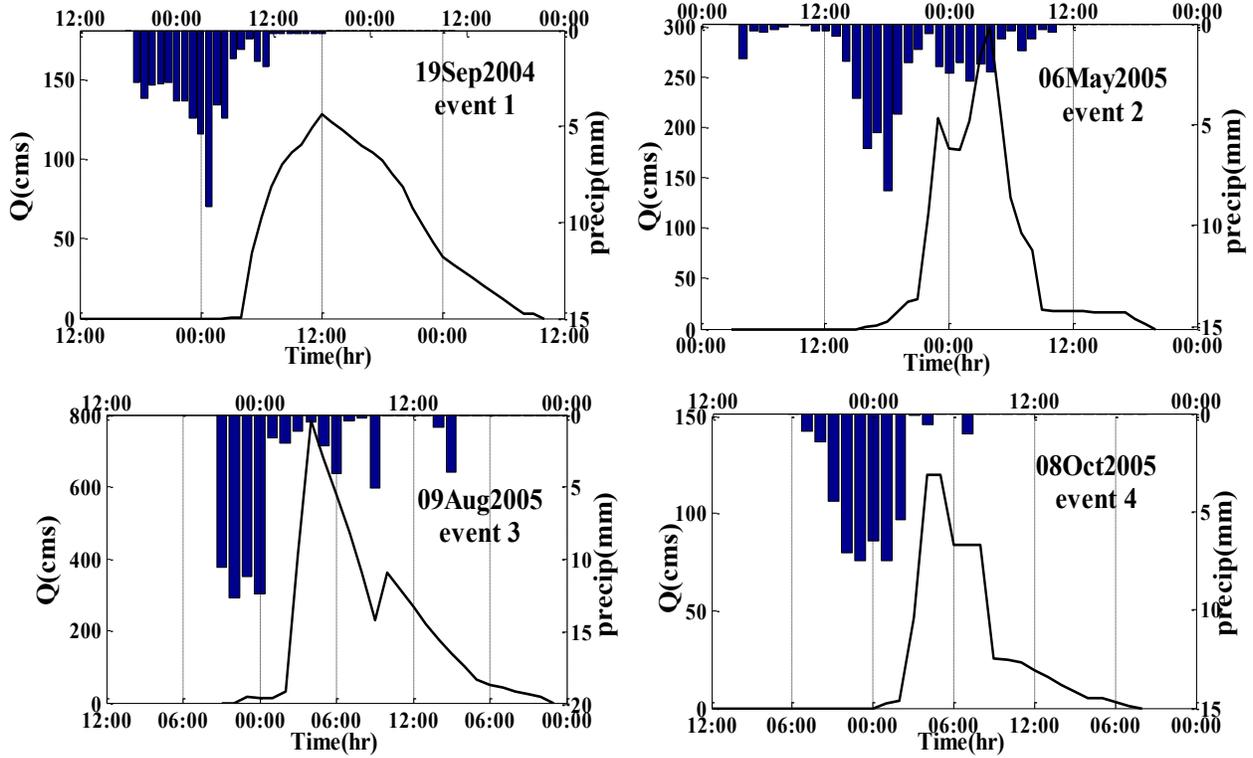


Fig. 2. Hydrographs and hietographs of four flood events in Tamar Basin (events 1-4).

## METHODOLOGY

### HEC-HMS Model of Study Area and Its Parameters

The HEC-HMS (HMS hereafter) is widely used as a standard and versatile model for hydrologic simulation (USACE, 2008). The model includes ten loss estimation methods, seven rainfall-runoff transformation methods and eight routing methods, from which SCS-CN, Clark hydrograph and Muskingum methods were selected, respectively. No base flow was considered in this study as it had an insignificant effect on flood flows.

The SCS-CN method has two parameters; curve number ( $CN$ ) and initial abstraction ( $I_a$ ). The method estimates the initial abstraction using the following equations:

$$I_a = \alpha \times S \quad (1)$$

where  $S$  (cm) is the basin maximum retention and  $\alpha$  is the loss coefficient. In this study, the loss coefficient was assumed to be between 0.15 and 0.25. Hence,  $CN$  and  $\alpha$  are the first and second sets of calibration parameters.

The Clark unit hydrograph method also requires the identification of two parameters, including time of concentration ( $T_c$ ) and storage coefficient ( $R$ ). The following equations were used for calculating  $T_c$  based on the method of SCS synthetic unit hydrograph (Chow, 1988):

$$T_c = 1.67 * \frac{(L \times 3.28)^{0.8} \times (1000 / CN - 9)^{0.7}}{1900y^{0.5}} \quad (2)$$

where  $L$  is the length of river in meter, and  $y$  is the slope of basin. The storage coefficient

and time of concentration are related as follows (Timothy, 2000):

$$\frac{R}{R + T_c} = C_s \quad (3)$$

Considering the constant  $C_s$  as a calibration parameter with values between 0.2-0.6, we also calibrate the storage coefficient parameter. Two parameter sets of the Muskingum routing method are  $x_m$ , which varies between 0.2-0.5, and  $K_m$  varying between 0.5 -3.5.

Therefore, each sub-basin has three parameters resulting in 21 parameters to which 6 routing parameters (two for each reach) are added. The total number of calibration parameters will therefore be equal to 27. Table 2 represents the parameters and their upper and lower bounds.

**Table 2.** Upper and lower bounds of the calibration parameters.

Parameter	Parameter Index	Sub-basin	Upper Bound	Lower Bound
		Sub-basin1	91	60
		Sub-basin2	91	61
		Sub-basin3	87	58
$CN_1 - CN_7$	1-7	Sub-basin4	85	60
		Sub-basin5	84	50
		Sub-basin6	91	70
		Sub-basin7	91	70
$\alpha_1 - \alpha_7$	8-14	7 Sub-basins	0.15	0.45
$Cs_1 - Cs_7$	15-21	7 Sub-basins	0.20	0.6
$X_m$	22-24	3 Reaches	0.20	0.5
$K_m$	25-27	3 Reaches	0.5	3.5

### Particle Swarm Optimization Algorithm

The PSO is a population-based optimization technique introduced by Eberhart and Kennedy (1995), motivated by collective and social behaviour of bird flocking or fish schooling (Parsopoulos, 2002).

The PSO algorithm is initialized with a population (swarm) of random solutions (particles) and searches for optima by updating particles' locations (values) within the parameters space through determining velocity and position vectors as follows:

$$\begin{aligned} X_i &= [X_{i1}, X_{i2}, \dots, X_{in}] \\ V_i &= [V_{i1}, V_{i2}, \dots, V_{in}] \end{aligned} \quad (4)$$

The algorithm updates the particles' locations in each iteration through following two best particles, i.e. "Pbest" which is the best solution achieved so far by a particle and "Gbest" value that is the best solution obtained so far among all particles in the swarm. In other words, each particle moves toward the best particle of its neighbourhood, and is identified by two vectors; velocity vector and position vector. After determining Gbest and Pbest values, each particle updates its velocity and position according to Eqs. (5) and (6).

$$V_{ij}(t) = W \times V_{ij}(t-1) + C_1 \times rand \times (Pbest_{ij} - X_{ij}(t-1)) + \quad (5)$$

$$C_2 \times rand \times (Gbest_j - X_{ij}(t-1))$$

$$X_{ij}(t) = X_{ij}(t-1) + V_{ij}(t) \quad (6)$$

where  $i$  shows the particle's number in a swarm,  $j$  is the particle's dimension and  $t$  is the iteration number.  $V_{ij}$  and  $P_{ij}$  are particle's velocity and position, respectively.  $W$  is the weighting factor for balancing exploration and exploitation features of the

search algorithm.  $C_1$  and  $C_2$  are learning factors.

A shortcoming of the PSO is the stagnation of particles before a good or near-global optimum is reached. A strategy to drive lazy particles and let them explore better solutions is to use the turbulent PSO (Liu and Abraham, 2007), in which velocities of lazy particles of which the velocities decrease to a threshold  $V_C$  are updated using Eq. (7):

$$\hat{V} = \begin{cases} V_{ij} & \text{if } |V_{ij}| \geq V_C \\ u(-1,1) \frac{V_{\max}}{\rho} & \text{if } |V_{ij}| < V_C \end{cases} \quad (7)$$

where  $U(-1,1)$  is a random number,  $\rho$  is a scaling factor which controls the domain of particle's oscillation according to  $V_{\max}$ . We have also implemented the elitist-mutation strategy (Reddy and Kumar, 2007) by which the best solution in the swarm replaces the worst solution, after performing mutation operator on the best solution.

### Performance Criteria

As the performance criteria, four objective functions, each of which considers one aspect of a hydrograph, are used in this study. The Root Mean Square Error (RMSE), Eq. (8), is the first criterion which is affected by all discharge points of a hydrograph, yet more importance is given to higher flows.  $V_{error}$ , Eq. (9), is the second objective function which focuses on the flood volume. The correlation coefficient ( $R$ ), Eq. (10), is the next objective function, and is used to minimize the error in estimating time to peak.  $Peak$ , Eq. (11), is the last objective function which considers the peak flow while ignoring the flood volume and time to peak criteria.

$$RMSE = \frac{1}{N} \sum_{j=1}^N \left[ \frac{1}{n_i} \sum_{i=1}^n (Q_{o,i} - Q_{s,i})^2 \right]^{1/2} \quad (8)$$

$$V_{error} = \frac{1}{N} \sum_{i=1}^N \left| \frac{V_{o,i} - V_{s,i}}{V_{o,i}} \right| \quad (9)$$

$$R_j = 1 - \frac{\sum_{i=1}^n (Q_{s,i} - \bar{Q}_s)(Q_{o,i} - \bar{Q}_o)}{\sqrt{\sum_{i=1}^n (Q_{s,i} - \bar{Q}_s)^2 \times \sum_{i=1}^n (Q_{o,i} - \bar{Q}_o)^2}} \quad (10)$$

$$R = \frac{1}{N} \sum_{j=1}^N R_j$$

$$Peak = \sum_{i=1}^N \left| \frac{Peak_{o,i} - Peak_{s,i}}{Peak_{o,i}} \right| \quad (11)$$

where  $Q_o$  and  $Q_s$  are, respectively, the values of observed and simulated discharges,  $Peak_o$  and  $Peak_s$  are the peak flows of observed and simulated hydrographs, respectively.  $\bar{Q}_o$  and  $\bar{Q}_s$  are, respectively, the mean values of observed and simulated discharges.  $V_o$  and  $V_s$  are, respectively, the observed and simulated volumes of runoff and  $N$  is the total number of sampled discharges.

### Fuzzy Optimal Model (FOM)

Let the total number of objective functions be  $m$  and the total number of feasible alternatives be  $n$ . Then, the decision matrix represents  $X = (x_{ij})_{m \times n}$ , where  $x_{ij}$  is the  $i$ th objective values of alternative  $j$ . The decision matrix  $X$  is transformed into a matrix of membership degree as follows:

$$r_{ij} = \frac{(x_{i, \max} - x_{ij})}{(x_{i, \max} - x_{i, \min})} \quad R = (r_{ij})_{m \times n} \quad (12)$$

where  $X_{i \max} = \bigvee_{i=1}^n x_{ij}$ ,  $X_{i \min} = \bigvee_{i=1}^n x_{ij}$ . For cost type of the objective function, the

smaller its value, the greater is the membership (Figure 3).

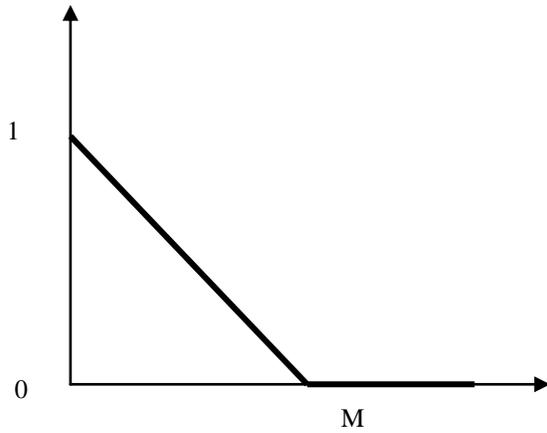


Fig. 3. The membership function based on the acceptable degree of the objective function.

For a multi-objective decision making problem, the “ideal” alternative is defined as:

$$G = \underbrace{(1,1,\dots,1)}_m^T \quad (13)$$

and the “non-ideal” alternative is defined as:

$$B = \underbrace{(0,0,\dots,0)}_m^T \quad (14)$$

To determine the optimal solution, one should choose an alternative which is closest to  $G$  and farthest from  $B$ . Therefore, the weighted distance is defined as:

$$D_j(w) = \sqrt{\sum_{i=1}^m [w_i (g_i - r_{ij})]^2} = \sqrt{\sum_{i=1}^m [w_i (1 - r_{ij})]^2} \quad (15)$$

$$d_j(w) = \sqrt{\sum_{i=1}^m [w_i (r_{ij} - b_i)]^2} \quad (16)$$

$$= \sqrt{\sum_{i=1}^m (w_i r_{ij})^2}$$

$$\sum_{i=1}^m w_i = 1 \quad (17)$$

$$f_j(u_j) = u_j^2 \sum_{i=1}^m [w_i (1 - r_{ij})]^2 + (1 - u_j)^2 \sum_{i=1}^m (w_i r_{ij})^2 \quad (18)$$

Consequently, the weighted distance should be minimized:

$$\text{Min}[f_j(u_j)] \quad 0 < u_j < 1 \quad j = 1, 2, \dots, n \quad (19)$$

$$\frac{df(u_j)}{du_j} = 0 \quad u_j = \left[ 1 + \frac{\sum_{i=1}^m [w_i (1 - r_{ij})]^2}{\sum_{i=1}^m (w_i r_{ij})^2} \right]^{-1} \quad (20)$$

According to the definition of membership functions in the fuzzy set theory, the bigger  $u_j$ , the better the alternative is; thus, the optimal order of alternatives is obtained.

## RESULTS

### Single-Objective Scenario

In the first stage, the joint-calibration of the first three events was conducted by using a single objective approach while considering the RMSE as the objective function. Equal degrees of importance were considered for all three events. The results in different tests showed that there was no unique set of parameters by which both events 1 and 3 can be calibrated. Consequently, an attempt was made to increase the number of parameters in order to enhance the model flexibility to simulate the flood hydrographs of both events 1 and 3 concurrently. This was undertaken through considering different loss coefficients for events 1 and 3. So, the total number of used parameters increased from 27 to 34, and the joint-calibration of the first three events was repeated.

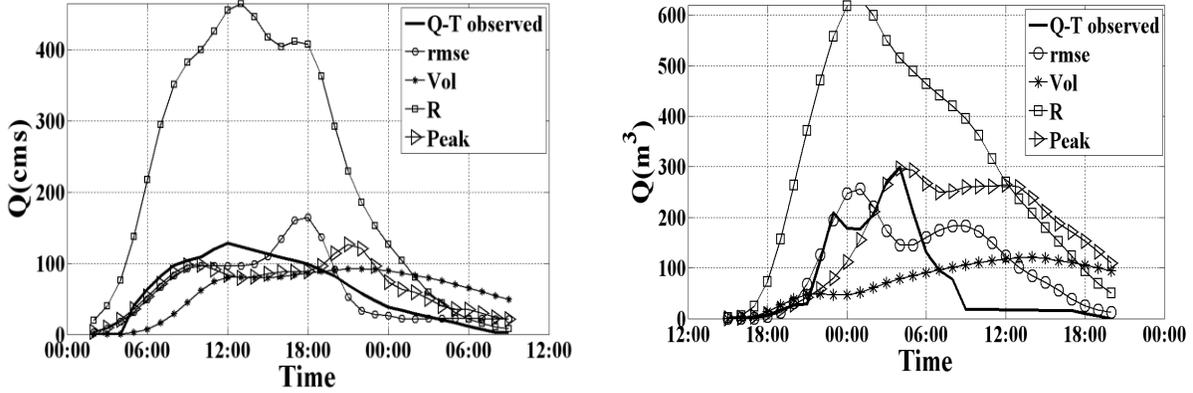


Fig. 4. Results of single-objective calibration of events 1 (left) and 2 (right) using four different criteria.

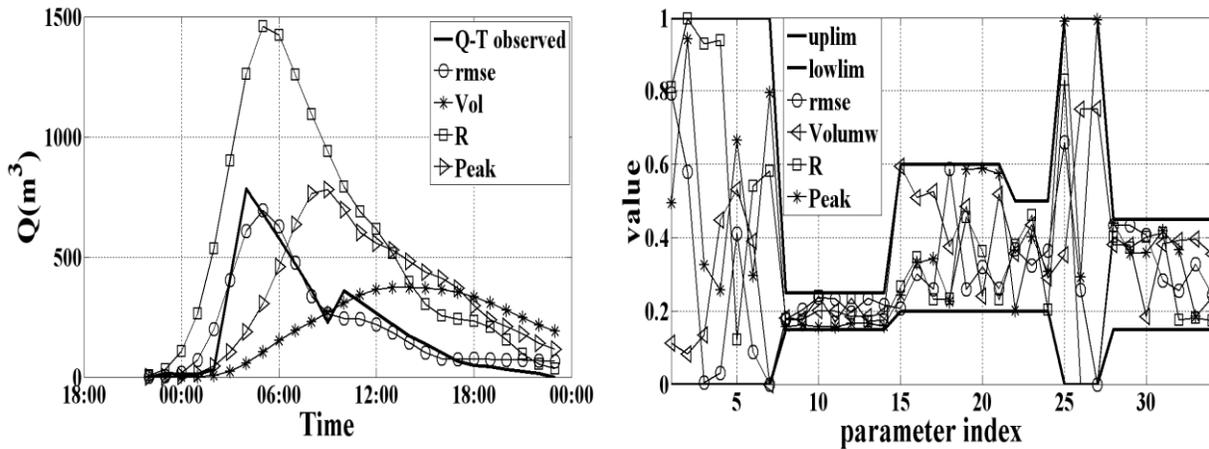


Fig. 5. Results of single-objective calibration of event 3 using four different criteria (left); and the parameter values and their upper and lower bounds (right).

Figures 4 and 5 show the observed and simulated hydrographs of the events. It is clear from the obtained results that the Peak function could only simulated the peak flow. Therefore, differences between the peak flows were negligible, while there was a significant difference in the shape and time to peak of simulated and observed hydrographs.

The objective function of *Vol* ( $V_{error}$ ) only considers the flood volume. So, the volume of floods was satisfactory simulated using this objective function. Figure 5 (right) shows the upper and lower bounds and parameter values resulted from calibration where the values of  $CN$  and  $K_m$  were normalized between 0 and 1. The result shows that the parameters obtained by using

different objective functions are quite different.

The results of the single-objective calibration showed that the RMSE was the only objective function showing relatively an acceptable performance since each of other objective functions focused on only one feature of a hydrograph. Non-uniqueness is another important implication resulted from the findings. It was seen that even in the case of using one objective function, it was not possible to find a unique set of parameter values.

### Multi-Objective Scenario

In this section, the FOM was used to combine three objective functions. In this algorithm, it is assumed that the preference

structure of decision makers is specified. Two different combinations of objective functions including ( $Peak, V_{error}, R$ ) and ( $RMSE, V_{error}, R$ ) were evaluated under scenarios of  $fuzzy_1$  and  $fuzzy_2$ , respectively. The weights assigned to each of objective functions were set to 0.4, 0.4 and 0.2 for the  $fuzzy_1$  scenario and 0.25, 0.5 and 0.25 for the  $fuzzy_2$  scenario, respectively. Figure 6 shows the simulated and observed hydrographs of the two mentioned scenarios.

Comparing the results obtained by the FOM with those resulted from the single-objective calibration shows that the multi-objective scenarios have performed much better than the single-objective scenarios. In the  $Fuzzy_1$  scenario, the flood volume of first event was simulated quite well, while significant differences are observed between recession curves, peak flows and time to peaks of the hydrographs. Nevertheless, the results are better than those of single-objective scenarios.

Comparing the results of jointly-event calibration in different scenarios revealed that event 2 with two close peaks as well as a fast decreasing recession curve was satisfactory simulated. So, the results were compared with the single event calibration in which the model has a greater degree of freedom in simulating each event. However, the result of single events was not satisfactory and close to those obtained in jointly-event calibrated scenarios.

The  $Fuzzy_1$  scenario simulated the peak flow of event 2 much better compared to the  $fuzzy_2$  scenario, and also outperformed event 3. This reveals the fact that selecting appropriate combinations of criteria has a significant influence on model results.

In order to compare the results of single-objective and multi-objective scenarios, we need to test how they perform in the verification stage.

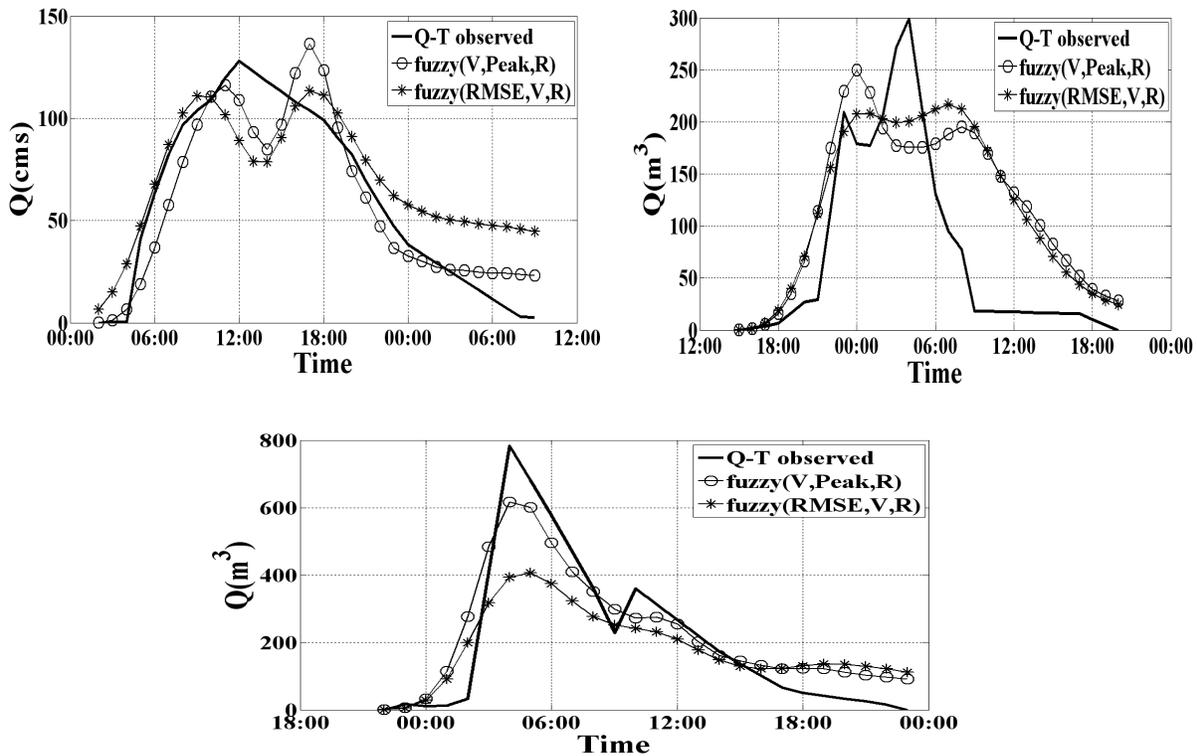


Fig. 6. The result of multi-objective calibration in scenarios of  $fuzzy_1$  and  $fuzzy_2$ .

**Verification**

In this stage, all the parameter sets obtained from single-objective and multi-objective scenarios were tested with respect to event 4. Six parameter sets were obtained, of which 4 parameter sets were from single-objective calibration and others from multi-objective calibration.

Figure 8 shows the simulated hydrograph for event 4 using 6 parameter sets mentioned above. One can see that all the parameter sets have performed poorly in simulating the peak flow. However, it may not be wise to judge based on the results presented in Figure 8 (right) because each parameter set is associated with its own basin initial condition that could be different from that of event 4.

Therefore, the initial abstraction coefficient values of each candidate set should have been recalibrated when they were used for simulating event 4 hydrograph. This recognizes the fact that each flood event could have its own basin initial condition.

In this regard, we only recalibrated initial abstraction coefficients and not the basin parameters. Recalibrating the initial abstraction coefficients of all the parameter sets could have been conducted by optimizing initial abstraction coefficients with respect to event 4. However, we multiplied them by a constant factor to make their simulated peak discharges almost equal to that of event 4. Figure 7 (left) shows the results after recalibrating the initial abstraction coefficient values.

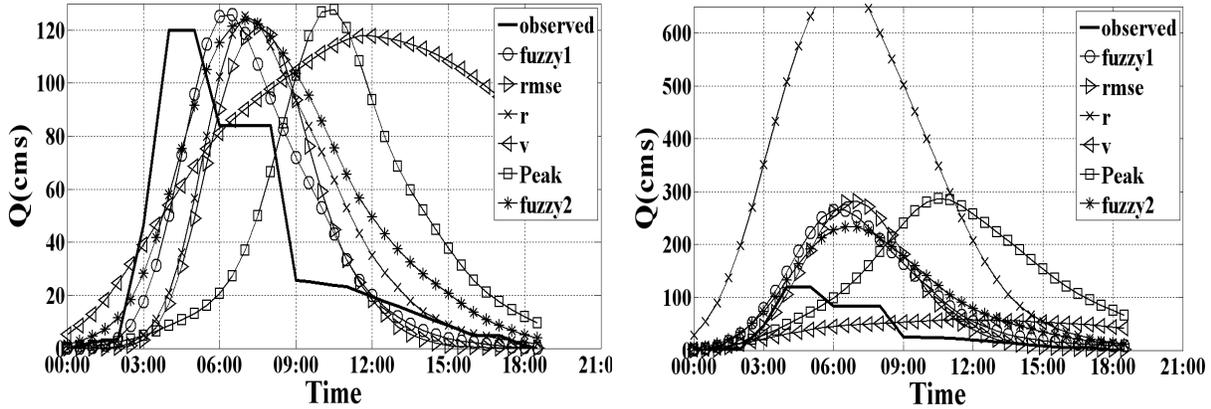


Fig. 7. Verification of single-objective and multi-objective calibration results before recalibration (right) and after recalibration (left) of initial abstraction coefficients.

Table 3. Initial abstraction values in single objective and multi-objective calibration after recalibration.

	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$	$\alpha_6$	$\alpha_7$	Acceptance State
RMSE	0.24	0.27	0.25	0.27	0.27	0.266	0.26	Y
$V_{err}$	0.42	0.49	0.56	0.47	0.56	0.52	0.52	N
R	0.7	0.7	0.96	0.68	0.94	0.68	0.71	N
Peak	0.11	0.11	0.12	0.12	0.11	0.11	0.116	Y
Fuzzy <sub>1</sub>	0.33	0.36	0.48	0.35	0.37	0.45	0.39	Y
Fuzzy <sub>2</sub>	0.35	0.38	0.34	0.34	0.36	0.36	0.35	Y

**Table 4.** Initial abstraction values in single objective and multi-objective calibration after recalibration.

	$CN_1$	$CN_2$	$CN_3$	$CN_4$	$CN_5$	$CN_6$	$CN_7$	$Km_1$	$Km_2$	$Km_3$
Fuzzy <sub>1</sub>	78.1	86.86	60.1	60.5	67.16	76.44	70.12	0.21	0.29	0.27
Fuzzy <sub>2</sub>	73.63	77.5	72.2	60.78	60.5	70.74	85.22	0.2	0.39	0.24
RMSE	75.84	68.07	75.52	60.76	54.42	77.94	70	0.2	0.4	0.31
Peak	75.7	88.7	67.8	65.3	73.1	72.84	86.8			
	$Cs_1$	$Cs_2$	$Cs_3$	$Cs_4$	$Cs_5$	$Cs_6$	$Cs_7$			
Fuzzy <sub>1</sub>	0.22	0.21	0.27	0.56	0.39	0.21	0.20			
Fuzzy <sub>2</sub>	0.22	0.21	0.23	0.56	0.23	0.36	0.22			
RMSE	0.23	0.27	0.2	0.21	0.22	0.24	0.35			
Peak	0.2	0.27	0.38	0.21	0.56	0.57	0.58			

One can see that the simulated hydrographs are now much closer to the observed hydrographs with a slight shift with respect to time to peak. Table 3 presents the recalibrated initial abstraction coefficients. For some parameter sets, the recalibration led, for at least one of the sub-basins, to an initial abstraction coefficient value beyond the extended physically meaningful range. Therefore, they were removed and only four parameter sets were remained including fuzzy<sub>1</sub>, fuzzy<sub>2</sub>, RMSE and  $V_{err}$ . Table 4 represents the parameter values of these sets, other than the recalibrated initial abstraction coefficients.

In order to compare the performance of multi-objective and single-objective calibration scenarios, the following two criteria were considered:

$$\varepsilon_v = \frac{V_o - V_s}{V_o} \times 100 \quad (21)$$

$$\varepsilon_Q = \frac{\sum_{i=1}^n (Q_{o,i} - Q_{s,i})}{\sum_{i=1}^N Q_{o,i}} \times 100\% \quad (22)$$

Eq. (21) assesses runoff volume error, while Eq. (22) deals with flow differences. Table 5 compares the results of four

parameter sets corresponding to both criteria. The results show that the fuzzy<sub>1</sub> scenario has a much better performance.

**Table 5.** Comparison of results of single-objective and multi-objective calibration regarding two criteria.

Objective Function	$\varepsilon_v$	$\varepsilon_Q$
RMSE	10.35	10.35
Peak	18.62	18.98
Fuzzy <sub>1</sub>	3.87	3.86
Fuzzy <sub>2</sub>	24.7	24.4

## SUMMARY AND CONCLUSION

This study presented the single-objective and multi-objective optimization algorithms for automatic calibration of the HEC-HMS rainfall-runoff model of the Tamar sub-basin of the Gorganroud river basin in north of Iran. A fuzzy optimal model was used to combine different criteria, and the particle swarm optimization algorithm was employed as the optimization algorithm. Three flood events were used for calibration and one for verification purposes. Four objective functions including root mean square error, percent error in peak flow, percent error in runoff volume, and correlation coefficient were used as the performance criteria.

The results of single-objective calibration of jointly-calibrated events showed that there was no single criterion that can represent all characteristics of runoff hydrographs. As a result, multi-objective calibration scenarios were also considered.

Six candidate parameter sets were recalibrated, tested and compared in the verification stage. After recalibration, only four of the candidate sets were remained. In order to compare the results of single objective with fuzzy multi-objective scenarios, their performances were assessed against two criteria. This comparison showed that the multi-objective approach outperformed the single objective method.

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