

## **Predicting Deficient Condition Performance of Water Distribution Networks**

**Abdy Sayyed, M.A.H.<sup>1</sup> and Gupta, R.<sup>2\*</sup>**

<sup>1</sup> Graduate Student, Civil Engineering Department, Visvesvaraya National Institute of Technology, Nagpur, India.

<sup>2</sup> Professor, Civil Engineering Department, Visvesvaraya National Institute of Technology, Nagpur, India.

Received: 09 May 2012;

Revised: 25 Oct. 2012;

Accepted: 25 May 2013

---

**ABSTRACT:** A water distribution network is subjected to various abnormal conditions such as pipe breaks, pump failures, excessive demands etc. in the design period. Under such conditions, the network may not be able to meet required demands at desired pressures, and becomes deficient. Traditional network analysis assumes nodal demands to be satisfied and available nodal pressures are calculated. However, assumption that demands are satisfied at all nodes is not true under deficient conditions. Therefore, under deficient conditions nodal demands and pressures are considered simultaneously through head-flow relationships to calculate available nodal flows. This type of analysis that determines available flows is termed as node flow analysis or pressure-driven or dependent wherein, outflows are considered as function of available pressure. Various node head-flow relationships (NHFR) have been suggested by researchers to correlate available flow and available pressure based on required flow and required pressure. Methods using these NHFRs have been classified herein as direct and indirect approaches. Applications of these approaches have been shown with two illustrative examples and results are compared.

**Keywords:** Node Flow Analysis, Pressure-Dependent Analysis, Water Distribution Networks.

---

### **INTRODUCTION**

A water distribution network (WDN) is designed to service consumers over a long period of time. The network is subjected to various abnormal conditions such as pipe breaks, pump failures, excessive demands etc. in the design period. The network may not be able to meet required demands at desired pressures under these abnormal conditions and become deficient. Its performance at any point of time under these

conditions can be obtained through its analysis. In traditional network analysis nodal demands are assumed to be satisfied and available nodal pressures are calculated. If the available pressures are more than desired ones the performance of a WDN is satisfactory; otherwise performance is unsatisfactory. However, assumption that demands are satisfied at all nodes is not true under deficient conditions. In such conditions, nodal demands may be satisfied fully, partially or not at all, depending upon

---

\* Corresponding author E-mail: drrajeshgupta123@hotmail.com

the available pressure. Therefore, under deficient conditions nodal demands and pressures are considered simultaneously through head-flow relationships to calculate available nodal flows. This type of analysis that determines available flows is termed as node flow analysis (NFA) in contrast to traditional analysis which determines nodal heads and termed as node head analysis (NHA) (Bhave, 1981, 1991). Such type of analysis is also termed as pressure-dependent or pressure-driven analysis as outflows are considered as function of pressure. In this paper, methodologies for pressure-deficient analysis have been categorized as direct and indirect approaches based on the methodology used to tackle node-flow relationship in solving the problem. Both types of approaches have been applied to a common problem and results are compared.

### NODE HEAD-FLOW RELATIONSHIPS

Available flow at a node under a deficient condition depends on available pressure. Hence, a relationship between flow and pressure at a node exists and is herein termed as node head-flow relationship (NHFR). In the analysis of network NHFR at different nodes must be satisfied along with usual node-flow continuity relationships and loop-head loss relationships.

Bhave (1981, 1991) was the first to propose a NHFR as shown in Figure 1. He considered only one hydraulic gradient level (HGL) to develop a NHFR. In obtaining the performance of network in which every outlet is considered, this HGL was taken as outlet level itself and referred as  $H^{\min}$  (Figure 1). Since velocity heads were neglected (as in NHA also), HGL at a node more than  $H^{\min}$  provided adequate flow (available flow,  $q^{\text{avl}} = \text{required flow, } q^{\text{req}}$ ); HGL value less than  $H^{\min}$  provided no flow

( $q^{\text{avl}} = 0$ ); and HGL value equal to  $H^{\min}$  provided partial flow ranging between no-flow and adequate-flow ( $0 < q^{\text{avl}} < q^{\text{req}}$ ). Gupta and Bhave (1996b) showed that for primary networks, in which demands at several outlets are lumped at a node, two HGL values are important in defining a NHFR. At some minimum HGL,  $H^{\min}$ , supply to the lowest outlet on secondary network would begin; and at some desirable HGL,  $H^{\text{des}}$ , all the outlets on secondary network would have adequate flows. However, Bhave's NHFR can be still used for obtaining performance of primary networks by suitably changing the value of  $H^{\min}$  (Gupta and Bhave, 1994; Ozger and Mays 2003; Ang and Jowitt, 2006). Gupta and Bhave (1994) suggested considering desirable heads at various nodes as  $H^{\min}$ ; thus a lower bound on available partial flows. Ozger and Mays (2003) suggested to use maximum outlet level in the locality served by a node as the  $H^{\min}$ . Ang and Jowitt (2006) mentioned that actual relationship between source head and outflows at each demand node is a bi-product of the analysis and used elevation of demand node itself as  $H^{\min}$ . The available flow at any demand node  $j$  is given by the alternate equations:

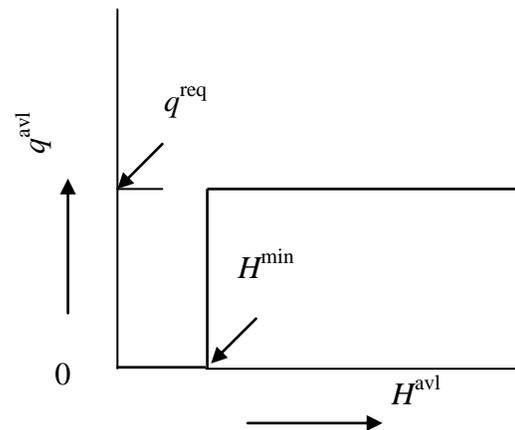


Fig. 1. Bhave's NHFR.

$$q_j^{avl} = q_j^{req} \text{ (adequate flow), if } H_j^{avl} > H_j^{min} \quad (1a)$$

$$0 < q_j^{avl} < q_j^{req} \text{ (no flow, partial flow or adequate flow), if } H_j^{avl} = H_j^{min} \quad (1b)$$

$$q_j^{avl} = 0 \text{ (no flow), if } H_j^{avl} < H_j^{min} \quad (1c)$$

Germanopoulos (1985) in describing a NHFR considered no flow for HGL value less than  $H^{min}$ ; and exponential increase of available flow beyond  $H^{min}$  as shown in Figure 2.

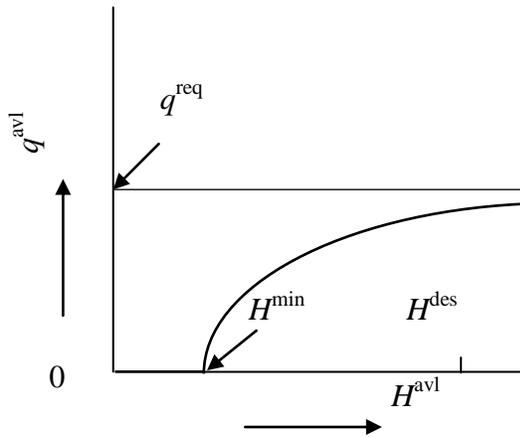


Fig. 2. Germanopoulos's NHFR.

$$q_j^{avl} = q_j^{req} \left[ 1 - 10^{-c_j \left( \frac{H_j^{avl} - H_j^{min}}{H_j^{des} - H_j^{min}} \right)} \right] \quad (2a)$$

$$\begin{aligned} &\text{(partial flow), if } H_j^{avl} > H_j^{min} \\ q_j^{avl} &= 0 \text{ (no flow), if } H_j^{avl} \leq H_j^{min} \end{aligned} \quad (2b)$$

It can be observed from Figure 2 that available flows are less than required flows even at HGL value more than  $H^{des}$  and the curve given by Eq. (2a) is asymptotic to  $q^{req}$  line. For higher values of  $c_j$ , the curve will reach  $q^{req}$  line rapidly.

Wagner et al. (1988) and Chandapillai (1991) suggested parabolic NHFR for HGL values between  $H^{min}$  and  $H^{des}$  as shown in Figure 3.

$$q_j^{avl} = q_j^{req}, \text{ if } H_j^{avl} \geq H_j^{des} \quad (3a)$$

$$q_j^{avl} = q_j^{req} \left( \frac{H_j^{avl} - H_j^{min}}{H_j^{des} - H_j^{min}} \right)^{n_j}, \quad \text{if } H_j^{min} < H_j^{avl} < H_j^{des} \quad (3b)$$

$$q_j^{avl} = 0, \text{ if } H_j^{avl} \leq H_j^{des} \quad (3c)$$

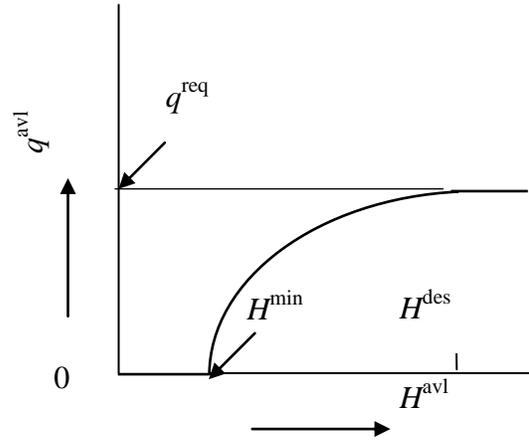


Fig. 3. NHFR by Wagner et al.

Fujiwara and Ganesharajah (1993) suggested a complex differentiable function of HGL to define NHFR as shown in Figure 4. Fujiwara and Li (1998) suggested approximate solution to differentiable function. However, these relationships lack a good hydraulic justification.

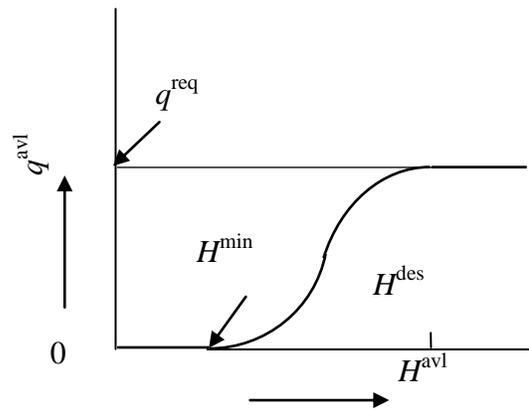


Fig. 4. Fujiwara & Ganesharajah NHFR.

$$q_j^{avl} = q_j^{req}, \text{ if } H_j^{avl} \geq H_j^{des} \quad (4a)$$

$$q_j^{avl} = q_j^{req} \frac{\int_{H_j^{min}}^{H_j^{avl}} (z - H_j^{min})(H_j^{des} - z) dz}{\int_{H_j^{min}}^{H_j^{des}} (z - H_j^{min})(H_j^{des} - z) dz}, \quad (4b)$$

if  $H_j^{min} < H_j^{avl} < H_j^{des}$

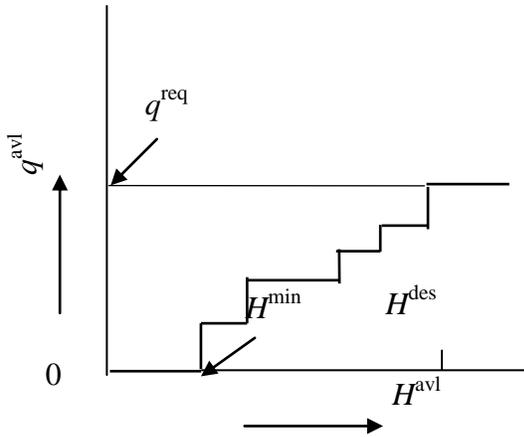
or

$$q_j^{avl} = q_j^{req} \frac{(H_j^{avl} - H_j^{min})^2 (3H_j^{des} - 2H_j^{avl} - H_j^{min})}{(H_j^{des} - H_j^{min})^3}, \quad (4b)$$

if  $H_j^{min} < H_j^{avl} < H_j^{des}$

$$q_j^{avl} = 0, \text{ if } H_j^{avl} \leq H_j^{des} \quad (4c)$$

Kalungi and Tanyimboh (2003) suggested a methodology in which a multiple-step type NHFR as shown in Figure 5 is generated internally. The number of steps and their sizes depended on number of sets of critical nodes and their HGL values. In this case, NHFR can be represented as (Bhave and Gupta, 2006).



**Fig. 5.** Kalungi and Tanyimboh NHFR.

$$q_j^{avl} = q_j^{req}, \text{ if } H_j^{avl} > H_j^{des} \quad (5a)$$

$$0 \leq q_j^{avl} \leq q_j^{req}, \text{ if } H_j^{min} \leq H_j^{avl} \leq H_j^{des} \quad (5b)$$

$$q_j^{avl} = 0, \text{ if } H_j^{avl} < H_j^{min} \quad (5c)$$

It should be noted that in NHFRs given by Eqs. (2a)-(2b), (3a)-(3c), or (4a)-(4c)

available flows can be obtained directly for any HGL value. Similar is the case with Eqs. (1a), (1c), (5a), and (5c). However, in NHFRs given by Eq. (1b) or Eq. (5b), available flows cannot be obtained directly and its maximum value is calculated either through optimization or through repeated analysis as described later.

## PROBLEM FORMULATION

Analysis problem is usually formulated in terms of unknown pipe discharges ( $Q$ ), unknown nodal heads ( $H$ ), loop flow corrections ( $\Delta Q$ ), or unknown pipe discharges and nodal heads ( $Q - H$ ). Since NFA involves a head-flow relationship at each demand node, a problem formulation in terms of  $H$  or  $Q - H$  can be easily solved as compared to other types of formulations. A general formulation of  $H$  equations for NFA is as follows:

$$\sum_{\substack{i \text{ connected to} \\ j \text{ through } x}} \left( \frac{H_i - H_j}{R_x} \right)^{1/p} - q_j^{avl} = 0, \quad (6)$$

for  $j = 1, \dots, J$

$$q_j^{avl} = q_j^{req} \phi(H_j^{avl}), \text{ for } j = 1, \dots, J \quad (7)$$

in which  $H_i$  and  $H_j$  = HGL values at the upstream node  $i$  and downstream node  $j$  of pipe  $x$ , respectively;  $R_x$  = resistance constant of pipe  $x$ ;  $p$  = an exponent, the value of which depends on head loss formula; and  $\phi(H_j^{avl})$  is a function of available HGL, the value of which lies between 0 and 1. Eq. (6) is a node flow continuity equation at demand node  $j$ ; and Eq. (7) is a set of NHFRs at demand nodes  $j$ , and one from this set would be applicable at node  $j$ .

The general formulation for  $Q - H$  equations for NFA is as follows:

$$\sum_{\substack{x \text{ connected} \\ \text{to } j}} Q_x - q_j^{avl} = 0, \text{ for } j = 1, \dots, J \quad (8)$$

$$H_i - H_j - R_x Q_x^n = 0, \text{ for } x = 1, \dots, X \quad (9)$$

$$q_j^{avl} = q_j^{req} \phi(H_j^{avl}), \text{ for } j = 1, \dots, J \quad (10)$$

in which  $Q_x$  = discharge in pipe  $x$ . Eq. (8) is node flow continuity equation at demand node  $j$ ; Eq. (9) is pipe head loss equation; and Eq. (10) is same as Eq. (7).

## NFA METHODS

NFA methods are classified herein, into two categories: (1) indirect approaches in which NHFRs do not provide direct calculation of  $q^{avl}$  for any  $H^{avl}$ ; and (2) direct approaches in which NHFRs provide direct calculation of  $q^{avl}$  for any  $H^{avl}$ . The NFA methods using NHFRs given by Eqs. (1a)-(1c) and (5a)-(5c) fall under indirect approaches and those using Eqs. (2a)-(2b), or (3a)-(3c), or (4a)-(4c) fall under direct approaches.

## INDIRECT APPROACHES

Since available flows cannot be obtained directly using NHFRs, the NFA problem is formulated and solved as an optimization problem (Bhave 1981, Tahar et al., 2002) to maximize total outflows. Alternatively, NFA problem is solved by repetitive use of traditional network solver (Bhave 1981; Ozger and Mays, 2003; Kalungi and Tanyimboh, 2003; Ang and Jowitt, 2006).

### Bhave's Method

Bhave's iterative NFA method is based on NHFRs as given by Eqs. (1a)-(1c) and shown in Figure 1. The method begins with certain assumptions regarding availability of flows at nodes, i.e. either  $q_j^{avl} = q_j^{req}$  (adequate flow node), or  $H_j^{avl} = H_j^{min}$  (critical node), or  $q_j^{avl} = 0$  (no flow node). Bhave (1981) suggested assuming adequate flow at all the nodes to start the iterative method. The network analysis is carried out and the

compatibility between assumed and obtained conditions at all the nodes is verified, i.e.  $H_j^{avl}$  should be more than  $H_j^{min}$  for the assumed adequate flow nodes,  $q_j^{avl}$  should be between 0 and  $q_j^{req}$  for assumed critical nodes; and  $H_j^{avl}$  should be less than  $H_j^{min}$  for assumed no flow nodes. If found compatible at all nodes, the NFA procedure is terminated; otherwise, these assumptions regarding availability of flows at different nodes are changed systematically [for details, refer Bhave (1981, 1991)] and network analysis is repeated. Thus, the method involves solving several NHA problems with different assumptions regarding availability of flow.

### Ozger and Mays Method

Ozger and Mays (2003) also used NHFR as shown in Figure 1. Their NFA method also starts with traditional NHA. Next, nodes at which pressures are insufficient to fully supply their demands are identified. Demands at these nodes are set to zero and an artificial reservoir at each of the pressure deficient node, with elevation equal to maximum outlet level, is introduced and connected through an infinitesimally short pipe with check valve that allows the flow from node to reservoir. Hydraulic analysis is carried out. If one or more artificial reservoirs are found to receive more water than their demands, those artificial reservoirs are removed and demands are restored. The procedure continues till no artificial reservoirs are found to receive water more than its demand. In comparison to Bhave's method, this method is slightly different in the sense that it allows only inflow to artificial reservoirs while Bhave's method allows both inflow and outflow at critical nodes (nodes at which artificial reservoirs are assumed).

### **Kalungi and Tanyimboh's Method**

Kalungi and Tanyimboh (2003) used multiple-step NHFR as shown in Figure 5 in their NFA method. Their method also starts with traditional NHA iteration. While in the methods of Bhawe as well as Ozger and Mays (2003), pressures at all the pressure deficient nodes are increased to their minimum HGL values, in the method of Kalungi and Tanyimboh pressure-deficient nodes are grouped together with nodes of same pressure contours in a set and their HGL values are set to an average value between consecutive pressure values. Hence, at all nodes either outflows or HGL values are made known to carry out iteration. The method systematically identifies no-flow, partial flow and key-partial flow nodes and terminates when there could be no more key-partial flow nodes.

### **Ang and Jowitt's Method**

Ang and Jowitt (2006) also used NHFR as shown in Figure 1. They suggested performing hydraulic analysis of the network with all the demands set to zero (i.e. calculation of static head at all demand nodes for zero demands). Next, artificial reservoirs are added at all nodes having positive static head with the same elevation as the demand node through small link with arbitrarily small resistance coefficient. Hydraulic analysis is carried out, and if one or more artificial reservoirs are found supplying water to distribution network, these artificial reservoirs are removed, and become no flow node. Next, all artificial reservoirs that have inflow greater than their specific demand are replaced with demand node of the stated demand. Hydraulic analysis of the updated network is carried out. If at any demand node, available head is found to be less than minimum head, artificial reservoir is added at this node, or else if there is any demand node with an outflow greater than its demand, artificial

reservoir is replaced by demand node with the stated demand. The analysis terminates when no such changes are required.

## **DIRECT APPROACHES**

Direct NFA approaches solve Eqs. (6) and (7) simultaneously (Gupta and Bhawe, 1996a; Tabesh et al., 2002) or Eqs. (8) to (10), simultaneously (Gupta et al., 2005; Wu et al., 2009; Giustolisi and Laucelli, 2011). In direct approaches Eq. (7) or Eq. (10) is replaced by set of alternate equations Eqs. (2a)-(2b), or (3a)-(3c), or (4a-4c). Gupta and Bhawe (1996b), and Bhawe (2003) compared various NHFRs and recommended use of Eqs. (3a)-(3c). Further, they showed that the value of  $n_j$  in Eq. (3b) lies between 1 and 2 and primarily depends on relative elevations of various outlets and frictional head loss requirement in the secondary network, which consist of small diameter pipes off-taking from the node of a primary network. Hence, a lot of field data is required to select the proper value of  $n$  at different nodes. In absence of such data, it was suggested to use an average value of  $n$  as 1.5 at all nodes. Further, in the absence of data for secondary network, the elevation of node itself was suggested as the value of  $H_j^{\min}$ .

Gupta and Bhawe (1996a) used Hardy Cross method for solving above formulation, while Tabesh et al. (2002) used Newton-Raphson method. Initially trial HGL values,  $H_j^{\text{avl}}$ , at all demand nodes are assumed and corresponding available nodal flows,  $q_j^{\text{avl}}$ , are obtained using Eq. (7). Next, corrections to the assumed HGL values are obtained so as to satisfy Eq. (6). The iterative method is continued, each time modifying the  $q_j^{\text{avl}}$  values through Eq. (7) for the corrected  $H_j^{\text{avl}}$  values, till the corrections become negligible. Convergence is slow in both Hardy Cross and Newton-Raphson

methods as compared to other methods even in case of traditional NHA. There are several measures by which convergence can be enhanced. (Bhave, 1981). Gupta and Bhave (1996a) used modified Hardy Cross method (Bhave, 1985) in which effect of correction at other node is taken into account along with some convergence acceleration factors to enhance the convergence. Tabesh et al. (2002) ensured faster convergence through step-length adjacent parameters.

Since NFA approaches are mainly used for predicting performance of a WDN under large number of conditions during its reliability analysis, direct approaches that require single NHA solution are more attractive than indirect approaches which require several NHA solutions (Gupta and Bhave, 2004).

Gupta et al. (2005) used gradient method with modified formulation to include alternate Eqs. (3a-3c). Wu et al. (2009) suggested extended global-gradient algorithm which can model fire demand as a volume-based demand as in node head analysis (demand dependent analysis). Giustolisi and Laucelli (2011) suggested enhanced global gradient algorithm to model network, for leakages and demands in pressure deficient conditions. Recently, Jinesh Babu and Mohan (2012) proposed an algorithm using artificial reservoirs (ARs) and artificial flow control valves (AFCVs) for solving NHFR shown in Figure 1. Artificial flow control valve restricts the outflows at node to their demand and prevents any back flow. The procedure is non-iterative and internally applies the NHFR and obtains the result in a single iteration.

Software based on Hardy Cross method prepared by Gupta and Bhave for NFA is used to obtain performance of a network using direct NFA approach and check convergence of problem for all-pipes-working condition and one-pipe-failure

conditions. Results so obtained are compared with those obtained by indirect approaches (Bhave 1981, Ozger and Mays 2003).

### ILLUSTRATIVE EXAMPLE I

A serial WDN consists of source node 0 and four demand nodes 1, 2, 3 and 4 as shown in Figure 6. The available HGL value at source node is 100 m, while the minimum required HGL values at nodes 1 through 4 are 90, 88, 90 and 85 m, respectively. The demand at nodes 1, 2, 3 and 4 are 2, 2, 3 and 4 m<sup>3</sup>/min respectively (including fire demand of 3 m<sup>3</sup>/min at node 4). The pipe numbers, length, diameter and Hazen-Williams coefficients are given in Table 1 in columns 1 through 4, respectively. EPANET is used to solve the problem using different methods by converting flows to L/min.

**Table 1.** Pipe data for illustrative example I.

Pipe number	Length m	Diameter mm	HW coefficient
(1)	(2)	(3)	(4)
1	1000	400	130
2	1000	350	130
3	1000	300	130
4	1000	300	130

Initially by assuming that demands are satisfied at all nodes (adequate-flow nodes), we carry out analysis and get HGL values at nodes 1 through 4 as 95.14, 88.71, 80.16 and 77.13 m, respectively. Since  $H^{avl}$  values are less than the corresponding  $H^{min}$  values at nodes 3 and 4 for the assumed adequate flow conditions NHFR (Eq. (1a)) are not satisfied. Hence, in the second NFA iteration nodes 1 and 2 are considered as adequate flow and nodes 3 and 4 are considered as critical nodes. Considering available flow at nodes 1 and 2 as 2 m<sup>3</sup>/min and HGL values at nodes

3 and 4 as 90 and 85 m, respectively, we get  $H^{avl}$  at nodes 1 and 2 as 97.05 and 93.62 m, respectively and  $q^{avl}$  at nodes 3 and 4 as -0.84, and 5.24 m<sup>3</sup>/min, respectively. Since  $H^{avl}$  at nodes 1 and 2 is more than corresponding  $H^{min}$  value, NHFR Eq. (1a) is satisfied at nodes 1 and 2. However,  $q^{avl}$  at node 3 is negative and at node 4 is more than  $q^{req}$  (surplus-flow) and therefore NHFR (Eq. (1b)) is not satisfied at these nodes. In the third iteration, node 3 is assumed as no-flow node and node 4 as adequate-flow node. After analysis, we get  $H^{avl}$  at nodes 1 through 4 as 97.30, 94.27, 91.24 and 88.21 m respectively. Since,  $H^{avl}$  at nodes 1, 2 and 4 is above the corresponding HGL, NHFR Eq. (1a) is satisfied at nodes 1, 2 and 4. However, NHFR (Eq. (1c)) is not satisfied at node 3, for the next iteration, node 3 is assumed as critical node. The  $q^{avl}$  at nodes 3 is 0.40 m<sup>3</sup>/min. The  $H^{avl}$  at nodes 1, 2 and 4 after four iterations is 97.05, 93.62, and 86.97 respectively. Since  $H^{avl}$  at nodes 1, 2 and 4 are more than corresponding  $H^{min}$  value, and  $q^{avl}$  at node 3 is in between 0 and  $q^{req}$ , NHFRs (Eq. (1a)) is satisfied at nodes 1, 2 and 4, and Eq. (1b) is satisfied at node 3. Thus, NHFRs are satisfied at all nodes and the NFA is completed.

When Ozger and Mays method is used for the above network, first step is identically carried out by considering all nodes as adequate flow nodes. The available pressure ( $H^{avl} - H^{min}$ ) at nodes 1 through 4 is 5.14, 0.71, -9.84 and -7.87 m respectively. The pressure at nodes 3 and 4 are found to be negative and hence an artificial reservoir is connected at these nodes with check valve of negligible resistance. The augmented network is analyzed. The  $q^{avl}$  at nodes 3 and 4 is 0.0 and 4.69 m<sup>3</sup>/min. The outflow at node 4 is more than the desired one; hence the artificial reservoir is removed from node 4 and normal demand is replaced. In third iteration, the  $H^{avl}$  at nodes 1, 2 and 4 is 97.05, 93.62, and 86.97 m,  $q^{avl}$  at node 3 is

0.40 m<sup>3</sup>/min. The NHFRs for indirect method is satisfied at all nodes, and the NFA solutions are reached.

Using Ang and Jowitt's method, initially analysis is carried out by assuming all nodal demands to be zero. The static head at nodes 1 through 4 are 10, 12, 10 and 15 m, respectively. Since head at all nodes are positive, artificial reservoirs are connected to each node with head same as respective node elevation using a pipe of negligible resistance. The second iteration is carried out, and the outflows at node 1 through 4 are found as 11.44, 7.99, -8.43 and 5.24 m<sup>3</sup>/min, respectively. As the outflow is negative at node 3, the artificial reservoir is removed from node 3 and demand is considered as zero. In the third iteration the available head at node 3 is found to be -3.5 m and zero at other nodes,  $q^{avl}$  at nodes 1 through 4 are 11.44, 2.06, 0 and 2.74 m<sup>3</sup>/min, respectively. Since the outflows at nodes 1 and 2 are more than the demand, the artificial reservoirs are removed from these nodes and full demand is restored. In the fourth iteration,  $q^{avl}$  at node 1 through 4 are 2, 2, 0, and 4.69 m<sup>3</sup>/min and available heads are 6.86, 5.15, -0.93 and 0. Thus,  $q^{avl}$  at node 4 is more than  $q^{req}$ . Therefore, artificial reservoir from node 4 is also removed and full demand is restored. In the fifth iteration, the available flows at nodes 1 through 4 are 2, 2, 0, and 4 m<sup>3</sup>/min, respectively. The available heads at nodes 1 through 4 are 7.30, 6.27, 1.24, and 3.21 m, respectively. The pressure is positive at node 3 where outflow is considered as zero, and hence node 3 is connected again to the artificial reservoir. In the sixth iteration the  $q^{avl}$  at node 3 is 0.40 m<sup>3</sup>/min, and  $H^{avl}$  at nodes 1 through 4 are 97.05, 93.62, 90 and 86.97 m, respectively. The above results are similar to the one obtained using Bhave's and Ozger and Mays method, described above and are compared iteration wise in Table 2.

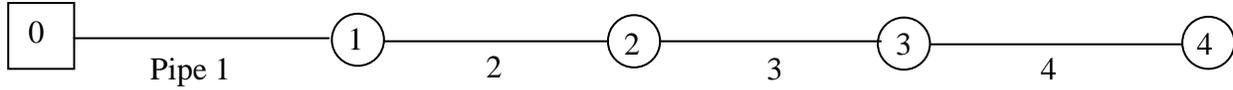


Fig. 6. A serial network for illustrative example I.

Table 2. Comparison of results of indirect approaches for illustrative example I.

Iteration		Bhаве's Method				Ozger and Mays Method				Ang and Jowitt's Method			
No.	Nodes→	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
	$q^{req}$	2	2	3	4	2	2	3	4	2	2	3	4
	$H_{min}$	90	88	90	85	90	88	90	85	90	88	90	85
1	$q^{avl}$	2	2	3	4	2	2	3	4	0	0	0	0
	$H^{avl}$	95.14	88.71	80.16	77.13	95.14	88.71	80.16	77.13	100	100	100	100
2	$q^{avl}$	2	2	-0.84	5.24	2	2	0	4.69	11.44	7.99	-8.43	5.24
	$H^{avl}$	97.05	93.62	90	85	96.86	93.15	89.07	85	90	88	90	85
3	$q^{avl}$	2	2	0	4	2	2	0.4	4	11.44	2.06	0	2.74
	$H^{avl}$	97.3	94.27	91.24	88.21	97.05	93.62	90	86.97	90	88	86.5	85
4	$q^{avl}$	2	2	0.4	4					2	2	0	4.69
	$H^{avl}$	97.05	93.62	90	86.97					96.86	93.15	89.07	85
5	$q^{avl}$									2	2	0	4
	$H^{avl}$									97.3	94.27	91.24	88.21
6	$q^{avl}$									2	2	0.4	4
	$H^{avl}$									97.05	93.62	90	86.97

**ILLUSTRATIVE EXAMPLE II (Ozger and Mays, 2003)**

A WDN shown in Figure 7 has two reservoirs RES1 and RES2 and 13 nodes 1 through 13. Pipe and node characteristics are given in Tables 3 and 4, respectively. Pipe numbers, length, diameter and Hazen-Williams coefficients are given in Table 3 in columns 1 through 4, respectively. Node numbers, elevation, and nodal demands are given in Table 4 in columns 1 through 3, respectively. The minimum HGL at a node is equal to node elevation below which no water is available. A minimum pressure threshold of 15 m is considered to satisfy the

full supply. Thus, Head loss is obtained by Hazen-Williams formula:

$$h = K (Q/C)^p (L/D^r) \tag{8}$$

in which  $h$  is head loss in pipe;  $L$ ,  $D$  and  $C$  are length, diameter and Hazen-Williams coefficient;  $Q$  is discharge in pipe; and  $K$ ,  $p$  and  $r$  are constants. For  $h$  and  $L$  in meters,  $Q$  in  $m^3/h$  and  $D$  in mm, the value of  $K$  is  $1.1466 \times 10^9$ . Value of  $p$  in the literature is considered as 1.85 or 1.852 (Savic and Walters, 1997) and that of  $r$  is 4.87. Herein, the value of  $p$  is taken as 1.85 in obtaining the performance of the network.

Ozger and Mays (2003) obtained the performance of system using NHFR of Figure 1 and given by Eqs. (1a-1c). Since a single HGL value is used in defining NHFR of Eqs. (1a-1c), they considered this HGL value as desirable HGL. Herein,  $H^{\min}$  is considered in Eqs. (1a-1c) as 15 m above

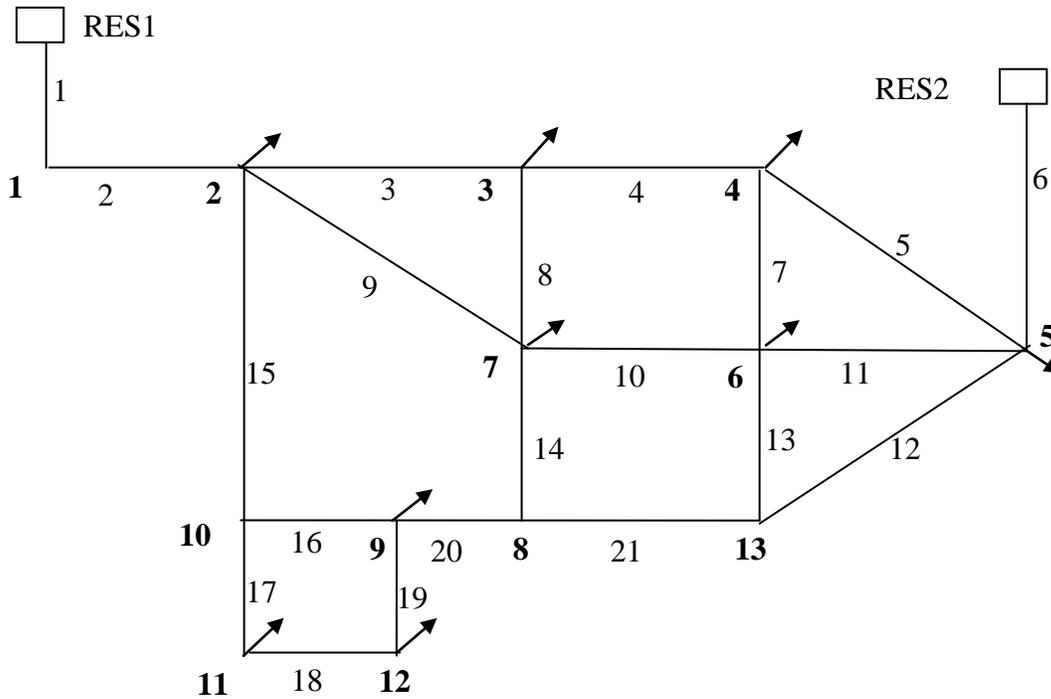
elevation and performance of network under failure of pipe 3 is obtained using Bhave's method. The available flow and available heads are given in columns 4 and 5 (Table 4), respectively. The deficient flows are shown by boldfaced.

**Table 3.** Pipe data for illustrative example II.

Pipe number	Length (m)	Diameter (mm)	HW coefficient
(1)	(2)	(3)	(4)
1	609.60	762	130
2	243.80	762	128
3	1524.00	609	126
4	1127.76	609	124
5	1188.72	406	122
6	640.08	406	120
7	762.00	254	118
8	944.88	254	116
9	1676.40	381	114
10	883.92	305	112
11	883.92	305	110
12	1371.60	381	108
13	762.00	254	106
14	822.96	254	104
15	944.88	305	102
16	579.00	305	100
17	487.68	203	98
18	457.20	152	96
19	502.92	203	94
20	883.92	203	92
21	944.88	305	90

**Table 4.** Node data and NFA results for failure of Pipe 3.

Node number	Elevation (m)	Demand (m <sup>3</sup> /h)	Using NHFRs (1a-1c)		Using NHFRs (3a-3c)	
			Available flow (m <sup>3</sup> /h)	Available HGL (m)	Available flow (m <sup>3</sup> /h)	Available HGL (m)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
RES1	60.96	0.00	-1165.70	60.960	-1293.64	60.960
RES2	60.96	0.00	-1214.49	60.960	-1404.87	60.960
1	27.43	0.00	0.00	60.587	0.00	60.508
2	33.53	212.40	212.40	60.433	212.40	60.322
3	28.96	212.40	212.40	46.861	<b>192.44</b>	41.895
4	32.00	640.80	<b>161.95</b>	47.000	<b>486.84</b>	41.933
5	30.48	212.40	212.40	50.446	212.40	47.195
6	31.39	684.00	<b>492.59</b>	46.390	<b>545.32</b>	41.938
7	29.56	640.80	640.80	46.550	<b>587.70</b>	42.655
8	31.39	327.60	<b>270.11</b>	46.390	<b>270.23</b>	42.628
9	32.61	0.00	0.00	53.451	0.00	51.547
10	34.14	0.00	0.00	54.871	0.00	53.433
11	35.05	108.00	108.00	51.582	<b>103.44</b>	49.108
12	36.58	108.00	<b>69.50</b>	51.580	<b>94.59</b>	48.874
13	33.53	0.00	0.00	48.360	0.00	44.693



**Fig. 7.** Network for illustrative example II.

Next, performance of this network is obtained using direct approach under failure of pipe 3 by considering NHFR as given by Eqs. (3a-3c). Value of exponent  $n$  is taken as 1.5. The  $H^{\min}$  value is taken as node elevation and  $H^{\text{des}}$  value is taken as 15 m above node elevation. Available nodal flows and HGL values are given in columns 6 and 7 (Table 4), respectively. The deficient flows are shown by boldface.

Following can be observed from Table 4.

1. There are no no-flow nodes and number of partial flow nodes obtained using NHFRs (1a-1c) are four; while that using NHFRs (3a-3c) are seven.

2. Total supply to network under failure of pipe 3 for NHFRs (1a-1c) and NHFRs (3a-3c) are 2380.19 m<sup>3</sup>/h and 2698.51 m<sup>3</sup>/h, respectively. Thus, available flows obtained by using NHFRs (1a-1c) are less as compared to that obtained by using NHFRs (3a-3c). This is because while using NHFRs (1a-1c) minimum HGL is taken as 15 m above node elevation to allow any flow. However, NHFRs (3a-3c) allowed outflows at pressure more than node elevation and at desirable head full demand is considered to be satisfied.

It should however be remembered that when nodal demands are lumped at the nodes of WDNs, the prediction of deficient-condition performance is rather approximate. While NHFRs (1a-1c) with minimum head at a node as desirable head provides lower bound on predicted flows, NHFRs (1a-1c) with minimum head equal to node elevation will provide upper bound on predicted flows. NHFRs that consider two heads predict performance in a better way and the direct method of solving requires less computational time and effort.

## SUMMARY AND CONCLUSIONS

NFA is a basic tool for obtaining performance of WDNs under deficient

conditions during reliability analysis. In NFA, node head-flow relationship is additionally satisfied along with usual node-flow continuity relationships and loop head loss relationships. Indirect approaches handle NHFRs externally and make use of traditional network solver. However, in the direct approaches NHFRs are considered simultaneously with node-flow continuity relationships and pipe-head loss relationships. Formulation of problem with unknown nodal heads (H equations) or unknown pipe discharges and nodal heads (Q-H equations) is best suited for direct approaches.

## REFERENCES

- Ang, W.K. and Jowitt, P.W. (2006). "Solution for water distribution systems under pressure-deficient conditions", *J. Water Resources Planning and Management, ASCE*, 132(3), 175-182.
- Bhave, P.R. (1981). "Node flow analysis of water distribution systems", *J. Transportation Engineering, ASCE*, 107(4), 457-467.
- Bhave, P.R. (1985). "Rapid convergence in hardy cross method of network analysis", *J. Indian Water Works Association*, 16(1), 1-5.
- Bhave, P.R. (1991). *Analysis of flow in water distribution networks*, Technomic Pub. Co., Lancaster, Pennsylvania, USA
- Bhave, P.R. (2003). *Optimal design of water distribution networks*, Alpha Science International Ltd., Pangbourne, England.
- Bhave, P.R. and Gupta, R. (2006). *Analysis of Water Distribution Networks*, Narosa Publishing House Pvt. Ltd., New Delhi, India.
- Chandapillai, J. (1991). "Realistic simulation of water distribution systems", *J. Transportation Engineering, ASCE*, 117(2), 258-263.
- Fujiwara, O. and Ganesharajah, T. (1993). "Reliability assessment of water supply systems with storage and distribution networks", *J. Water Resources Research*, 29(8), 2917-2924.
- Fujiwara, O. and Li, J. (1988). "Reliability analysis of water distribution networks in consideration of equity, redistribution, and pressure dependent demand", *J. Water Resources Research*, 34(7), 1843-1850.

- Germanopoulos, G. (1985). "A technical note on the inclusion of pressure dependent demand and leakage terms in water supply network models", *Civil Engineering Systems*, 2(3), 171-179.
- Giustolisi O. and Laucelli D. (2011). "Water distribution network pressure-driven analysis using the enhanced global gradient algorithm (EGGA)", *J. Water Resources Planning and Management, ASCE*, 137(6), 498-510.
- Gupta, R. and Bhave, P.R. (1994). "Reliability analysis of water distribution systems", *J. Environmental Engineering, ASCE*, 120(2), 447-460.
- Gupta, R. and Bhave, P.R. (1996a). "Reliability-based design of water distribution systems", *J. Environmental Engineering, ASCE*, 122(1), 51-54.
- Gupta, R. and Bhave, P.R. (1996b). "Comparison of methods for predicting deficient network performance", *J. Water Resources Planning and Management, ASCE*, 122(3), 214-217.
- Gupta, R. and Bhave, P. R. (2004). "Comments on 'redundancy model for water distribution systems' by P. Kalungi and T.T. Tanyimboh", *Reliability Engineering & System Safety*, 86(3), 331-333.
- Gupta, R. and Bhave, P.R. (2004). "Redundancy-based strengthening and expansion of water distribution networks", *Proceedings of 6<sup>th</sup> International Conference on Hydroinformatics*, Singapore.
- Gupta, R., Awale, A., Markam, A. and Bhave, P.R. (2005). "Node flow analysis of water distribution networks using gradient method", *Proceedings of National Conference on Advances in Water Engineering for Sustainable Development*, Indian Institute of Technology Madras, Chennai, 207-214.
- Jinesh Babu, K.S. and Mohan S. (2012). "Extended period simulation for pressure-deficient water distribution network", *J. Computing in Civil Engineering, ASCE*, 26(4), 498-505.
- Kalungi, P. and Tanyimboh T.T. (2003). "Redundancy model for water distribution systems", *Reliability Engineering and System Safety*, 82(3), 275-286.
- Ozger, S.S. and Mays, L.W. (2003). "A semi-pressure-driven approach to reliability assessment of water distribution networks", *Proceedings of 30<sup>th</sup> IAHR World Congress*, Thessaloniki, 345-352.
- Rossman, L.A. (2000). *EPANET user's manual*, Risk Reduction Engineering Laboratory, U.S. Environmental Protection Agency, Cincinnati.
- Tabesh, M., Tanyimboh, T.T. and Burrows, R. (2002). "Head-driven simulation of water supply networks", *International J. Engineering*, 15(1), 11-22.
- Tahar, B., Tanyimboh, T.T. and Templeman, A.B. (2002). "Pressure-dependent modelling of water distribution systems", *Proceedings of 3<sup>rd</sup> International Conference on Decision Making in Urban and Civil Engineering*, London.
- Todini, E. (2003). "A more realistic approach to the extended period simulation of water distribution networks", *Advances in Water Supply Management*, Maksimovic, C., Butler, D. and Memon, F.A. (Eds.), Swets and Zeitlinger Publishers, Balkema, Lisse, The Netherlands, 173-184.
- Wagner, J.M., Shamir, U. and Marks, D.H. (1988). "Water distribution reliability: simulation method", *J. Water Resources Planning and Management, ASCE*, 114(3), 276-294.
- Wu, Z.Y., Wang, R.H., Walski, T.M., Yang, S.Y., Bowdler, D. and Baggett, C.C. (2009). "Extended global-gradient algorithm for pressure-dependent water distribution analysis", *J. Water Resources Planning and Management, ASCE*, 135(1), 13-22.