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The Effect of Spatial Variability and Anisotropy of Soils on Bearing Capacity of Shallow Foundations

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ABSTRACT: Naturally occurred soil deposits inherit heterogeneity and anisotropy in their strength properties. The main purpose of this paper is to model the soil stratum with anisotropy consideration and spatially varying undrained shear strength by using random field theory coupled with finite difference numerical analysis to evaluate their effect on the bearing capacity of the shallow foundations. In the present study, undrained shear strength of soil is considered as a stochastic variable and is assumed to be log-normally distributed and spatially correlated throughout the domain. Two kinds of anisotropy of cohesion are incorporated in the analyses. As the first kind, mechanical anisotropy of cohesion was taken into account by generalizing the conventional isotropic Mohr-Coulomb failure criterion to the anisotropic one, and the second kind is the heterogeneity anisotropy associated with difference in the correlation structure of the cohesion data in the horizontal and vertical directions considered by a special anisotropic correlation function. The results showed the importance of different components of anisotropy and the stochastic variation of shear strength parameters. Mechanical anisotropy and the spatial variability of the cohesion showed that they have significant effects on the bearing capacity of the shallow foundations and their negligence will lead to an under-conservatism.

Keywords: Anisotropy, Heterogeneity, Random Field, Shallow Foundations, Undrained Shear Strength.

INTRODUCTION

The ultimate bearing capacity of the strip footing on the homogeneous and isotropic soils has been extensively studied in several studies. The process of formation and deposition of natural deposits leads to spatial variability and anisotropy in the soil properties which are neglected in the conventional methods. Considerable works have been done with regard to the influence of anisotropy and non-homogeneity on the bearing capacity of clays (Raymond 1967; Reddy and Srinvasan, 1967, 1971; Davis and Christian 1971; Davis and Booker, 1973; Livneh and Greenstein, 1973; Salencon 1974a, 1974b). Most of these studies found that the anisotropy and non-homogeneity have a considerable influence on the bearing capacity of clays. Raymond (1967) studied

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the bearing capacity of footings and embankments on the heterogeneous clays through the slip circle method. He presented a dimensionless plot of failure criteria for footings. If a footing is analyzed based on his assumption and the result is compared with the failure criteria, the footing is theoretically considered safe or unsafe.

Davis and Christian (1971) used a new description of anisotropic cohesive strength to provide a simple solution for the bearing capacity of a strip footing. Davis and Booker (1973) showed that by means of the theory of plasticity, solutions are obtained for solving the problem of bearing capacity of the clay (φ = 0) which is inhomogeneous in the vertical direction only. It is shown that the rate of increase of cohesion with depth plays the same role as density plays in the bearing capacity of homogeneous cohesive-frictional soils.

On the other hand, however, very few attempts have been made to study the effect of anisotropy and non-homogeneity on the bearing capacity of c-φ soils. Reddy and Srinivasan (1970) studied the effect of anisotropy and non-homogeneity on the bearing capacity of c-φ soils including $\varphi = 0$ condition of soils. In their study, they used the method of characteristics to obtain the bearing capacity of shallow foundations on the heterogeneous and anisotropic soils. Salencon (1974) and Salencon et al. (1976) presented an analysis for the bearing capacity of c-φ-γ soil taking a linear variation of cohesion with depth. Meyerhof (1978) obtained the bearing capacity for soils exhibiting anisotropy in friction by the conventional Terzaghi's type approach using two extreme values of φ for the outer zones and an equivalent φ for the radial shear zone.

Limit analysis and in particular the upper bound analysis has long been used as a convenient tool for solving problems that involve anisotropy and non-homogeneity of soils. Chen (1975) considered the applications of limit analysis to geotechnical problems. He utilized the limit analysis theory for the bearing capacity of footings on a single and two layered soil, considering both the anisotropy and heterogeneity. He compared the ratios of N_c values, those obtained by his own and Salencon (1974), for heterogeneous and homogeneous deposits. The ratios obtained by him are found slightly higher than those of Salencon (1974). Adopting a Prandtl failure mechanism, Reddy and Rao (1982) used the limit analysis (upper bound) to obtain the bearing capacity of strip footing on c-φ soils, exhibiting anisotropy and non-homogeneity in cohesion.

The purpose of this paper is to investigate the effect of inherent variability of the shear strength and two distinct types of anisotropy on the bearing capacity of the shallow foundations. Mechanical anisotropy of cohesion attributed to the directional variability of cohesion and the directionality of correlation structure was investigated to see their effect on the bearing capacity of the shallow foundations.

For these reasons, the undrained shear strength is assumed as a log-normally distributed and spatially correlated variable. Mechanical anisotropy, denoted by C_H/C_V is taken into account by generalizing the conventional Mohr-Coulomb yield criterion. The analyses were conducted with Fast Lagrangian Analysis Code FLAC 5.0 which produces random finite difference model (RFDM) for anisotropic bearing capacity problem.

INHERENT VARIABILITY

The spatial variation in property C with the depth of z can be decomposed into a trend function t and the fluctuation component w (Phoon et al., 1999):

$$C(z) = t(z) + w(z) \tag{1}$$

where *C(z)* is in situ cohesion, *t(z)* is deterministic trend function, and *w(z)* is the off-trend function or the fluctuation component which fluctuates around the deterministic trend function. Deterministic trend can be estimated from the reasonable amount of in situ data, whereas the fluctuation can be characterized as a random variable having a zero mean and non-zero variance. Figure 1 schematically illustrates different components of the inherent variability.

Deterministic Variation of Cohesion

The deterministic part of the soil properties can be assumed to have either a linear variation with the depth or a quadratic variation (Kenarsari, et al., 2012). Linear variation is usually reasonable for normally consolidated deposits while over-consolidated alluvial deposits exhibit quadratic trend. However, in this study linear trend was chosen for its simplicity. Thus, the cohesion at depth z from the surface is given by Salencon et al. (1976):

$$C_V = C_{V_0} + \lambda z \tag{2}$$

where C_{V0} is the cohesion in the vertical direction at z = 0, and λ is the strength density, which means the increasing rate of the shear strength with depth.

Stochastic Variation of Cohesion

In the present study, for the stochastic variation of the soil cohesion, log-normal distribution is proposed because of two reasons; first, there is no possibility of the existence of negative values, and the second reason is its simple relationship with the normal distribution (Harr, 1987). The cohesion is assumed to be characterized by correlated log-normally distribution using the aid of most recognized representative statistical parameters which are the mean value, the standard deviation, and the correlation length. In practice the dimensionless coefficient of variation (*COV*) is used instead of standard deviation which can be defined as the standard deviation divided by the mean. Typical values for the *COV* of the undrained shear strength have been suggested by several investigators (Lee, *et al*., 1983; Duncan, 2000). The Suggested values are based on the in situ or laboratory tests and the recommended range is 0.1-0.5 for the *COV* of the undrained shear strength. The third important attribute of a random field is its correlation structure.

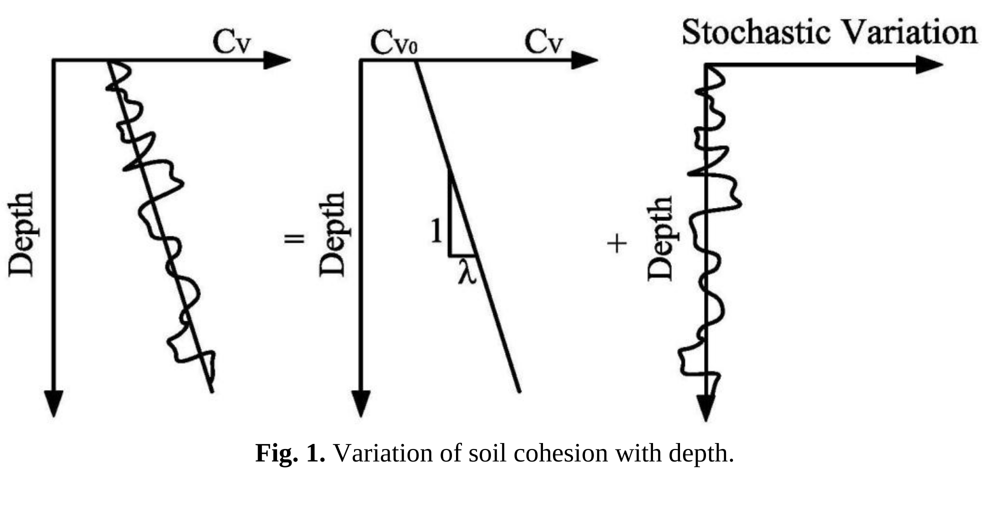

Fig. 1. Variation of soil cohesion with depth.

It is obvious that if two samples are close to each other, they will be usually more correlated compared to the case when they are widely separated. Choleski decomposition technique was adopted to produce the covariance matrix and it was then used to generate the correlated data. The Choleski technique as discussed in Nash (1979) is based on decomposing a symmetric, positive, and definite matrix into a lower triangular matrix. Since the cohesion field is log-normally distributed, taking its logarithm yields a normally distributed random field. The values of cohesion are realized from:

$$\ln c = L.\varepsilon + \mu_{\ln c} \tag{3}$$

where μ_{lnc} is the mean of *ln c*, ε is a Gaussian vector field (having zero mean and unit variance) and *L* is the lower triangular matrix defined by:

$$\mathrm{A} = \mathrm{LL}^{\mathrm{T}} \tag{4}$$

where *A* is covariance matrix which bears the heterogeneity anisotropy of soil stratum to be discussed shortly. The anisotropic covariance matrix is given by (Vanmarcke, 1983):

$$A(\Delta x, \Delta y) = \sigma^2_{\ln c} \exp(-2\sqrt{(\frac{\Delta x}{\theta_H})^2 + (\frac{\Delta y}{\theta_V})^2}) \tag{5}$$

where σ^2_{lnc} is the variance of *ln c*, θ_H and θ_V are autocorrelation length or the scale of fluctuation in horizontal and vertical direction, respectively, Δx and Δy are horizontal and vertical lag distances respectively.

The correlation length is the parameter which describes the degree of correlation of a soil property, and is defined as the distance beyond which the random values will be no more correlated at all. It should be noted that in the case of a large correlation length, the random field tends to be smooth, and conversely, when it is small, the random field tends to be rough (Griffith and Fenton, 2001).

Figure 2 illustrates a flowchart of the Monte Carlo simulation scheme by random finite difference modeling through repeated analyses.

ANISOTROPY

Anisotropy is the property of being directionally dependent as opposed to isotropy which implies identical properties in all directions. It can be defined as a difference, when measured along different axes, in materials' physical or mechanical properties.

This study covers two major sources of anisotropy, namely heterogeneity and mechanical anisotropy. Heterogeneity anisotropy reflects the difference in correlation length in the horizontal and vertical directions. It is indeed the directionality of the correlation structures. Mechanical anisotropy on the other hand, is the non-uniqueness of the horizontal and vertical deformation or the strength parameters. Detailed description and literature review on this issue are provided in the subsequent sections.

Heterogeneity Anisotropy

A distinction is usually made between a scale of fluctuation in the vertical and horizontal directions which is called heterogeneity anisotropy. Available studies strongly show that a much higher horizontal degree of correlation exists in comparison to the vertical direction, due to the process of deposition. The values of θ_V are generally between 0.5 and 2 m, whereas the corresponding values for θ_H are generally of the order of 10-30 m (Cherubini, 2000). The value of θ_H/θ_V, called Heterogeneity

Anisotropy Factor ($A.F_H$), has been reported for clays to be of the order of 9 (Vanmarcke, 1997), 10 (Soulie, et al., 1990), and 13 (Phoon and Kulhawy, 1999).

Mechanical Anisotropy

During the deposition and subsequent consolidation, the particles of uncemented sedimentary soils develop an interparticle equilibrium that leads to particular ranges of possible soil fabrics. The conditions applied during these processes, control the particles' orientations and contact configurations which rule the soil response during the subsequent changes in the stresses or strains.

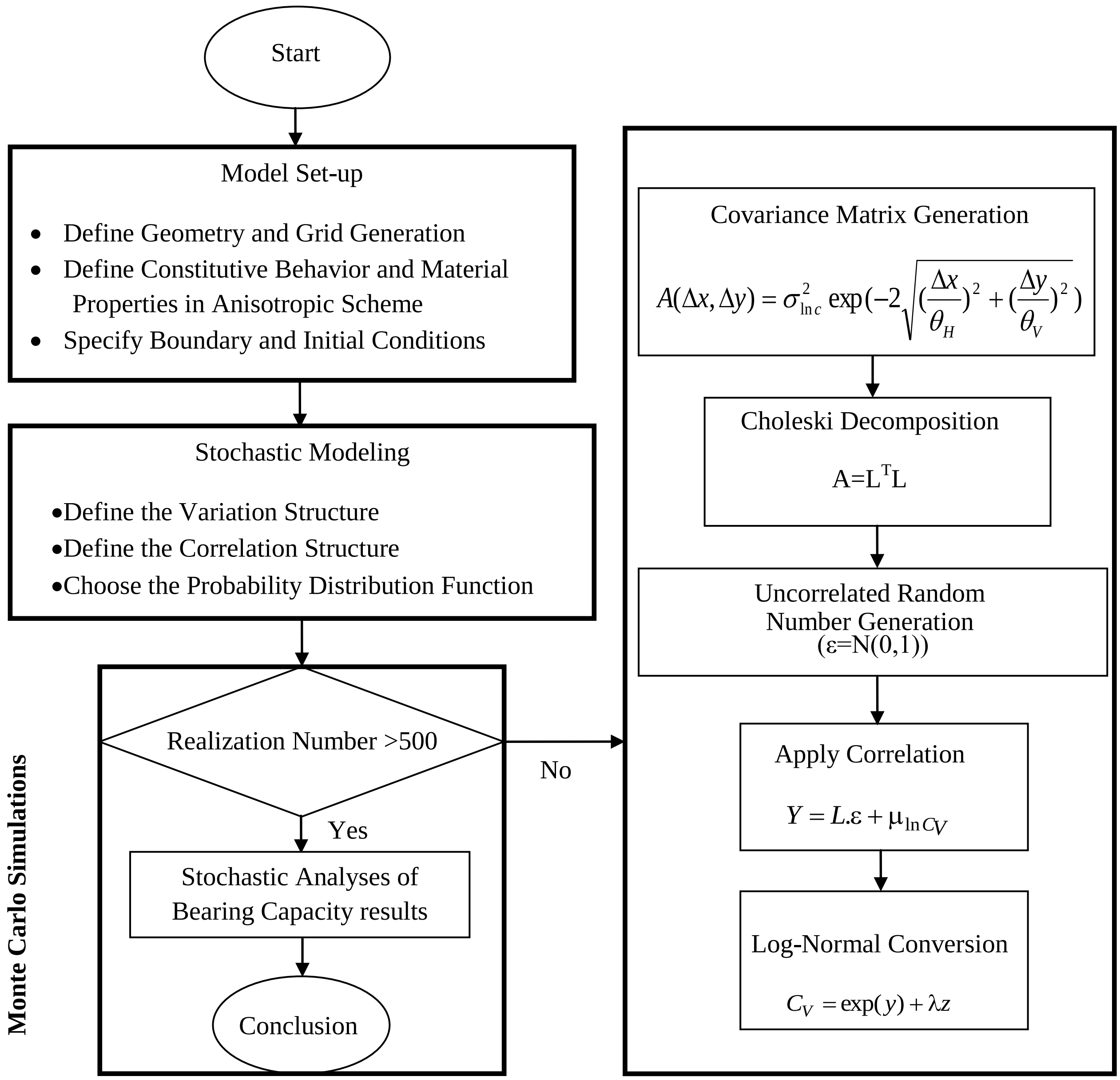

Fig. 2. Flowchart for Monte Carlo simulation adopted in current study.

Because the sediments fall under the gravity, their particle contacts are directionally dependent or anisotropic, and their response depends on the direction of the applied changes of the stress or strain. Casagrande and Carrillo (1944) first recognized the need to decompose the anisotropy components of sediments into “Inherent” and “Induced” groups. The soils are most likely to possess a combination of inherent and induced anisotropy which is referred to as “Initial anisotropy” in the literature, and is the one that the geotechnical engineer has to deal with.

The above mentioned type of anisotropy, hereinafter referred to as “mechanical anisotropy” is related to the mechanical behavior of the natural alluvial deposits. Mechanical behavior is represented by either deformation or the strength parameters. According to Mohr-Coulomb’s failure criterion, the soil strength is described by two parameters: cohesion c and friction angle φ. As far as the mechanical anisotropy is concerned, various researchers e.g. Duncan and Seed (1966) and Mayne (1985) concluded that φ exhibits only a modest anisotropy and is quite independent from the load direction. On the other hand, undrained shear strength and cohesion are found to be highly dependent on the stress paths and the type of the test used for measuring the shear strength parameters. However, anisotropic behavior has been measured in the consolidated and drained shear test and it should be noted that the slightly higher friction angle occurs when the major principal stress coincides with the direction of sand deposition, rather than when the major principal stress acts perpendicular to the direction of formation.

Since the early 1940’s, attempts have been made to quantify the degree of anisotropy of the soil cohesion. Casagrande and Carillo (1944) proposed that the soil cohesion in any direction in the vertical-horizontal plane can be expressed in terms of the cohesion in the principal directions:

$$C_i = C_H + (C_V - C_H)\sin^2 i \qquad (6)$$

where C_V and C_H are the shear strengths obtained from compression in the vertical and horizontal direction respectively, C_i is the shear strength in the *i* direction at which *i* represents the inclination of the major principal direction with respect to the horizontal direction.

The ratio C_H/C_V is found to be constant for a given soil (Lo, 1965) and is denoted by $A.F_M$ (Mechanical Anisotropy Factor). One of the purposes of this paper is to provide a numerical method which can be used to calculate the bearing capacity of the shallow foundations on soils the cohesion of which varies with direction. To achieve this objective, conventional Mohr-Coulomb yield criterion was generalized to include the effect of strength anisotropy which is exhibited by the variation of the cohesion with direction.

Soils sediments are often deposited vertically and then subjected to equal horizontal stresses, therefore they are often assumed to have a symmetrical vertical axis, with the horizontal axis representing the axis of transverse isotropy. Therefore, a common way to represent their anisotropic elastic response is to assume cross anisotropy. The elastic constants for a cross-anisotropic material are thus commonly taken as E_V, E_H, ν_{VH}, ν_{HV}, ν_{HH}, G_{VH}, and G_{HH}.

However, due to symmetry requirement it can be shown that:

$$\frac{\nu_{VH}}{E_V} = \frac{\nu_{HV}}{E_H} \qquad (7)$$

and

$$G_{HH} = \frac{E_H}{2(1+\nu_{HH})} \qquad (8)$$

in which E_V is Young's modulus in the depositional direction, E_H is Young's modulus in the plane of deposition, G_{VH} is shear modulus in the depositional direction, G_{HH} is shear modulus in the plane of deposition, ν_{VH} is Poisson's ratio for straining in the plane of deposition due to stress acting in the direction of deposition, ν_{HV} is Poisson's ratio for straining in the direction of deposition due to stress acting in the plane of deposition and ν_{HH} is Poisson's ratio for straining in the plane of deposition due to stress acting in the same plane.

The total number of the independent unknowns is then reduced from twenty one to only five (from full-anisotropic to cross-anisotropic material).

Soils normally possess inherent anisotropy and therefore even isotropically consolidated samples often have anisotropic stiffness characteristics. Table 1 shows the cross-anisotropic elastic stiffness ratios G_{HH}/G_{VH} and E_H/E_V for several materials, including the glass ballotini. It can be observed that the stiffness anisotropy exists under the isotropic stresses ($K=\sigma_3/\sigma_1=1$). Most materials tend to show higher normal stiffness in the vertical than in the horizontal direction when $K<<1$, but the opposite is true when $K>>1$.

Gibson (1974) derived the following bounding values for the stiffness ratio (E_H/E_V) based on the argument that strain energy function cannot be negative in an elastic material due to the thermodynamic requirements:

$$0 \leq \frac{E_H}{E_V} \leq 4 \qquad (9)$$

Gibson (1974) used three different values for the ratio of shear modulus to vertical Young's modulus (G_{VH}/E_V), namely, $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{1}{2}$.

MODIFIED MOHR-COULOMB MODEL

The failure envelope for this model corresponds to a Modified Mohr-Coulomb criterion (shear yield function) with tension cutoff (tensile yield function). The shear flow rule is non-associated and the tensile flow rule is associated.

Table 1. Stiffness anisotropy of different materials.

Researchers	$K=\sigma_3/\sigma_1$	G_{HH}/G_{VH}	E_H/E_V	Material type*
Jamiolkowski et al. (1995)	0.55	1.21 and 1.88	-	Panigaglia and Pisa Clay (N)
Belloti et al. (1996)	0.5	0.96	0.82	Ticino Sand (R)
	1.0	1.20	1.21	
	1.5	1.26	1.55	
	2.0	1.45	1.9	
Jovicic and Coop (1998)	1.0	1.9 and 1.6	-	Kaolin (R) and London Clay (N)
Pennington et al. (1997)	0.45	1.22 and -	-	R and N Gault clay
	1.0	1.45 and 1.8	-	
	1.5	1.55 and 2.1	-	
	2.0	1.65 and 2.3	-	
Kuwano (1999)	0.45	0.85,1.1 and 0.75	0.5,0.6 and 0.48	Ham River Sand, Dunkerque Sand and Glass ballotini
	1.0	1.1,1.3 and 1.05	0.85,0.98 and 0.84	
	2.0	1.38,1.65 and 1.4	1.23,1.42 and 1.3	

*R/N=Reconstituted/Natural

In the finite difference implementation of this model, principal stresses σ_1, σ_2, and σ_3 are used; the out-of-plane stress (σ_{zz}) is recognized as one of these stress components meeting the plane strain condition. The principal stresses and principal directions are evaluated from the stress tensor components and ordered so that (recall that compressive stresses are negative):

$$\sigma_1 \le \sigma_2 \le \sigma_3 \tag{10}$$

The corresponding principal strain increments Δe_1, Δe_2, and Δe_3 are decomposed as follows:

$$\Delta e_i = \Delta e_i^e + \Delta e_i^p \quad i = 1,2,3 \tag{11}$$

where the superscripts *e* and *p* refer to elastic and plastic parts, respectively; and the plastic components are non-zero only during the plastic flow. The incremental expression of Hooke's law in terms of the principal stress and strain has the form:

$$\begin{aligned}\Delta\sigma_1 &= \alpha_2\Delta e_1 + \alpha_4\Delta e_2 + \alpha_3\Delta e_3\\ \Delta\sigma_1 &= \alpha_5\Delta e_1 + \alpha_6\Delta e_2 + \alpha_5\Delta e_3\\ \Delta\sigma_1 &= \alpha_3\Delta e_1 + \alpha_4\Delta e_2 + \alpha_2\Delta e_3\end{aligned} \tag{12}$$

where

$$\begin{aligned}\alpha_1 &= \frac{1}{1-2\nu_{HV}\nu_{VH}-2\nu_{HV}\nu_{VH}\nu_{HH}-\nu_{HH}\nu_{HH}}\\ \alpha_2 &= \alpha_1(1-\nu_{VH}\nu_{HV})E_H,\\ \alpha_3 &= \alpha_1(\nu_{HH}+\nu_{VH}\nu_{HV})E_H,\\ \alpha_4 &= \alpha_1\nu_{VH}(1+\nu_{HH})E_H,\\ \alpha_5 &= \alpha_1\nu_{HV}(1+\nu_{HH})E_V,\\ \text{and }\ \alpha_6 &= \alpha_1(1-\nu_{HH}\nu_{HH})E_V.\end{aligned} \tag{15}$$

with the ordering convention of Eq. (10), the failure criterion may be represented in the plane (σ_1,σ_3) as illustrated in Figure 3.

The failure envelope is defined from point A to point B by the Modified Mohr-Coulomb yield function:

$$f^s = \sigma_1 - \sigma_3 N_\varphi + 2C_\theta\sqrt{N_\varphi} \tag{13}$$

where

$$N_\varphi = \frac{1-\sin\varphi}{1+\sin\varphi} \tag{14}$$

and from B to C by a tension yield function of the form:

$$f^t = \sigma^t - \sigma_3 \tag{15}$$

where φ is friction angle, σ^t the tensile strength and C_θ the cohesion in inclined plane that is obtained by:

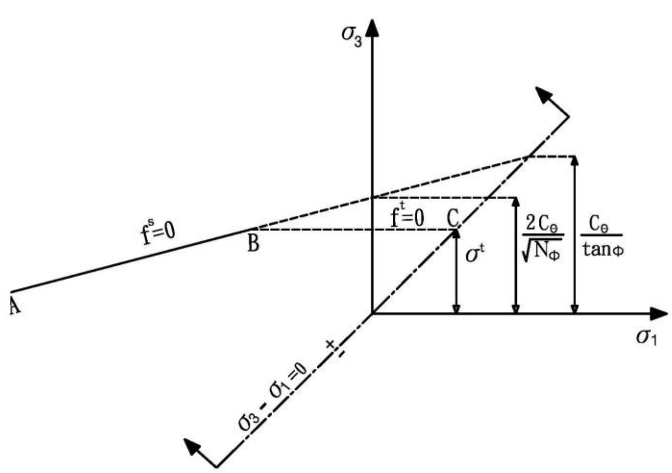

Fig. 3. Modified Mohr-Coulomb yield criterion.

$$C_\theta = C_H + (C_V - C_H)\sin^2(45 + \frac{\varphi}{2} + \theta) \quad (16)$$

where

$$\theta = 0.5\tan^{-1}(\frac{2\tau_{xy}}{\sigma_y - \sigma_x}) \quad (17)$$

NUMERICAL MODELING OF ANISOTROPIC BEARING CAPACITY PROBLEM

In this study, the effect of the cohesion anisotropy (mechanical and heterogeneity), stochastic and deterministic non-homogeneity on the bearing capacity of rough footing underlain by natural alluvial deposit is investigated by employing the finite difference method using FLAC^{2D}. The finite difference method is perhaps the oldest numerical technique used for solving the sets of differential equation, given initial values and/or boundary values. The prediction of collapse loads under the steady plastic flow condition can be difficult for a numerical model to simulate accurately (Sloan and Randolph, 1982). The bearing capacity is dependent on the steady plastic flow beneath the footing. The FISH programming facility embedded in FLAC^{2D} was utilized to consider the effect of heterogeneity and anisotropy of cohesion on the bearing capacity of the shallow foundations. Modified Mohr-Coulomb constitutive model as explained in detail was adopted to seek for the failure.

The footing is incrementally displaced vertically into the soil mass by applying a downward velocity field to the area representing the footing. The value of velocity increment is 2.5×10^{-5} m/step, which is the value of the total displacement required for the failure divided by the number of the essential steps. The bearing capacity will be calculated using a FISH function, an internal programming option of FLAC^{2D} which computes the vertical load at each step. The calculated vertical load at the final step is the bearing capacity of the footing.

The problem used for the analyses is a 0.5 m wide strip footing (in plane strain condition) bearing on a shallow stratum supported by a rigid and perfectly rough base (Figure 4). The rough strip footing was simulated by fixing the x-velocity to zero for the gridpoints representing the footing base. The horizontal extent of the stratum is set at 3m and the depth of stratum is 1m. The vertical boundary is assumed to be perfectly smooth and rigid. The uniform mesh as shown in Figure 4 is used. The finite difference mesh consists of 341 nodes and 300 rectangular elements.

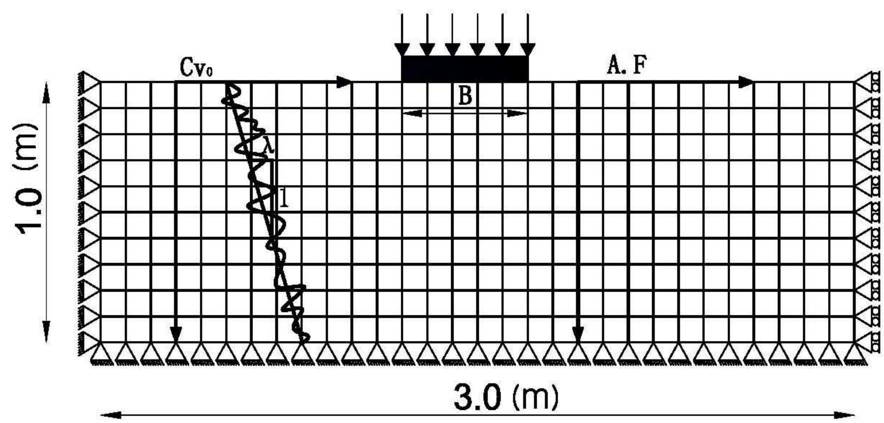

Fig. 4. Finite difference model used in Random Finite Difference Method (RFDM) analyses.

Deterministic Bearing Capacity

Adopting a FISH program development by authors in drained condition, values of the normalized drained bearing capacity pressure, $q' = q/C_{V0}$, have been obtained. The input parameters varied according to Table 2 where B and γ are the width of foundation and the unit weight of soil, respectively.

Furthermore, to simplify the analysis, the soil is assumed weightless. Under this assumption, the well-known bearing capacity relationship is simplified to $q = C_{V0}N_c$. Therefore, the trend of variation of the bearing capacity factor N_c, is studied and the results are plotted.

Table 2. Parameters in deterministic bearing capacity analysis of shallow foundation.

Parameter	Values Considered
φ°	0, 5, 10, 15, 20, 25
$G=\gamma B/C_{H0}$	0, 2
$A.F_M=C_H/C_V$	0.5, 1, 1.5, 2
$\nu=\lambda B/C_{H0}$	0, 0.25 , 0.5, 0.75, 1, 1.25

Stochastic Bearing Capacity

The calculation of the stochastic bearing capacity of the shallow footing overlaying as a spatially variable natural alluvial deposit is performed in an undrained condition by another FISH program that as illustrated before, is a combination of the finite difference method and the random field theory. The stiffness parameters of soil such as poisson's ratio (ν_{HV}, ν_{VH} and ν_{HH}) is assumed constant throughout the domain while the undrained Young's modulus, undrained shear strength in the horizontal and vertical direction, and the shear modulus are considered heterogeneous. Undrained Young's modulus is assumed to be fully correlated to the undrained shear strength by assuming E_{uv}/C_{uv} ratio of 800 in both horizontal and vertical plane.

The effect of COV_{Cuv}, θ_V, $A.F_H$, and $A.F_M$ on the stochastic bearing capacity is investigated and these parameters are presented in Table 3.

Table 3. Parameters in stochastic bearing capacity analyses of shallow foundation.

Parameter	Values Considered
COV_{Cuv}	0.1, 0.25, 0.5, 0.75
θ_V (m)	0.50
$A.F_H= \theta_H/\theta_V$	1,2,5,10
$A.F_M=C_{uH}/C_{uV}$	0.5, 1, 1.5, 2

Zhalehjoo et al. (2012) showed that for small values of the foundation width, the strength density has no significant effect on the bearing capacity of the shallow strip footings. Therefore, the value of the strength density is taken constant at 1 kPa/m.

For each set of considered COV_{Cuv}, θ_V, $A.F_H$, and $A.F_M$ values, Monte Carlo simulation has been executed which involved 500 realizations of the shear strength random field and the subsequent numerical analysis of the bearing capacity. Mean ultimate bearing capacity (μq_{ult}) and the coefficient of variation of the result ($COVq_{ult}$) were then evaluated for different sets of stochastic parameters.

RESULTS AND DISCUSSION

In order to validate the results of the numerical FDM analysis, the bearing capacity in full homogeneous and isotropic conditions was calculated and compared with those acquired from different classic methods and bearing capacity calculation formulations. Table 4 provides the comparative data regarding the verification of the FDM method capability in the bearing capacity calculation adopted in this study. The results of N_c-values obtained from the current study are fairly acceptable when compared to those of well-known classic limit equilibrium or limit analysis methods.

Values of q' have been obtained for φ = 10 and 20 degrees in Figures 5a and 5b. These figures revealed that the variation of q´ with ν is almost linear for the range of parameters considered. To validate the result obtained from these analyses, a comparison was made between the current numerical results and those reported by Reddy and Rao (1982) that showed a good agreement. Their assumptions about anisotropy and non-homogeneity of cohesion are similar to those considered in this paper.

Figure 6 reveals the effect of anisotropy presented by $A.F_M$, non-homogeneity presented by ν (ν = 0 and ν = 0.5 simulate homogeneous and non-homogeneous condition respectively), and the soil friction angle on the bearing capacity factor, by N_c. It is evident that with an increase in each of these factors, N_c becomes greater.

Table 4. Comparison between N_C–values for G=0.0 and ν=0.0.

Researchers	Applied Method	N_c-Values	
		φ =10°	**φ =20°**
Terzaghi*	Limit Equilibrium	9.30	17.00
Meyerhof*	Stress Characteristics	8.00	14.50
Reddy and Srinvasan (1970)	Method of Characteristics	9.30	17.00
Salencon (1974 b)	Limit analysis (upper bound and lower bound)	8.35	14.84
Chen (1975)	Limit analysis (upper bound)	8.34	14.80
Reddy and Rao (1982)	Limit analysis (upper bound)	8.34	14.83
Current study	Finite Difference Method (FDM)	8.52	14.94

*From Reddy and Srinvasan (1970)

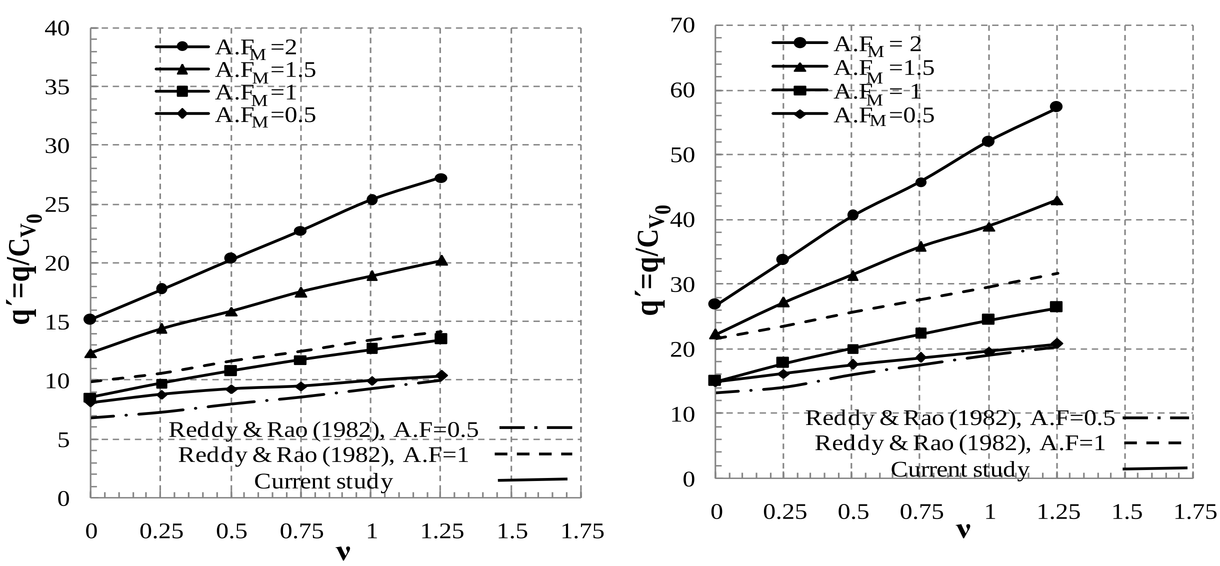

Fig. 5. Variation of q′ with ν for different $A.F_M$ values at; a) φ=10°and b) φ=20°.

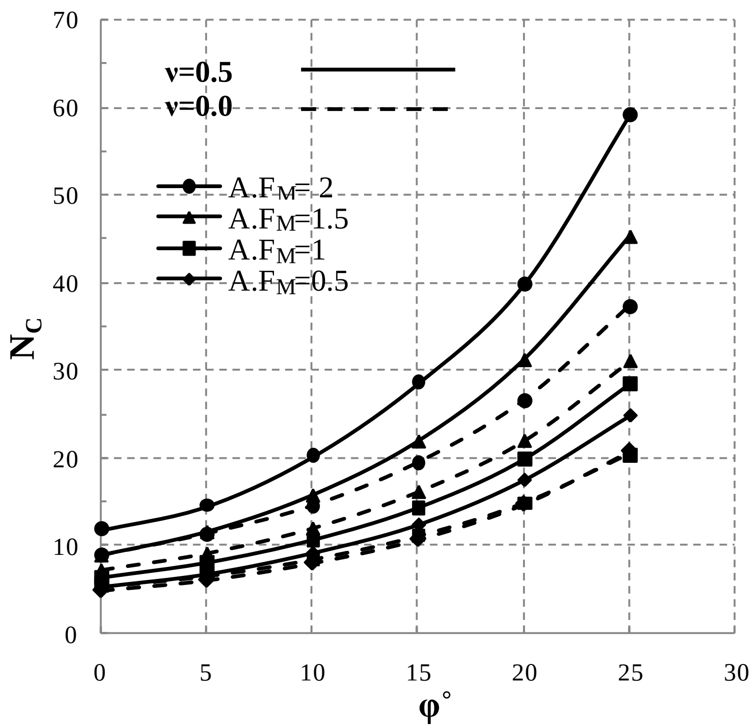

Fig. 6. Variation of N_C with angle of friction for homogeneous and non-homogeneous conditions.

The results obtained by the stochastic modeling for each set of parameters are depicted in Figures 7 and 8. In Figure 7, assumed values of the correlation length in the vertical and horizontal directions are different for each chart. It is evident that $A.F_H$ has no significant effect on the mean ultimate bearing capacity ($\mu_{q_{ult}}$). Also the effects of COV_{Cuv} and $A.F_M$ are demonstrated in each chart. It is clear that with increasing $A.F_M$, the ultimate bearing capacity of the shallow foundation also increases. This finding is in clear conformity to those of the deterministic analyses as provided in Figures 5 and 6. However, it varies inversely with the increase of COV_{Cuv}. This implies that including more variation in the cohesion values results in more possibility of the weak zones formation in the underlying stratum and a reduction in the mean bearing capacity of the overlying foundation is then expected.

Although it is stated that $A.F_H$ does not have a significant effect on the mean ultimate bearing capacity of the shallow foundations laying on the spatially variable natural alluvial deposits, it is evident in Figure 8 that it has an increasing effect on the bearing capacity. This behavior is expected when referring to Fenton and Griffiths (2007). While maintaining the normalized vertical correlation length (θ_V/B) constant at 1, this incremental behavior was pointed out to be related to the weakest path issue. When the correlation length is very high, the soil properties become spatially constant, albeit still random from realization to realization. In this case, because the soil properties are spatially constant, the weakest path returns to the log-spiral. On the other hand, for the case of intermediate correlation lengths, it allows enough spatial variability for a failure surface which deviates somewhat from the log spiral and it is not too long, rendering less bearing capacity.

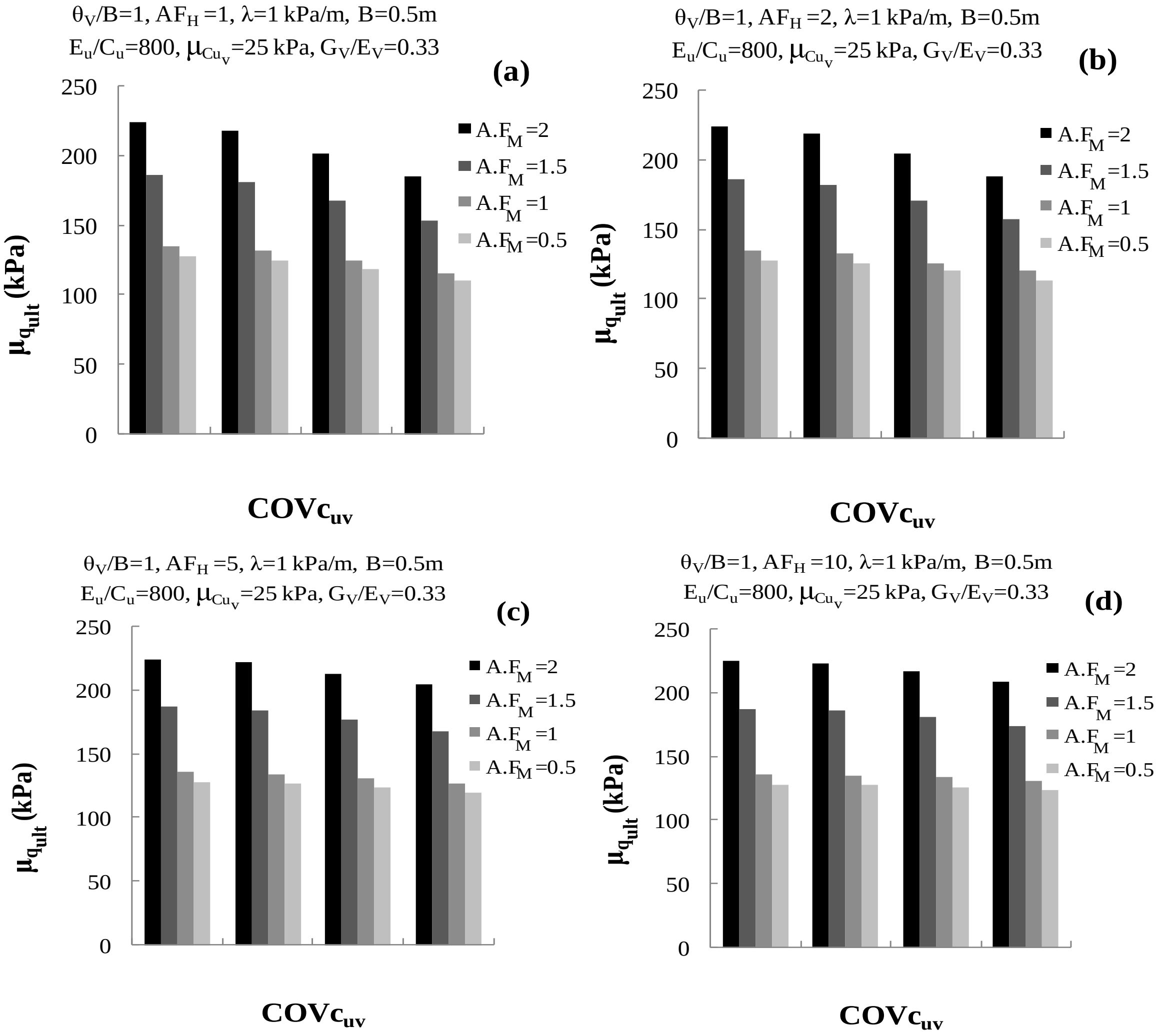

Fig. 7. Effect of A.F$_M$ and COV$_{Cuv}$ on mean ultimate bearing capacity of shallow foundations.

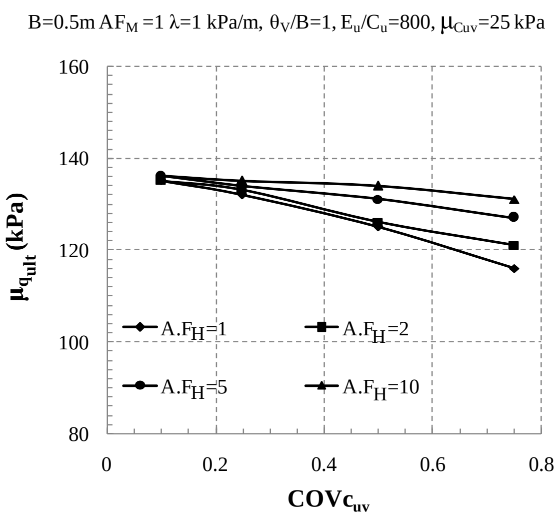

Fig. 8. Effect of heterogeneity anisotropy on ultimate bearing capacity.

CONCLUSIONS

This paper presents the effects of two types of anisotropy of cohesion on the ultimate bearing capacity of a shallow footing embedded on the heterogeneous alluvial deposits. The model is based on the numerical simulations using the lagrangian explicit finite difference code $FLAC^{2D}$.

In the first type of anisotropy, the conventional isotropic Mohr-Coulomb failure criterion was generalized to an anisotropic one, and the following results were obtained:

In the mechanical anisotropy consideration, the effect of $A.F_M$ was investigated on the bearing capacity of the shallow foundation for different strength parameters (v and φ). It was shown that the mechanical anisotropy ($A.F_M$) has an increasing effect on the bearing capacity while maintaining the other parameters constant. Strength density formulized in a dimensionless form (v) and the internal friction angle showed a similar effect on the bearing capacity as expected from the literature review.

As far as the heterogeneity anisotropy is concerned, the cohesion field was assumed to be a random variable which is log-normally distributed with different correlation distances in the vertical and horizontal directions. The effect of heterogeneity anisotropy ($A.F_H$) on the bearing capacity of the shallow foundation resting on a spatially variable deposit was investigated for different sets of stochastic parameters. In other words, heterogeneity anisotropy is a stochastic parameter studied jointly with other affecting inherent phenomena. The coefficient of variation of the undrained cohesion was shown to induce uncertainty in the bearing capacity results while decreasing the mean ultimate bearing capacity. Heterogeneity anisotropy ($A.F_H$) was found to increase the bearing capacity of the shallow foundation, however the extent of improvement is not too much. This means that neglecting the spatial variability of the soil properties leads to an overestimation in the bearing capacity prediction.

In summary, for the range of parameters considered, the results clearly show that the anisotropy and non-homogeneity of soils have a considerable influence on the bearing capacity of the shallow foundations. Among all, the mechanical anisotropy and the coefficient of variation of the strength parameters are more highlighted to influence the bearing capacity calculation.

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