

## Probabilistic Assessment of Pseudo-Static Design of Gravity-Type Quay Walls

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**ABSTRACT:** Failure of the quay walls due to earthquakes results in severe economic loss. Because of hazards threatening such inexpensive nodes of national and international transportation networks, seismic design of quay walls is still an evolving topic in marine structural engineering. This study investigates the sensitivity of the gravity-type quay wall stability respect to uncertain soil and seismic properties using ultimate limit-state pseudo-static design process. Stability is defined in terms of safety factor against sliding (*sfs*), overturning (*sfo*) and exceeding bearing capacity (*sfb*). In order to assess the forces exerting on quay walls, to be more accurate, pore water pressure ratio, horizontal and vertical inertia forces, fluctuating and non-fluctuating components of hydraulic and soil pressure were considered. It was found that the increase of water depth in front of the quay, vertical and horizontal seismic coefficients, and pore water pressure ratio play important roles in reduction of all mentioned safety factors. Increase of specific weight of the rubble mound, backfill and foundation soil, friction angle of wall-foundation/seabed interface and wall back-face/backfill interface and friction angle of backfill soil, lead safety factors to magnify. A comprehensive sensitivity analysis was also performed using the tornado diagrams. Results of this study could give designers insights into the importance of uncertain soil and seismic factors, in order to choose geometry of the design in a way that its analysis and assessment is less relied on severely uncertain parameters and to introduce more reliable and economic quay walls.

**Keywords:** Quay Wall, Safety Factor, Seismic Design, Stability, Ultimate Limit-State, Uncertainty.

### INTRODUCTION

Past experience demonstrated that quay walls are susceptible to severe damage during earthquakes. Significant damage was observed not only in the case of a strong earthquake such as the 1995 Hyogoken-Nanbu earthquake, but also under moderate

earthquakes (Werner, 1998). For these reasons, seismic design of quay walls is still a developing issue in the marine structural engineering literature.

Several researchers have studied safety factors and performance of gravity-type quay walls in probable or observed seismic conditions.

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Choudhury and Ahmad (2007, 2008) studied safety factors of gravity-type quay walls against sliding and overturning with pseudo-dynamic and pseudo-static methods. They examined the effect of changes in soil properties and seismic coefficients on safety factors in two conditions of high and low porous backfills. Further, a couple of studies have investigated the effect of density and modification of foundation and backfill soils on liquefaction phenomenon, stability and residual displacement of quay walls in Kobe port in Japan as reference model of quay walls (Iai et al., 1998; Chen, 2000; Alyami et al., 2007 and 2009).

Lee and Mosalam (2006) studied the effect of porosity of backfill and foundation soils on stability and performance of wall via centrifuge tests. Kim et al. (2005) revealed the high sensitivity of quay wall stability to friction angle of quay wall base and seabed and necessity of accurate estimation of friction coefficient.

Notwithstanding a plethora of research in this area, as far as the researcher is concerned, there exist gaps for probabilistic assessment of stability considering forces and design coefficients in a comprehensive approach fashion.

In the context of seismic design of quay walls, a variety of techniques and analyses have been developed. Although these analyses are carried out deterministically, the uncertainty associated with the characteristics of scenario earthquakes undermines these approaches. In addition, the behavior of quay walls is significantly governed by the properties of structural materials and soils. Although the properties of the material of well-constructed structures can be assumed to be deterministic with some tolerable limits of variation, most of the parameters controlling the properties of soils are of a random nature, and consequently, uncertainty exists in the seismic response.

Therefore, realistic assessment of the seismic stability of the quay walls requires a probabilistic approach which is based on an appropriate treatment of uncertainty of soil properties and ground motion variability.

Uncertainty in the loss estimation of the structural system, mainly due to uncertainties in the ground motion, structural and soil properties, can be costly because it is directly related to the repair cost. Thus, it is very important to identify and rank the sources of uncertainty according to their relative influence on the stability of the structure (Kramer and Elgamal, 2001 and Jones et al., 2002).

In this study, the sensitivity of a typical reference model gravity-type quay wall design to uncertain soil characteristics, seismic scenario and water depth in front of the quay has been investigated through the ultimate limit-state pseudo-static process. Ultimate limit-state design can be employed in different failure and stability assessment conditions, here, to identify and rank the significant sources of basic uncertainties, safety factors against failures due to sliding, overturning and bearing capacity of the quay wall are introduced as seismic demands.

The sensitivity analysis is performed in a probabilistic framework and the propagation of basic uncertainties is investigated using the mean values and coefficient of uncertain parameters by means of the tornado diagram.

Studying the influence of uncertainty of different soil properties and earthquake scenario on safety factors help designers choose geometry of the design in a way that its analysis and assessment is less relied on severely uncertain parameters, to introduce more reliable designs.

## **GENERAL DEFINITION OF THE PROBLEM**

A typical gravity-type quay wall, as shown in Figure 1, has been investigated.

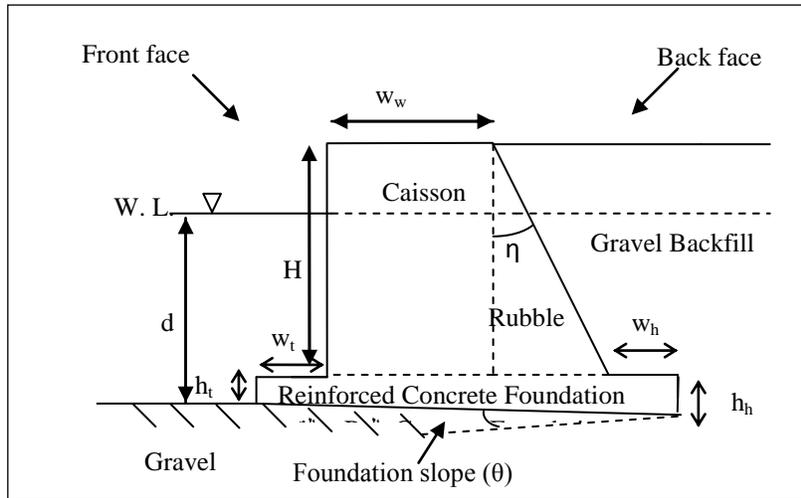


Fig. 1. Cross section of a typical quay wall.

Adapted hypothetical geometrical non-dimensional values are as,  $h_h=h_t=0.03H$ ,  $w_w=0.3H$ ,  $w_t=0.1H$ ,  $w_h=0.25H$ ,  $\eta=20^\circ$ ,  $\theta=0^\circ$ . Since the present study is not geometrically parametric, these dimensions are all considered constant. Table 1 lists mean value of parameters for which uncertainty and sensitivity assessments are performed. Hypothetical values for material properties are obtained from available data and literature on real material (Quinn, 1972 and Sowers, 1993) and about the geometry of the structure, its dimensions are adapted from observing dimensions of constructed real quay walls in for example, Japan and south Iran.

Conventional positive direction of horizontal and vertical components of acceleration coefficients are assumed to provide the worst condition during analyses

(Ebeling and Morrison, 1992). Quay wall is assumed to be located on a gravel bed and retain a gravel backfill. In order to stiffen the backfill, rubble mound has been devised in back face of the wall.

### ASSESSMENT OF THE FORCES AND DESIGN CRITERIA

#### Forces

Typical loads exerted on the wall and their approximate application points, during earthquakes are demonstrated in Figure 2. External forces in seismic condition consist of active earth pressure, inertia, weight, hydrostatic and hydrodynamic water pressure.

Table 1. Hypothetical values adopted for analysis.

Unit weight of saturated backfill gravel ( $\gamma_{sg}$ )	Unit weight of saturated seabed gravel ( $\gamma_{sg}$ )	Unit weight of saturated rubble ( $\gamma_{sr}$ )	Water depth in front of the quay (d)	Horizontal seismic coefficient ( $k_h$ )
-backfill ( $kN/m^3$ ) 20	-seabed ( $kN/m^3$ ) 20	22 ( $kN/m^3$ )	0.85H	0.35
Internal friction angle of gravel backfill ( $\phi$ )	Friction angle between wall and backfill ( $\delta_b$ )	Friction angle between footing and its bed ( $\delta_f$ )	Pore water pressure ratio ( $r_u$ )	Vertical seismic coefficient ( $K_v$ )
47.5°	24°	47.5°	0.175	0.175

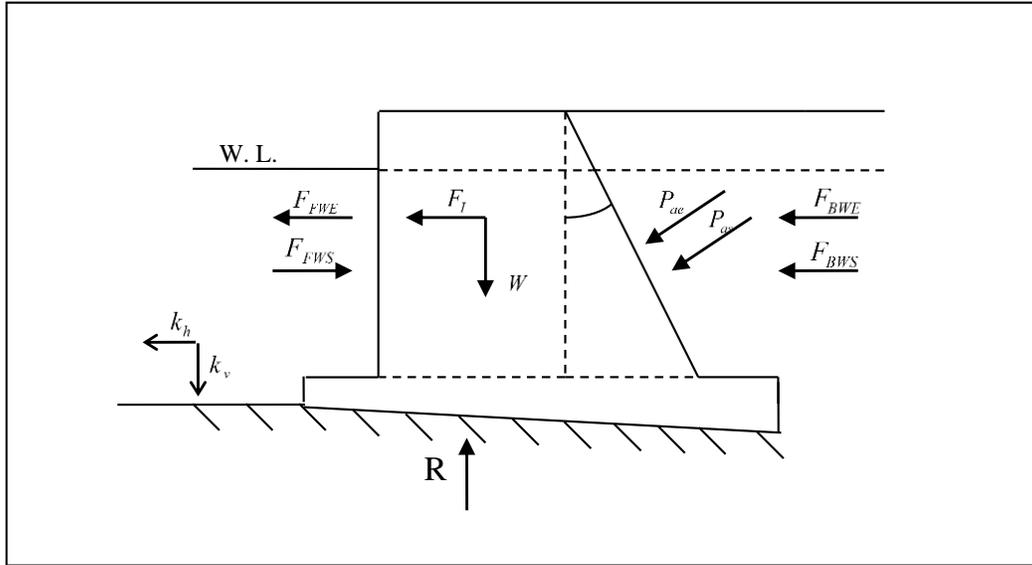


Fig. 2. Forces exerted on quay wall in seismic condition.

The formulation for forces and their application points are demonstrated in next sections. Modifications for seismic coefficient and unit weights due to existence of excess pore pressure in seismic condition and partial submergence of back fill soil should be devised utilizing formulae proposed by Ebeling and Morrison (1992) and PIANC (2001).

Soil pressure: In order to calculate active soil pressure, an alternative to Mononobe-Okabe method was proposed by Mylonakis et al. (2007), is utilized as follows:

$$\psi = a \tan[k_m / (1 - k_v)] \quad (1)$$

$$k_{ae} = \frac{\cos \eta}{\cos \delta_b \cos^2 \eta} \times \left[ \frac{1 - \sin \phi \cos(\Delta_2 - \delta_b)}{1 + \sin \phi \cos \Delta_1} \right] \exp(-2\lambda \tan \phi) \quad (2)$$

$$\Delta_1 = a \sin[\sin(\eta + \psi) / \sin \phi] \quad (3)$$

$$\Delta_2 = a \sin[\sin \delta_b / \sin \phi] \quad (4)$$

$$2\lambda = \Delta_2 - (\Delta_1 + \delta_b) - 2\eta - \psi \quad (5)$$

Mylonakis et al. (2007) have proved that their proposed method in active condition is more precise than that of Mononobe-

Okabe's, but this is vice versa for the passive state.

According to Kim et al. (2004), active-soil pressure and its application points are divided into two fluctuating and non-fluctuating components. Fluctuating component of soil pressure is:

$$P_{ae} = 0.5 * (K_{ae} - K_{as}) (H + hh)^2 \gamma (1 - k_v) \quad (6)$$

It is oriented at an angle  $\delta_b$  to the normal along the back of the wall at a height equal to  $0.55(H + hh)$  (Seed and Whitman, 1970). Non-fluctuating component of soil pressure is  $p_{as}$  and its application point is calculated using Coulomb theory (Kramer, 1996).

Water pressure: Water pressure is also divided into two fluctuating and non-fluctuating components. Fluctuating component of water pressure acting at  $0.4d$  above the base of the wall is calculated by the modified version of Westergaard method proposed by Zangar (1953) for inclined surfaces. Non-fluctuating component is hydrostatic water pressure. It acts at  $d/3$  above the base of the wall and is calculated as:

$$F_{WS} = 0.5 * \gamma_w * d^2 \quad (7)$$

$$sfs = (VF * \tan \delta_f) / (HF) \quad (10)$$

Inertia and weight forces: Inertia force of each part of the wall system is calculated by multiplying unit weight of each field by its volume and horizontal seismic coefficient. For calculating effective weight, the effect of buoyancy is accounted for by subtracting unit weight of water from saturated unit weights of materials. Vertical acceleration is also considered in calculations. Application points of these body forces are centre of gravity for each geometry condition.

**Design Criteria and Stability Evaluation**

Safety factor against failure due to exceeding bearing capacity:

Gravity-type quay walls have shallow foundation ( $\frac{D_f}{B} \leq 2.5$ ) and their factor of safety against failure due to exceeding bearing capacity of the subsoil, is (Bowles, 1996):

$$sfb = \frac{q_{ult}}{(\sigma_{max} - \gamma D_f)} - \gamma D_f < \sigma_{max} \quad (8)$$

Safety factor against overturning:

$$sfo = \frac{PM}{AM} \quad (9)$$

*PM* (Passive Moments) is the sum of vertical component of soil pressure, weight force of the system which will be destabilized (soil wedge and wall's system) and static water pressure in front of the wall and *AM* (Active Moments) consists of: horizontal component of soil pressure, inertia force of the system which will be destabilized (soil wedge and wall's system), static and dynamic water pressure in back of the wall and dynamic water pressure in front of the wall.

Safety factor against sliding:

**SEISMIC AND SENSITIVITY ANALYSIS**

All seismic design procedures involve uncertainty about loads and material properties apart from quality control of construction procedures. Thus without considering uncertainty of material properties, the sole use of a sophisticated analysis tool does not guarantee a safe design. In the following, the effects of uncertainty in material properties as well as ground motion and water depth in front of the quay are presented.

**Characterization of Uncertain Properties**

For evaluating the effect of uncertainty in the stability of quay walls, uncertainty associated with material properties has been represented by assigning a mean and standard deviation in terms of coefficient of variation for each parameter. These mean and standard deviation values of material properties have been chosen from the range of values suggested in the literature (e.g. Porter and Beck, 2002)

Although there are so many geotechnical and seismic parameters for the quay wall involved in this problem, ten parameters related to the saturated soil, friction angles of soils with their interface, seismic coefficients, water depth in front of the quay and pore water ratio were identified as key parameters for the uncertainty analysis. The selection of these parameters was based on the outcome of previous research and engineering judgments.

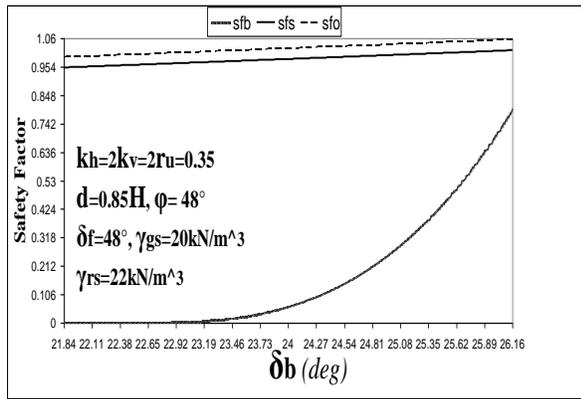
For the seismic analysis whose results should be used in sensitivity analysis, all uncertain parameters are assumed as random variables and for each of these variables, two extreme values corresponding to the assumed upper and lower bounds of their

probability distribution were selected. One can observe that these extreme values come from the normal distribution assumption, mean plus standard deviation and mean minus standard deviation, respectively, representing their upper and lower bounds. Using these two extreme values for a selected random variable, the safety factors were calculated using analytical pseudo-static solution, while all other variables have been assumed to be deterministic with values equal to their mean value.

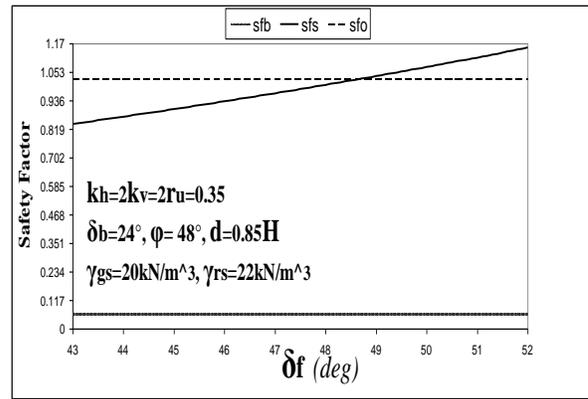
Mean values for all uncertain parameters are listed in Table 1. The coefficient of variation (CoV) for all parameters

considered here is assumed to be approximately the same, with the value of either 9% or 12%. 9% is chosen for soil properties and water depth in front of the quay about which the designers are less uncertain and 12% is chosen for seismic coefficients and pore water pressure ratio since they are naturally more indeterminate.

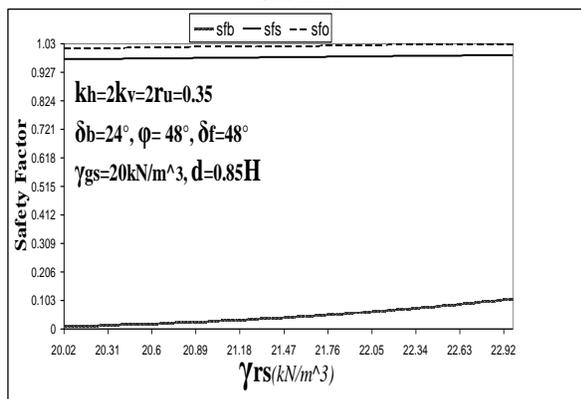
Variations of safety factors due to different uncertain parameters are indicated in Figure 3. As shown in Figure 3, *sfb* values are multiplied by 100 in order to be magnified and be shown in the same axis as *sfo* and *sfs*.



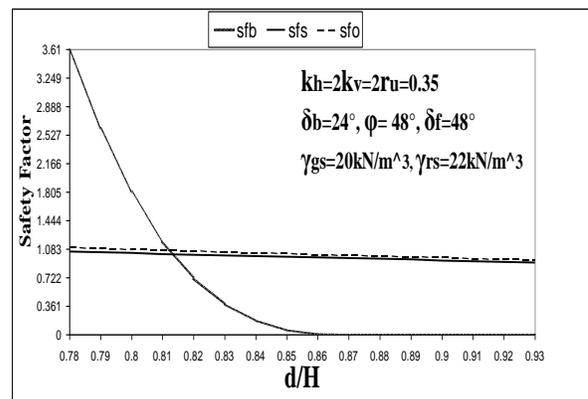
a) Safety factor variations due to uncertainty in value of friction angle between wall back-face with its backfill



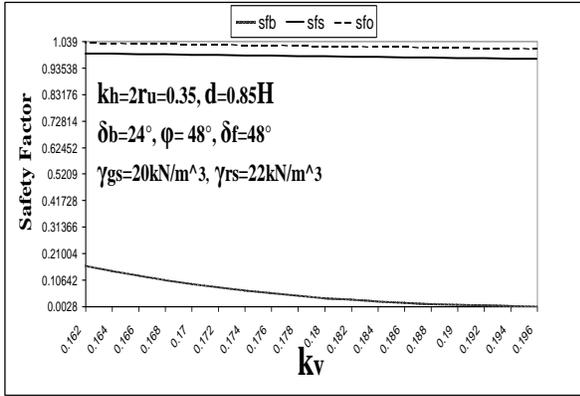
b) Safety factor variations due to uncertainty in value of friction angle a wall-base and seabed



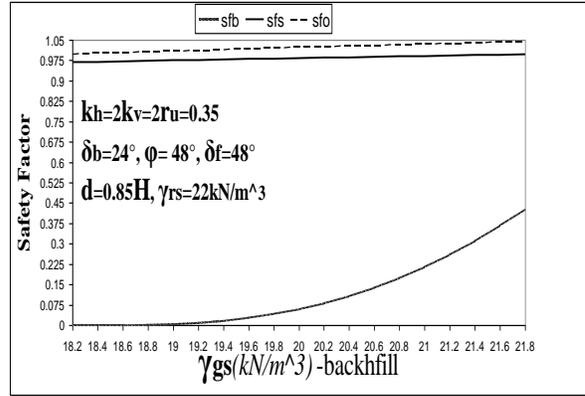
c) Safety factor variations due to uncertainty in value of specific weight of rubble mound gravel



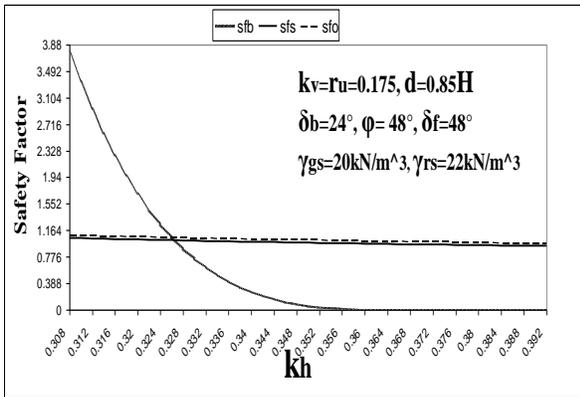
d) Safety factor variations due to uncertainty in value of water depth (non-dimensional)



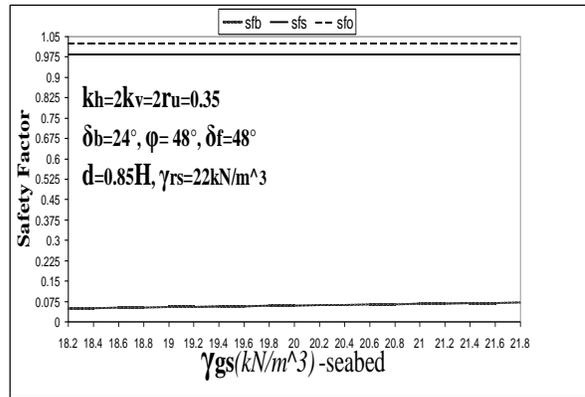
e) Safety factor variations due to uncertainty in value of vertical seismic coefficient



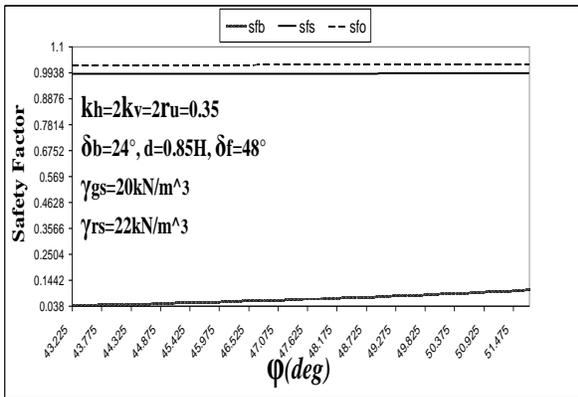
f) Safety factor variations due to uncertainty in value of specific weight of backfill gravel



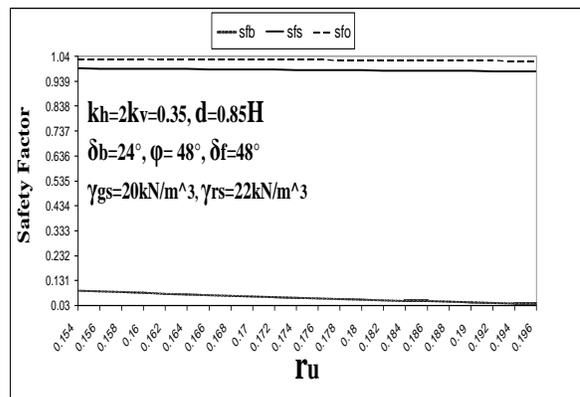
g) Safety factor variations due to uncertainty in value of horizontal seismic coefficient



h) Safety factor variations due to uncertainty in value of specific weight of seabed gravel



i) Safety factor variations due to uncertainty in value of pore water pressure ratio



j) Safety factor variations due to uncertainty in value of internal friction angle of gravel backfill

Fig. 3. Safety factor variations due to uncertain design parameters.

It can be observed that the increase of water depth in front of the quay, vertical and horizontal seismic coefficients, pore water pressure ratio play important roles in the reduction of all-mentioned safety factors. Results caused by the increase of specific weight of the rubble mound, back fill and foundation gravel, friction angle for wall-foundation/seabed interface and wall back-face/backfill interface and friction angle of backfill soil, lead safety factors to magnify.

Perceiving carefully the Figure 3, readers can understand more about trends and slopes of the aforementioned increases and decreases in safety factors' values. For instance, it can be concluded that when  $AM > PM$ , the whole system overturns and definition of factor of safety against exceeding bearing capacity will be pointless, according to this fact, values of  $sf_b$  equals zero.

For the reason that calculation of  $sf_b$  is implicitly involved with exerted forces on the wall, more than other safety factors, its curves provide higher degree of nonlinearity in their trends in comparison to other safety factors.

### Method of Sensitivity Analysis

Reducing the number of uncertain parameters cuts down the computational effort and cost. To this end, those parameters with associated ranges of uncertainty that lead to relatively insignificant variability in stability should be identified and then be treated as deterministic parameters by fixing their values at their best estimate, such as the mean.

There are various methods for ranking uncertain parameters according to their sensitivity to safety factors, such as tornado diagram analysis, FOSM (First Order-Second Moment) analysis, and Monte Carlo simulation (Porter *et al.* 2002; Lee and Mosalam, 2006).

The tornado diagram analysis was used here because of its simplicity and efficiency to identify sensitivity of uncertain parameters. After choosing a band of values for each random variable, safety factors corresponding to each random variable's range of values were calculated. The difference of the maximum and minimum safety factors' values for each selected band, was termed as swing of the response. This calculation procedure was repeated for all random variables and the three considered design criteria alike. Finally, these swings were plotted in 3 tornado diagrams (Figure 4) from top to the bottom in a descending order according to their size as to demonstrate the contribution of each variable relative to the safety factor under the consideration.

In Figure 4, the vertical line in the middle of tornado diagram indicates the value of target safety factor corresponding to the calculations using the mean values of all random variables and the length of each swing (horizontal bar) represents the variation in the safety factor due to the variation in the respective random variable.

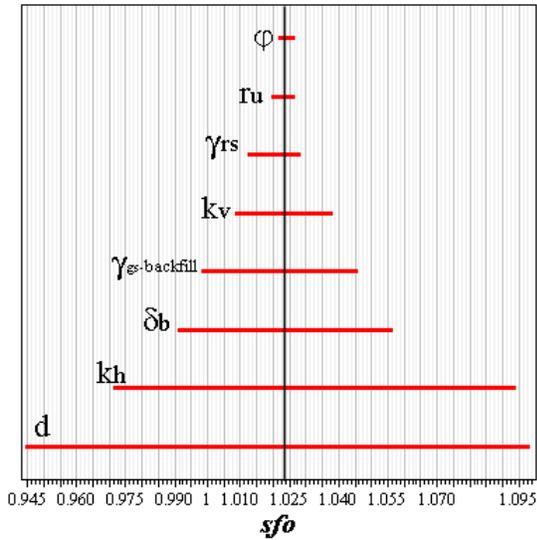
It is noteworthy that longer swing implies that the corresponding variable has larger effect on the response than those with shorter swing. According to the tornado diagrams, safety factors against failures due to sliding and overturning are both mostly sensitive to uncertainty in values of water depth, horizontal seismic coefficient, friction angle between wall back-face and its backfill, specific weight of backfill soil, vertical seismic coefficient, specific weight of rubble mound, pore water pressure ratio, and internal friction of backfill soil, respectively.

Safety factor against overturning follows the same behavior except that the order of water depth and horizontal seismic coefficient and also the order of pore water pressure ratio and internal friction of backfill

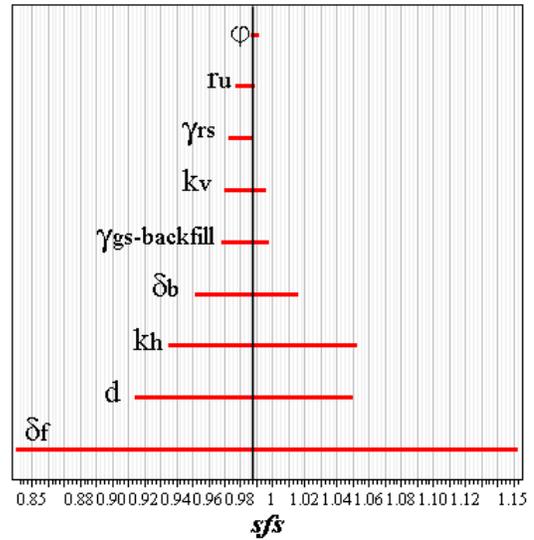
soil are exchanged. Only  $sfb$  is sensitive to specific weight of seabed, and only  $sfs$  is sensitive to friction angle between wall-base and seabed.

It can be noticed in Figure 4 that the water depth and horizontal seismic coefficient are the two greatest contributors of the safety factors' variability. It can also

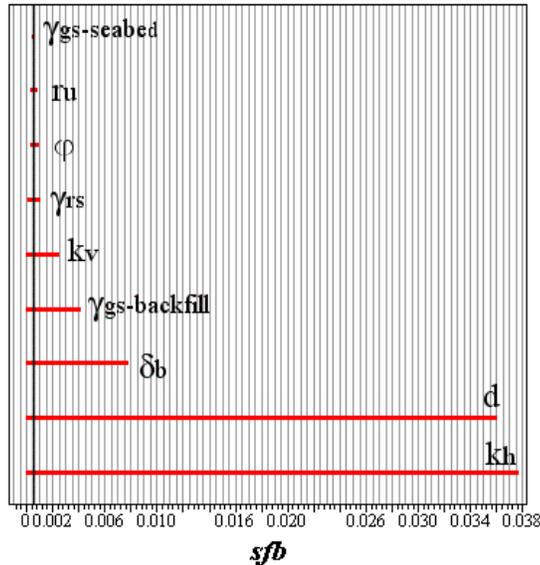
be seen in this figure that nearly all swings are asymmetric about against the vertical line. This skewness of the safety factor distributions implies that the problem is highly nonlinear. In other words, the same degree of positive and negative change in these parameters does not produce the same amount of variation in safety factors.



a) Safety factor against overturning



b) Safety factor against sliding



c) Safety factor against failure due to exceeding bearing capacity

Fig. 4. Tornado diagrams for safety factors (a-c).

## CONCLUSIONS

Through the use of safety factors as a measure of seismic demand, the relative significance of each uncertain parameter to the seismic demand was identified and ranked. In addition, the response variability due to ground motions and water depth in front of the quay was studied.

To investigate the sensitivity of the stability of quay walls with respect to uncertainties of ten design parameters, tornado diagram analyses were conducted. It was found that the uncertainties in the water depth and seismic coefficient are the two parameters contributing to the variance of safety factors more than other parameters. In addition, it was revealed that the CoV of safety factors due to ground motion variability is comparable to the maximum CoV of safety factors due to uncertainty in material properties alone, making seismic design of structures more crucial.

For further studies, assessments may be performed by dynamic analysis or using more appropriate probabilistic methods, in order to provide more comprehensive information about sensitivity analysis of quay walls to different uncertain design parameters.

## NOMENCLATURE

$k_h, k_v$ : Horizontal and vertical seismic coefficients of the system

$F_{FWS}, F_{FWE}, F_{BWE}, F_{BWS}$ : Hydrodynamic and hydrostatic forces,  $F, B, S, E$  stand for in front of the wall, back of the wall, static force and dynamic force, respectively

$F_I$ : Inertia force of the system under consideration for pseudo-static analysis

$H, w_w$ : Wall height and width

$h_h, w_h$ : Heel height and width

$h_t, w_t$ : Toe height and width

$HF, VF$ : Resultant vertical and horizontal forces

$k_m$ : Modified seismic acceleration coefficient for partially submerged condition

$k_{as}, k_{ae}$ : Effective seismic coefficients in static and dynamic conditions

$PM, AM$ : Resultant of stabilizing and destabilizing moments

$P_{as}, P_{ae}$ : Active soil pressure exerting on the system in static and dynamic conditions.

$W$ : Weight force of the system under consideration for pseudo-static analyze

$\bar{\gamma}$ : Weighed unit weight based on the volume of soil in the failure wedge above and below the phreatic surface

$\gamma_w$ : Unit weight of water equal to 10 ( $kN/m^3$ )

$\gamma'$ : Unit weight of soil in saturated condition ( $\gamma - \gamma_w$ )

$\eta$ : Rubble mound inclination angle with vertical line

$\theta$ : Inclination angle of wall foundation with horizontal line

$R$ : Resultant reaction force of seabed to foundation

$\sigma_{max}$ : Maximum stress exerting from wall and its foundation to seabed

$D_f$ : Buried depth of foundation

$q_{ult}$ : Ultimate bearing capacity of the seabed soil

$\psi$ : Seismic inertia angle

$\Delta_1, \Delta_2$ : Caquot angles

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