

The Effect of Dynamic Permeability on Velocity and Intrinsic Attenuation of Compressional Waves in Sand

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ABSTRACT: Stress waves contain useful information about the properties of porous materials; they can be recovered through different non-destructive testing methods such as crosswell, vertical seismic profile, borehole logging as well as sonic tests. In all these methods, it is crucial to assess the effects of frequency on wave attributes including velocity and intrinsic attenuation. The dependency of permeability on frequency which is known as dynamic permeability and its effects on wave attributes of compressional waves are investigated in the present paper. Utilizing the dispersion relation derived for compressional waves, it is shown how the velocity and intrinsic attenuation of waves propagated in water saturated sand may be influenced by dynamic permeability. In low frequency range (viscous dominated flow regime), the dynamic permeability behaves like Darcy steady-state permeability and its effects on wave attributes are negligible. However, deviations from Darcy permeability start to occur at higher frequencies. Therefore, it is important to know how dynamic permeability controls the behavior of wave velocity and intrinsic attenuation in relatively high frequencies. For example, it is demonstrated that neglecting dynamic permeability results in overestimation of velocities of fast and slow waves in high frequency ranges (inertia dominated flow regime).

Keywords: Attenuation, Compressional Waves, Dynamic Permeability, Sand, Velocity.

INTRODUCTION

The study of wave propagation and attenuation in fluid saturated porous media has captured considerable attention in a number of diverse fields such as exploration geophysics, ultrasonic testing of porous structures as well as seismology. Studying the attributes of waves such as velocity and attenuation is of great significance since they carry valuable information about the solid bulk properties also in addition to properties

like porosity and permeability of porous media.

To model the propagation of waves in porous solids saturated with fluid, Biot's theory of porous media (Biot, 1956a, b) has been extensively used by many researchers (Theodorakopoulos and Beskos, 2006; Lo et al., 2006; Ghasemzadeh and Abounouri, 2012). Biot's formulation employs complex viscosity function to describe the frequency-dependent combined effects of inertial forces and viscous forces which dominate the fluid flow in high and low frequency ranges,

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respectively. This function was introduced to evaluate the viscous fluid flow under an oscillatory pressure gradient. Later, Johnson et al. (1987) used energy flux consideration at micro-scale and proposed a more realistic description of the frequency-dependent interaction between the fluid and solid phases by employing dynamic permeability parameter. They showed that permeability of porous media depends on frequency of the oscillating flow. Numerical calculations of dynamic permeability for different microstructures were performed by several authors (Sheng et al., 1988; Chapman and Higdon, 1992). The validity of dynamic permeability was experimentally tested as well. Charlaix et al. (1988) analyzed the frequency dependence of the dynamic permeability utilizing direct measurements in fluid saturated samples of sintered spherical beads and sintered crushed glass. Johnson et al. (1994) used superfluid He II to probe the dynamic permeability of porous media. Experimental results revealed excellent agreement with the predictions made via the theoretical model.

In the present study, constitutive equations governing the behavior of saturated porous media (Smeulders, 1992; Verruijt, 2010) are exploited to derive a complex dispersion relation for compressional waves. This dispersion yields the velocity and intrinsic attenuation of compressional waves. This paper is novel in the sense that it takes dynamic permeability into consideration in the dispersion relation as well as presenting comparisons between different cases of dynamic permeability and steady state permeability. Wave propagation utilizing dynamic permeability is pivotal for analyzing seismic data as a function of frequency. Unfortunately, many studies neglect the effect of dynamic permeability (see e.g., Kim et al., 2002; Lo et al., 2008). Therefore, it is crucial to conduct a study to investigate the importance of dynamic

permeability. Our study on the effects of dynamic permeability is performed on water saturated sand over a wide range of excitation frequencies. The results are compared with models that ignore dynamic permeability effects in their formulations. It is demonstrated how neglecting dynamic permeability leads to miscalculation of compressional wave velocity and intrinsic attenuation in high frequency range.

FIELD EQUATIONS

Assume a homogeneous and isotropic fluid-filled porous medium with a porosity of n (pore volume fraction). We rewrite the momentum conservation of solid phase presented by Smeulders (1992) to include viscoelastic properties of the solid frame. This equation can be written in terms of effective stress σ' and fluid pressure p as follows:

$$-\frac{\partial \sigma'}{\partial x} - (\alpha^* - n) \frac{\partial p}{\partial x} = \frac{\partial}{\partial t} (\rho_{11} v + \rho_{12} w) + b_0 (v - w) \quad (1)$$

where v and w represent the average velocity of solid and fluid particles, respectively. The viscous dissipation factor b_0 related to the porosity n , steady-state Darcy permeability k_0 and the fluid viscosity η by the relation of $b_0 = \eta n^2 / k_0$. The density coefficients are expressed in terms of solid mass density ρ_s and fluid mass density ρ_f :

$$\rho_{11} = (1 - n) \rho_s - \rho_{12} \quad (2)$$

$$\rho_{12} = -n \rho_f (\tau - 1) \quad (3)$$

where the tortuosity of the pores $\tau \geq 1$ is a dimensionless and purely geometrical quantity. The density coefficient ρ_{12} represents a mass coupling parameter between the solid and the fluid. This

coupling effect is due to tortuosity of the porous network. Iversen and Jørgensen (1993) obtained the empirical relation below for sandy sediments:

$$\tau = 1 + 2(1 - n) \quad (4)$$

The above relation is used to calculate tortuosity in the present study. The coefficient α in Eq. (1) is defined by:

$$\alpha^* = 1 - \frac{C_s(1 + 2i\xi_s)}{C_p} \quad (5)$$

where ξ_s represents hysteretic damping and $i = \sqrt{-1}$. Compressibility moduli of solid frame and solid grains are represented by C_p and C_s , respectively. it is worth mentioning that hysteretic ξ_s damping is introduced with the aim of considering viscoelastic behavior of the solid frame which leads to attenuation caused by grain to grain contact.

The equation for momentum conservation of pore-fluid phase is written as follows (Smeulders, 1992):

$$-n \frac{\partial p}{\partial x} = \frac{\partial}{\partial t} (\rho_{12}v + \rho_{22}w) - b_0(v - w) \quad (6)$$

where

$$\rho_{22} = -n\rho_f - \rho_{12} \quad (7)$$

Eq. (6) is actually the extension of Darcy's law since this equation can take into account the case of oscillation fluid motion in high excitation frequencies. Bear in mind that the first term on the right of Eq. (6) includes unsteady interaction originating from the pore fluid being accelerated in the tortuous porous network. Therefore, the study of wave propagation in both low frequency range (viscous dominated regime) as well as high frequency range (inertia dominated regime) will be possible. Since

fluid pressure gradients are either balanced by viscous forces (low frequency range) or by inertia forces (high frequency range), Smeulders (1992) obtained the transition frequency ω_c for which viscous forces are equal to inertia forces acting on the pore fluid. This frequency is defined by:

$$\omega_c = \frac{\eta n}{\tau \rho_f k_0} \quad (8)$$

Frequency range of seismic waves, e.g., produced by earthquakes is often below rollover frequency ω_c while the frequency used in sonic and ultrasonic tests are above this limit.

A second set of basic equations is provided by the equations of conservation of mass in porous media. These equations must be written for the fluid and the solid phases. The first equation is the storage equation which is derived from the mass conservation of both solid and fluid phases (Verruijt, 2010):

$$\alpha^* \frac{\partial w}{\partial x} + S_p \frac{\partial p}{\partial t} = - \frac{\partial [n(w - v)]}{\partial x} \quad (9)$$

where storativity of the pore spaces S_p is defined by:

$$S_p = nC_f + (\alpha^* - n)C_s \quad (10)$$

where C_f represents the fluid compressibility which may also include the compression of gas bubbles in it.

The widely accepted interpretation of Eq. (9) is that the compression of the porous medium consists of the compression of solid particles and of the pore fluid, plus the amount of fluid expelled from an element by flow.

Finally, the equation of conservation of mass for the solid frame is defined by Verruijt (2010):

$$m_v^* \frac{\partial \sigma'}{\partial t} = - \frac{\partial v_s}{\partial x} \quad (11)$$

where m_v can be redefined for viscoelastic case by:

$$m_v^* = \frac{3C_p}{(3 + 4GC_p)(1 + 2i\xi_s)} \quad (12)$$

where G represents shear modulus of porous medium.

COMPRESSIONAL DISPERSION RELATION

Let us assume the $\exp i(\omega t - kx)$ dependence for all relevant parameters as follows:

$$u = \hat{u} \exp i(\omega t - k_p x) \quad (13)$$

where $u_0 = [\hat{v}, \hat{w}, \hat{p}, \hat{\sigma}']^T$ represents a transposed vector with four elements, with the accent circumflex referring to the representation in the frequency domain. Also, ω is angular frequency, k_p is wavenumber and $i = \sqrt{-1}$.

Substituting the periodic solution into basic Eqs. (1), (6), (9) and (11) results in four equations with four unknowns. Since it is mathematically more convenient to reduce them to two equations with two unknowns, the result is rearranged in terms of solid and fluid particle velocities. Therefore, the following system of homogeneous equations is obtained for the compressional waves:

$$\left[\rho \omega^2 - \mathbf{K}_p k_p^2 \right] \begin{Bmatrix} \hat{w} \\ \hat{v} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (14)$$

where the compressional pore-viscoelasticity stiffness matrix is defined by:

$$\mathbf{K}_p = \begin{pmatrix} R & Q \\ Q & P \end{pmatrix} \quad (15)$$

where the parameters P , Q and R are as follows:

$$P = \frac{1}{m_v} + \frac{(\alpha^* - n)^2}{S_p} \quad (16)$$

$$Q = \frac{n(\alpha^* - n)}{S_p} \quad (17)$$

$$R = \frac{n^2}{S_p} \quad (18)$$

The frequency-dependent density matrix in Eq. (14) is defined by:

$$\rho = \begin{pmatrix} a_{22} & a_{12} \\ a_{12} & a_{11} \end{pmatrix} \quad (19)$$

where

$$a_{11} = \rho_{11} - b_0 i \omega^{-1} \quad (20)$$

$$a_{12} = \rho_{12} + b_0 i \omega^{-1} \quad (21)$$

$$a_{22} = \rho_{22} - b_0 i \omega^{-1} \quad (22)$$

To obtain a non-zero solution from the Eq. (14), the determinant of the system must be equal to zero which results in the below dispersion relation for compressional waves:

$$k_p^4 r_1 - k_p^2 r_2 \omega^2 - r_3 \omega^4 = 0 \quad (23)$$

where $r_1 = -Q^2 + PR$, $r_2 = a_{22}P - 2a_{12}Q + a_{11}R$ and $r_3 = a_{12}^2 - a_{11}a_{22}$. Bear in mind that the Eq. (23) is a quadratic equation and, therefore, it has two complex roots (wavenumbers) which are k_{p1} and k_{p2} :

$$k_{p1}^2 = \frac{(r_2 - \sqrt{r_2^2 + 4r_1r_3})\omega^2}{2r_1} \quad (24)$$

$$k_{p2}^2 = \frac{(r_2 + \sqrt{r_2^2 + 4r_1r_3})\omega^2}{2r_1} \quad (25)$$

Now we have derived complex wavenumbers of the compressional waves in the pore-viscoelastic media which can yield the intrinsic attenuation and velocity. From the above equations, it is concluded that there are two damped compressional waves in the fluid saturated porous media. The wave with the higher velocity and in-phase solid and fluid displacements is called a fast wave (k_{p1}) while the wave with the lower velocity and out-of-phase displacements is called a slow wave. The slow wave has more attenuative behavior vis-à-vis the fast wave. This is because the movements of solid and fluid parts are not in phase for slow wave.

DYNAMIC PERMEABILITY

Considering the $\exp(i\omega t)$ dependence for velocities and pressures in Eq. (6), the equation of conservation of the fluid momentum in the case of rigid porous medium will reduce to:

$$-\partial_x \hat{p} = \left(\frac{\eta\phi}{k_0} - i\omega\tau\rho_f \right) \hat{w} \quad (26)$$

The above equation implies that the fluid momentum equation is actually superposition of the low and high frequency ranges. This equation is a very simplified description of the frequency-dependent dissipation mechanism. A more realistic description was proposed by Johnson et al. (1987) using energy flux consideration on the microscale. They introduced the concept of dynamic permeability or, alternatively, dynamic tortuosity as follows:

$$-\partial_x \hat{p} = \frac{\eta\phi}{\hat{k}(\omega)} \hat{w} \quad (27)$$

$$-\partial_x \hat{p} = -i\omega \hat{\tau}(\omega)\rho_f \hat{w} \quad (28)$$

where the dynamic permeability $\hat{k}(\omega)$ and dynamic tortuosity $\hat{\tau}(\omega)$ are defined by Johnson et al. (1987):

$$\hat{k}(\omega) = k_0 \left[\left(1 - \frac{iM}{2} \frac{\omega}{\omega_c} \right)^{1/2} - i \frac{\omega}{\omega_c} \right]^{-1} \quad (29)$$

$$\hat{\tau}(\omega) = \tau \left[1 - i \frac{\omega_c}{\omega} \left(1 + \frac{iM}{2} \frac{\omega}{\omega_c} \right)^{1/2} \right] \quad (30)$$

Eqs. (27) and (28) are actually alternative descriptions of the same physics. The similarity parameter M in the above equations is given by:

$$M = \frac{8\tau k_0}{n\Lambda^2} \quad (31)$$

where the characteristic length scale of the pore size is represented by Λ . The similarity parameter M is expected to be approximate to 1 for most porous media (Johnson et al., 1987; Allard et al., 1998), so we assume the same value in present study.

The effect of dynamic permeability can be introduced via the viscous dissipation factor. Thus, substitution of Eq. (29) to Eq. (27) or, alternatively, Eq. (30) to Eq. (28) can be used to yield the effect of frequency-dependent dissipation mechanism for deformable porous media. Here, the equation for dynamic permeability is used, therefore, the Eq. (29) is substituted into Eq. (27) and after forming a comparable equation with Eq. (26), the effect of dynamic permeability can be incorporated by replacing b_0 with $\hat{b}(\omega)$:

$$\hat{b}(\omega) = b_0 \left(1 + i \frac{M}{2} \frac{\omega}{\omega_c} \right)^{1/2} \quad (32)$$

Replacing the frequency-dependent viscous dissipation factor $\hat{b}(\omega)$ in Eqs. (20) to (22) instead of frequency-independent factor b_0 allows us to consider the effect of dynamic permeability $\hat{k}(\omega)$ in dispersion relations.

NUMERICAL RESULTS AND COMPARISON

In our study, we consider the properties of the Sand used in Lo et al. (2006). Water is used as a fluid that saturates the sand. The properties of the sand and the water are presented in Table 1 and Table 2, respectively.

Table 1. Material properties of Sand.

Parameter	Symbol	Value
Solid mass density	ρ_s	2650 kg m ⁻³
Porosity	n	0.437
Intrinsic permeability	k_0	5.946×10 ⁻¹² m ²
Frame compressibility	C_p	0.0283 MPa ⁻¹
Frame shear modulus	G	13.3 MPa ⁻¹
Grains compressibility	C_s	0.0286 GPa ⁻¹

Table 2. Material properties of Water.

Parameter	Symbol	Value
Frame compressibility	ρ_f	997 kg m ⁻³
Frame shear modulus	C_f	0.444 GPa ⁻¹
Grains compressibility	η	1×10 ⁻³ Pa.s

Before studying the effect of dynamic permeability on wave velocity and intrinsic attenuation, we demonstrate the frequency-dependent behavior of the dynamic permeability. Figure 1 shows the magnitude of the dynamic permeability $|\hat{k}(\omega)|$ divided by Darcy permeability k_0 as the function of dimensionless frequency ω/ω_c . It is revealed that that the magnitude of the dynamic permeability reduces to the Darcy permeability in the low frequency range; yet, for $\omega/\omega_c \geq 1$, the value of dynamic permeability gradually starts to drop. Keep in mind that using Eq. (8), the value of transition frequency ω_c for water saturated sand is found to be 34673 rad.s⁻¹ (5518 Hz).

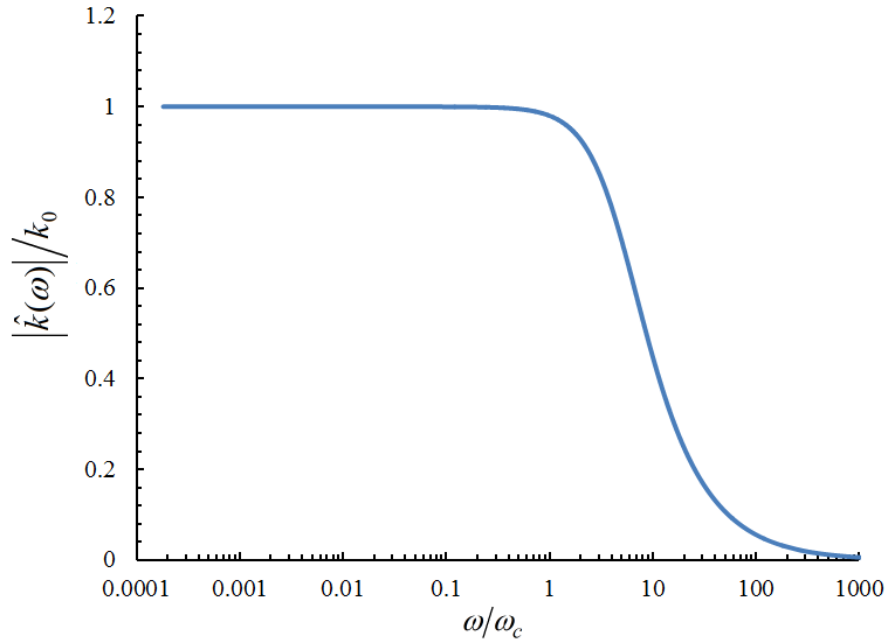


Fig. 1. Frequency-dependent permeability.

The frequency-dependent behavior of the wave velocity and intrinsic attenuation in fluid saturated poroelastic media is studied for three different cases including:

- Case I: Taking into consideration the dynamic permeability $k(\omega)$ by using dissipation factor $\hat{b}(\omega)$.
- Case II: Neglecting the dynamic permeability $k(\omega)$ by using dissipation factor b_0 .
- Case III: Taking into consideration the same methodology as of Biot (1956a, b) which does not include dynamic permeability. This method has been exploited by several researchers throughout the years (see e.g., Berryman, 1980; Kim et al., 2002; Lo et al., 2008). The equations used for case III are described in Appendix A.

Figures 2 and 3 show the velocity of fast and slow compressional waves versus dimensionless frequency ω/ω_c . The real part of complex compressional wavenumbers $k_j (j = p_1, p_2)$ is used to plot the wave

velocity $c_j = \omega/\text{Re}(k_j)$ of fast and slow waves. Figures 2 and 3 reveal that the slow wave velocity is very dispersive (frequency-dependent) at low frequency range while the fast wave velocity is frequency-independent. As shown in Figures 2 and 3, the velocities of fast and slow waves are exactly equal for all three cases in the low frequency range ($\omega/\omega_c \ll 1$) where viscous forces are dominant. The wave velocities of plotted cases start to diverge from each other when frequencies are higher than the transition frequency ω_c . This is due to the fact that dynamic permeability deviates from Darcy permeability in high frequency ranges (Figure 1). Bear in mind that the wave velocity reaches its peak value at $\omega/\omega_c = 1000$. We come to the conclusion that neglecting dynamic permeability (Cases II and III) leads to a considerable overestimation of the velocity of fast and slow waves in high frequency ranges where inertial forces are dominant.

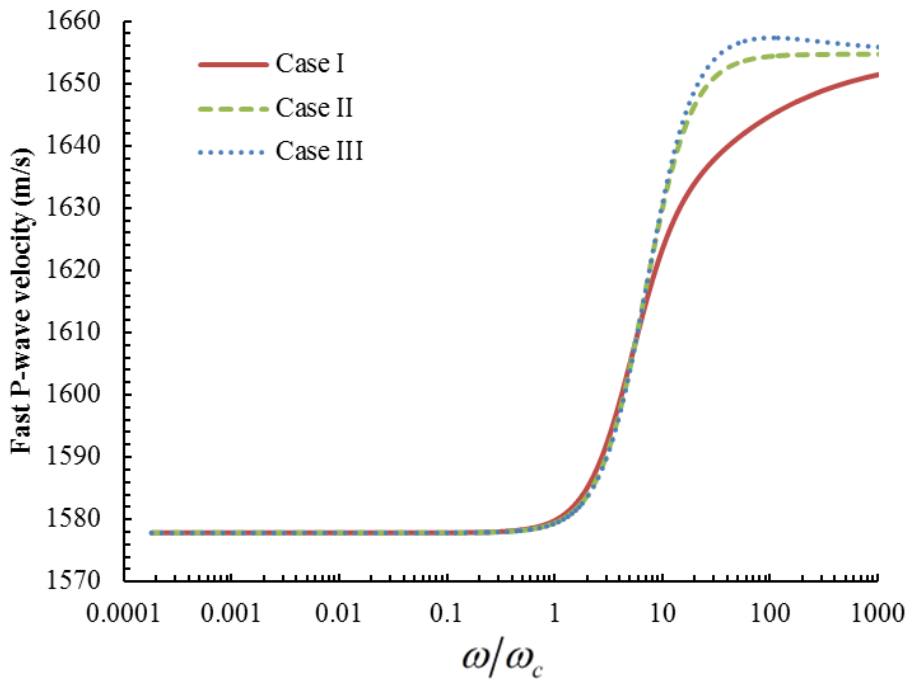


Fig. 2. Fast wave velocity versus dimensionless frequency.

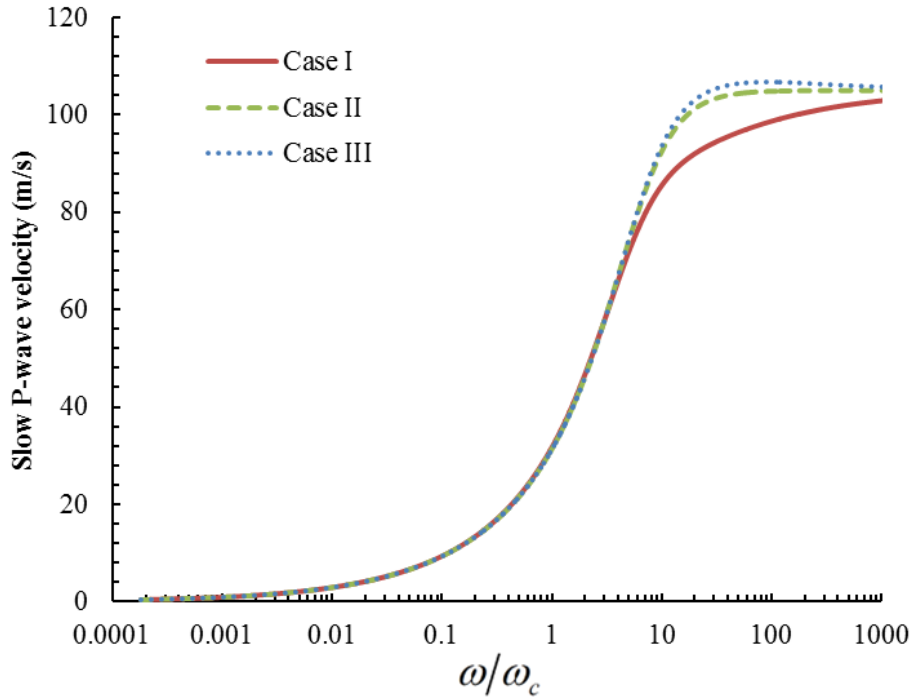


Fig. 3. Slow wave velocity versus dimensionless frequency.

Figures 4 and 5 show dimensionless intrinsic attenuation $Q_j^{-1} = 2 |\text{Im}(k_j)/\text{Re}(k_j)|$ of the fast and slow waves which are derived from real and imaginary parts of the complex compressional wavenumbers k_j . The maximum intrinsic attenuation value of fast wave occurs at the vicinity of the transition frequency ($\omega/\omega_c=1$), while the peak attenuation of slow wave occurs at $\omega/\omega_c=0.0001$. It is illustrated that the magnitude of intrinsic attenuation of the considered wave modes are almost the same in low frequency range for cases I, II and III, but the difference is very significant for frequencies higher than transition frequency ω_c . Fast wave intrinsic attenuations of cases II and III (Figure 4) are larger compared to Case I where frequencies are in the $1 \leq \omega/\omega_c \leq 50$ range. However, in Case I, intrinsic attenuation is larger than the others at $\omega/\omega_c \geq 50$. Similar conclusions can also be drawn for the slow wave (Figure 5), but the

effect of dynamic permeability is not that significant. Pay attention that the maximum value of intrinsic attenuation for the fast wave is approximately 33% and 30% larger for the cases II and III when compared to Case I. Therefore, neglecting the effects of dynamic permeability can lead to serious errors in the calculation of wave intrinsic attenuation particularly at relatively high excitation frequencies. The distinction between high and low frequencies' behavior for the considered cases is due to the fact that the drag exerted by the solid on the fluid is dominated by inertial forces in high frequency ranges but by viscous effects in low frequency ranges. The high-frequency properties are dependent on the value of the dynamic permeability $k(\omega)$ while low-frequency properties are dependent on the value of k_0 .

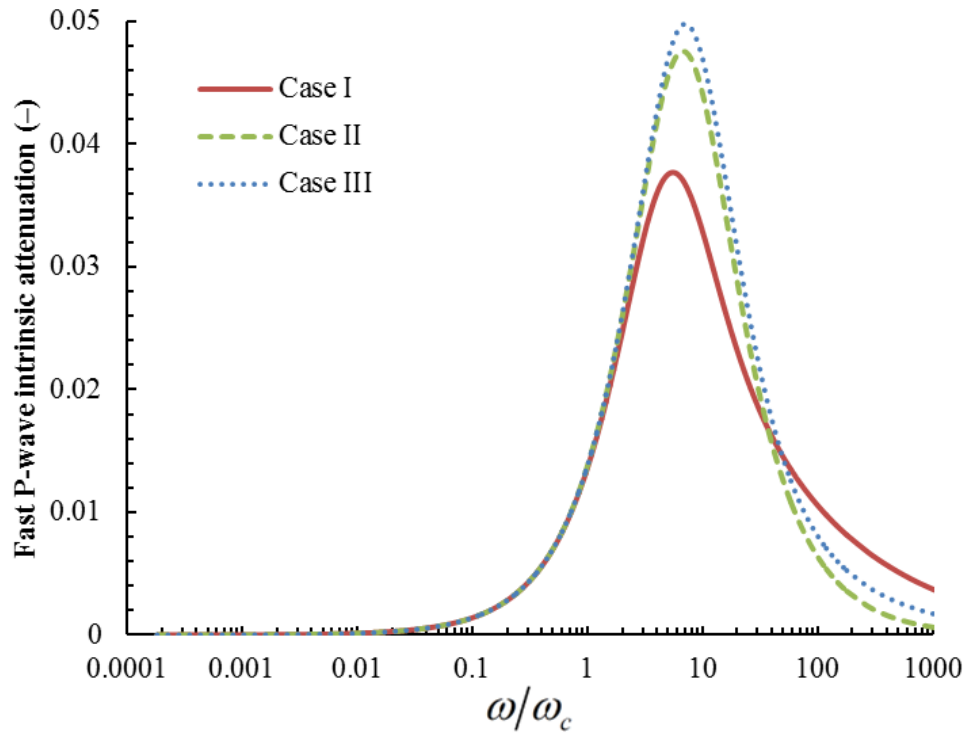


Fig. 4. Fast wave intrinsic attenuation versus dimensionless frequency.

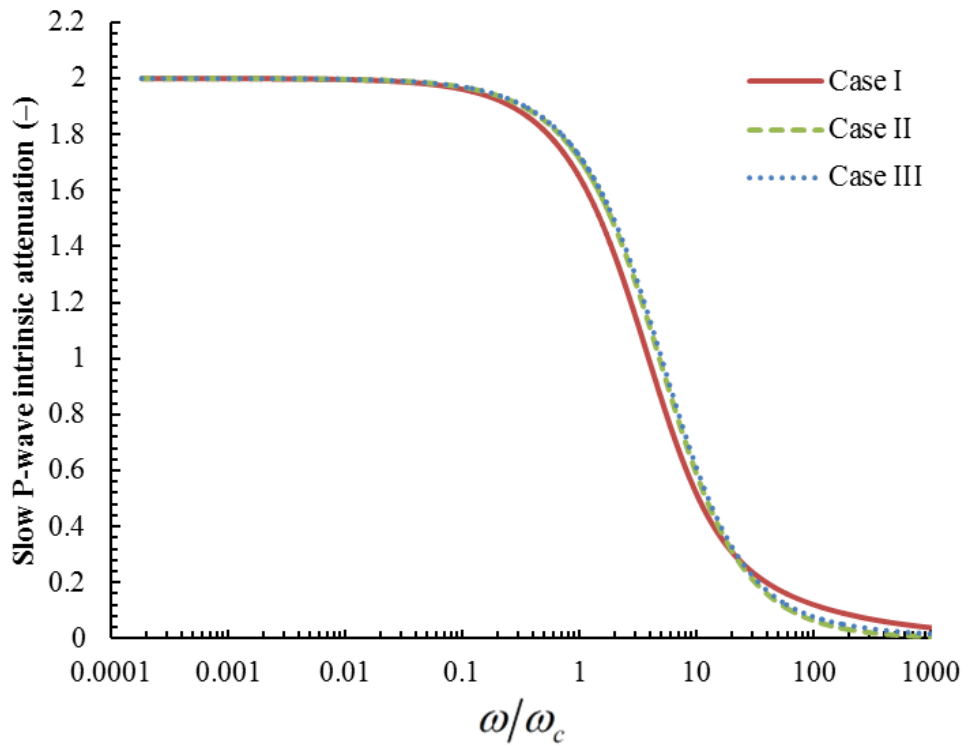


Fig. 5. Slow wave intrinsic attenuation versus dimensionless frequency.

CONCLUSIONS

The main purpose of the paper was to study the effects of dynamic permeability on wave attributes including velocity and intrinsic attenuation. Therefore, an analytical closed-form dispersion relation was derived to calculate compressional wave velocity as well as intrinsic attenuation in full frequency range (from low-frequency viscosity dominated to high-frequency inertia-dominated behavior). Numerical results revealed how neglecting the effects of dynamic permeability can result in miscalculation of compressional waves' attributes specially attenuation of the fast wave at frequencies higher than transition frequency or in vicinity of the transition frequency. However, it was found that for frequencies smaller than the transition frequency, inertia terms may be neglected, and the dynamic permeability can be replaced by the Darcy permeability. Therefore, considering dynamic permeability effects is essential for data interpretation of non-destructive testing methods (e.g., sonic and ultra-sonic tests) of porous materials where the utilized frequencies are relatively high and inertia forces dominate the fluid flow.

NOMENCLATURE

English symbols

- c = Wave velocity
- C_p = Compressibility of solid frame
- C_f = Compressibility of fluid
- C_s = Compressibility of grains
- G = Shear modulus of skeleton
- k_0 = Intrinsic permeability
- k_p = Compressional wave number
- M = Similarity parameter
- n = Porosity
- p = Fluid pressure

Q^{-1} = Intrinsic attenuation

v = Average velocity solid particles

w = Average velocity fluid particles

Greek symbols

η = Fluid viscosity

ξ_s = Hysteretic damping

ρ_f = Fluid density

ρ_s = Solid density

τ = Tortuosity

σ' = Effective stress

ω = Angular frequency

ω_c = Transition frequency

APPENDIX A.

Equations of Case III

Equations to compute compressional wave velocity and intrinsic attenuation of case III

$$\text{Det} \begin{bmatrix} \rho - H k_p^2 / \omega^2 & \rho_f - C k_p^2 / \omega^2 \\ \rho_f - C k_p^2 / \omega^2 & q_f - M k_p^2 / \omega^2 \end{bmatrix} = 0$$

where

$$H = \frac{1}{C_p} + \frac{4G}{3} + \frac{(1/C_s - 1/C_p)^2}{(D - 1/C_p)}$$

$$C = \frac{1}{C_s} \left(\frac{(1/C_s - 1/C_p)}{D - 1/C_p} \right)$$

$$M = \frac{(1/C_s)^2}{(D - 1/C_p)}$$

$$D = \frac{1}{C_s} (1 + n(C_f/C_s - 1))$$

$$\rho = (1 - n)\rho_s + n\rho_f$$

$$q_f = \frac{\tau\rho_f}{n} + \frac{iF(\zeta)\eta}{k_0\omega}$$

$$F(\zeta) = \frac{1}{4} \frac{\zeta T(\zeta)}{1 - 2T(\zeta)/i\zeta}$$

$$T(\zeta) = \frac{\text{ber}'(\zeta) + i \text{bei}'(\zeta)}{\text{ber}(\zeta) + i \text{bei}(\zeta)}$$

where $\zeta = (\omega/\omega_c)^{0.5}$ and functions $\text{ber}(\zeta)$ and $\text{bei}(\zeta)$ represent the real and imaginary parts

of the Kelvin function and $\text{ber}'(\zeta)$ and $\text{bei}'(\zeta)$ their derivatives.

REFERENCES

- Allard, J., Henry, M., Tizianel, J., Kelders, L. and Lauriks, W. (1998). "Sound propagation in air saturated random packings of beads", *Journal of Acoustical Society of America*, 104, 2004–2007.
- Berryman, J.G. (1980). "Confirmation of Biot's theory", *Applied Physics Letter*, 37(4), 382–384.
- Biot, M.A. (1956a). "Theory of Propagation of Elastic Waves in Fluid-Saturated Porous Solid: Low Frequency Range", *Journal of Acoustical Society of America*, 28, 168–178.
- Biot, M.A. (1956b). "Theory of Propagation of Elastic Waves in Fluid-Saturated Porous Solid: Higher frequency range", *Journal of Journal of Acoustical Society of America*, 28, 179–191.
- Chapman, A. and Higdon, J. (1992). "Oscillatory stokes flow in periodic porous media", *Physics of Fluids*, 4(10), 2099–2116.
- Charlaix, E., Kushnick, A. and Stokes, J. (1988). "Experimental study of dynamic permeability in porous media", *Physical Review Letters*, 61, 1595–1598.
- Ghasemzadeh, H. and Abounouri, A.A. (2012). "Effect of subsurface hydrological properties on velocity and attenuation of compressional and shear wave in fluid-saturated viscoelastic porous media", *Journal of Hydrology*, 460–461, 110–116.
- Iversen, N. and Jørgensen, B.B. (1993). "Diffusion coefficients of sulfate and methane in marine sediments: Influence of porosity", *Geochimica et Cosmochimica*, 57, 571–578.
- Johnson, D.L., Hemmick, D. and Kojima, H. (1994). "Probing porous media with first and second sound. i. dynamic permeability", *Journal of Applied Physics*, 76, 104–114.
- Johnson, D.L., Koplik, J. and Dashen, R. (1987). "Theory of Dynamic Permeability and Tortuosity in Fluid-Saturated Porous-Media", *Journal of Fluid Mechanics*, 176, 379–402.
- Kim, S.H., Kim, K.J. and Blouin, S.E. (2002). "Analysis of wave propagation in saturated porous media. I. Theoretical solution", *Computer Methods in Applied Mechanics and Engineering*, 191, 4061–4073.
- Lo, W.C., Sposito, G. and Majer, E. (2006). "Low-frequency dilatational wave propagation through fully-saturated poroelastic media", *Advances in Water Resources*, 29, 408–416.
- Lo, W.C., Yeh, C.L. and Jan, C.D. (2008). "Effect of soil texture and excitation frequency on the propagation and attenuation of acoustic waves at saturated conditions", *Journal of Hydrology*, 357, 270–281.
- Sheng, P. and Zhou, M.Y. (1988). "Dynamic permeability in porous medium", *Physical Review Letters*, 61, 1591–1594.
- Smeulders, D.M.J. (1992). "On Wave Propagation in Saturated and Partially Saturated Porous Media", PhD Thesis, Eindhoven University of Technology.
- Theodorakopoulos, D.D. and Beskos, D.E. (2006). "Application of Biot's Poroelasticity to Some Soil Dynamics Problems in Civil Engineering", *Soil Dynamics and Earthquake Engineering*, 26, 666–679.
- Verruijt, A. (2010). *An Introduction to Soil Dynamics*, Springer, New York.