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Safety Analysis of the Patch Load Resistance of Plate Girders: Influence of Model Error and Variability

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ABSTRACT: This study aims to undertake a statistical study to evaluate the accuracy of nine models that have been previously proposed for estimating the ultimate resistance of plate girders subjected to patch loading. For each model, mean errors and standard errors, as well as the probability of underestimating or overestimating patch load resistance, are estimated and the resultant values are compared one to another. Prior to that, the models are initially calibrated in order to improve interaction formulae using an experimental data set collected from the literature. The models are then analyzed by computing design factors associated with a target risk level (probability of exceedance). These models are compared one to another considering uncertainties existed in material and geometrical properties. The Monte Carlo simulation method is used to generate random variables. The statistical parameters of the calibrated models are calculated for various coefficients of variations regardless of their correlation with the random resistance variables. These probabilistic results are very useful for evaluating the stochastic sensitivity of the calibrated models.

Keywords: Calibration, Monte Carlo, Patch Loading, Plate Girder, Uncertainty.

INTRODUCTION

Slender steel plate girders are often subjected to concentrated loads commonly named as patch loading. Concentrated loads acting in the plane of the web are often resisted by web stiffeners. However, situations arise in which concentrated loads are applied through a relatively thin flange to unstiffened parts of the web e.g. wheel loads on crane gantry girders and roller loads during the launching of plate and box girder bridges (Roberts and Shahabian, 2000). Thus, the calculation of the patch load resistance of plate girders becomes of significant importance for economical and safety reasons.

During the past decades, various models have been proposed for computing the resistance of web panels of plate girders subjected to patch loading (see for example, Lagerqvist and Johansson, 1996; Graciano and Johansson, 2003; Davaine and Aribert, 2005; Graciano and Casanova, 2005; Chacón et al., 2011 and 2012). Because of the complexity of the behavior of the web element, most of the patch load resistance models have been developed based on the

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empirical or semi-empirical approach. To make an empirical formula reliable, it has to be calibrated or verified using a wide range of experimental conditions. Since limited experimental conditions have been used in the development of most of these models, the value of associated coefficients in these models may not be optimal. More precisely, their values could have been different if identified on a wider experimental set. Therefore, the coefficients in all models should be calibrated prior to the application of the models (Rattanapitikon, 2007; McCabe et al., 2005).

The need to incorporate uncertainties in an engineering design has long been recognized. One can identify the following sources of uncertainty (Der Kiureghian and Ditlevsen, 2009):

1. Uncertainty inherent in geometry, material properties, boundary conditions, and so on.
2. Uncertainty resulting from the type of the probabilistic distribution selected for input variables.
3. Uncertainty resulting from physical models selected.
4. Uncertainty resulting from the measurement of observations.

Probabilistic analysis allowing the estimation of the reliability of a design, considers the stochastic variability of the data (Paola, 2004). Direct Monte Carlo simulation technique is the most prevalent probabilistic approach when the complexity of the problem prevents the development of analytical modelling. Radlinska et al. (2007) demonstrated how variability would influence model predictions. They used the Monte Carlo approach to quantify the level of uncertainty resulted from material variability in the model predictions.

Sensitivity analysis is the study of how the variations in the output of a model (load carrying capacity, stress state, etc.) can be apportioned, qualitatively or quantitatively, to different sources of variation, and of how a given model depends upon the quality of the information fed into it. Sensitivity analysis can be generally divided into two groups: (i) deterministic sensitivity analysis and (ii) stochastic sensitivity analysis (Kala and Kala, 2010). Compared to the deterministic sensitivity analysis (parametric study), the stochastic sensitivity analysis provides more extended information about the problem under study.

For instance, the stochastic sensitivity analysis was performed to study the resistance of thin-walled steel members (Kala, 2005). Similarly, an imperfection sensitivity analysis of plate girder webs subjected to patch loading was conducted by Graciano et al. (2011).

The study of uncertainty effects resulted from the variation in material and geometrical properties on the performance of patch load resistance models may be considered as an original topic, which can potentially furnish a ground to have a safe design for plate girders subjected to patch loading. To the best of authors' knowledge, no previous work has been conducted on the assessment of patch load resistance models involving randomness in system parameters.

Considering the above, this study aims to evaluate the accuracy of nine existing models in estimating the resistance of plate girder webs subjected to patch loading. The experimental results collected from the literature are used to examine the models. The first step in this study is to compute the model calibration factors to enhance the accuracy of model predictions in estimating the patch load resistance of plate girders on the full data set. To evaluate the accuracy of the models for the preliminary design purposes, the calibrated models are analyzed and design ratios and design factors associated with a specific probability of exceedance (*i.e.* risk level) are computed. By obtaining "design models" their efficiency/quality in terms of economical consequences is compared one to another. Figure 1 shows a schematic procedure required in the solution of the problem.

Given that deterministic analysis cannot provide complete information about the accuracy of the models, probabilistic analysis is used to evaluate the performance of the calibrated models in the stochastic field. To do so, the material and geometrical properties of plate girder tests are supposed to be uncertain and have a Gaussian or Log-normal distribution.

The models are finally compared in this stochastic frame, where material and geometrical parameters are considered as random variables. Monte Carlo simulation is used with different Coefficients of Variations (COVs) considering correlations between the random variables. To evaluate the performance of the calibrated patch load resistance models, the stochastic sensitivity analysis of these models is conducted to obtain the probabilistic results for the random resistance variables.

PATCH LOADING

During past decades, numerous tests have been performed by several researchers to provide a better understanding of material and geometric parameters on failure mode of plate girders subjected to patch loading (Figure 2). After extensive theoretical and experimental investigations (Roberts and Rockey, 1979; Kutmanova and Skaloud, 1992; Markovic and Hajdin, 1992; Roberts and Newark, 1997) it has been concluded that the patch load resistance of plate girders (P_u) may depend on the web thickness (t_w), web depth (d_w), web width (b_w), flange thickness (t_f), flange width (b_f), load length (c), Young's modulus (E), web yield stress (σ_w) and flange yield stress (σ_f). However, recent research works performed by Chacón et al. (2010) showed the flange yield stress (σ_f) does not play a mechanical role in the resistance to patch loading.

A number of different models have been proposed for predicting the resistance of plate girders subjected to patch loading, as seen in Table 1.

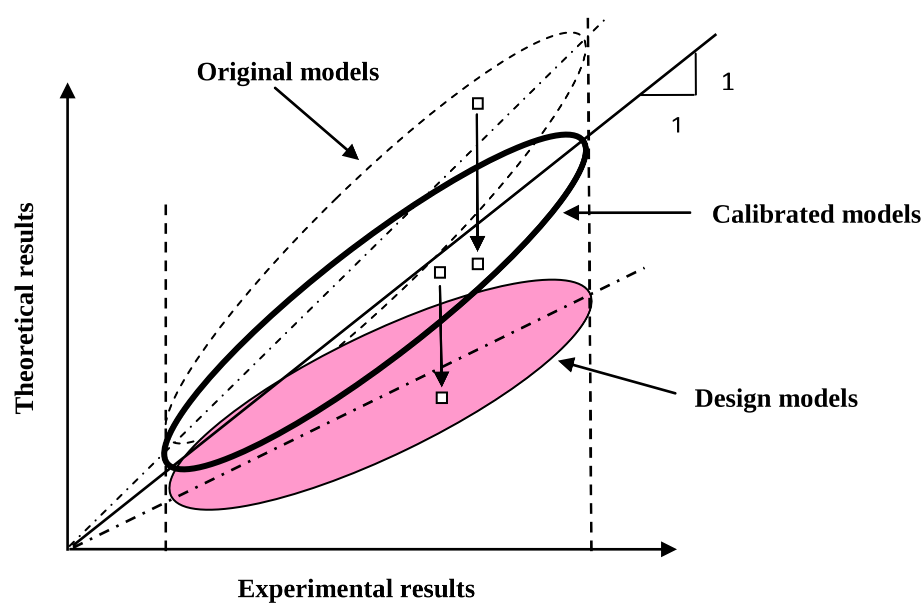

Fig. 1. Schematic procedure of the study.

Table 1. Models for patch load resistance of plate girders.

Formula	Ref.	Number of variables
$Pu_1 = 9000 t_w^2$	(Markovic and Hajdin, 1992)	1 (t_w)
$Pu_2 = t_w^2 \sqrt{E \sigma_w}$	(Markovic and Hajdin, 1992)	3 (t_w, E, σ_w)
$Pu_3 = 0.75 t_w^2 \sqrt{E \sigma_w \frac{t_f}{t_w}}$	Roberts and Rockey, 1979)	4 (t_w, E, σ_w, t_f)
$Pu_4 = 0.55 t_w^2 \sqrt{E \sigma_w \frac{t_f}{t_w}} \left(0.9 + 1.5 \frac{c}{d_w}\right)$	(Markovic and Hajdin, 1992)	6 (t_w, E, σ_w, t_f, c, d_w)
$Pu_5 = 10000 t_w^2 \left(1.2 + \frac{5 I_f}{4 I_w} \frac{d_w}{t_w} \left(1 + \frac{c}{d_w}\right)^2 \left(0.85 + \frac{b_w}{100 d_w}\right)\right)$	Roberts and Rockey, 1979)	6 (t_w, I_f, I_w, c, d_w, b_w)
$Pu_6 = 19.45 t_w^2 \sigma_w \left(1 + 0.004 \frac{c}{t_w}\right) \left(\frac{I_f}{t_w^4}\right)^{0.1}$	(Markovic and Hajdin, 1992)	4 (t_w, I_f, c, σ_w)
$Pu_7 = 12.6 t_w^2 \sigma_w \left(1 + 0.004 \frac{c}{t_w}\right) \left(\left(\frac{I_f}{t_w^4}\right) \sqrt{\frac{\sigma_f}{240}}\right)^{0.153}$	(Kutmanova and Skaloud, 1992)	5 (t_w, σ_w, I_f, c, σ_f)
$Pu_8 = 0.5 t_w^2 \sqrt{E \sigma_w \frac{t_f}{t_w}} \left(1 + \frac{3c}{d_w} \left(\frac{t_w}{t_f}\right)^{1.5}\right)$	(Markovic and Hajdin, 1992)	6 (t_w, E, σ_w, t_f, c, d_w)
$Pu_9 = 1.1 t_w^2 \sqrt{E \sigma_w} \left(\frac{t_f}{t_w}\right)^{0.25} \left(1 + \frac{(c + 2t_f) t_w}{d_w t_f}\right)$	(Roberts and Newark, 1997)	6 (t_w, E, σ_w, t_f, c, d_w)

Since the number of model parameters varies between one (model 1) and six (models 4, 5, 8 and 9), the complexity of relationship given in Table 1 is variable. It can be expected that the higher the number of parameters, the better the model's ability to fit an experimental data set. However, the "practical cost", due to the necessity of identifying input parameters, as well as the sensitivity to uncertainty also depends on the number of parameters.

EXAMINATION OF EXISTING MODELS

In order to evaluate the accuracy of the existing models to estimate the patch load resistance of plate girders, the results computed using the models (P_u) are compared with the results obtained from the experiments (P_{ex}). An extensive literature survey has been performed for experimental results of plate girders subjected to patch loading (Roberts and Rockey, 1979; Kutmanova and Skaloud, 1992; Markovic and Hajdin, 1992; Roberts and Newark, 1997). In calculating the theoretical patch load resistance of plate girders, determined in accordance with the existing models, the mean of Young's modulus E was taken as 205 GPa.

For each model j (j = 1, 2, …, 9) and each experiment i (i = 1, 2, …, 116), the ratio $r_{j,i} = \dfrac{P_{u\,j,i}}{P_{ex_i}}$ is calculated. In order to further assess the accuracy of the models, from the 9×116 data set of $r_{j,i}$ values arithmetic mean m, standard deviation s, average relative error $\overline{Er}$ (Eq. (1)), R-squared value R^2 and root mean square error $RMSE$ (Eq. (2)) are computed for each model. The results are presented in Table 2.

$$\overline{Er} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\left|P_{ex_i} - P_{u\,j,i}\right|}{P_{ex_i}}\right)\times 100$$

(j = 1, 2, …, 9) (1)

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(P_{ex_i} - P_{u\,j,i}\right)^2}$$

(j = 1, 2, …, 9) (2)

where n is the number of experiments, which in this case is 116.

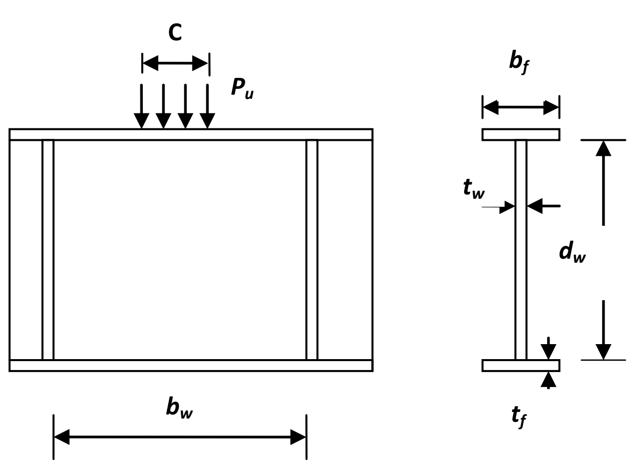

Fig. 2. Patch loading and girder dimensions.

Table 2. Statistical parameters of the patch load resistance models.

Model ratio	$\frac{P_{u1}}{P_{ex}}$	$\frac{P_{u2}}{P_{ex}}$	$\frac{P_{u3}}{P_{ex}}$	$\frac{P_{u4}}{P_{ex}}$	$\frac{P_{u5}}{P_{ex}}$	$\frac{P_{u6}}{P_{ex}}$	$\frac{P_{u7}}{P_{ex}}$	$\frac{P_{u8}}{P_{ex}}$	$\frac{P_{u9}}{P_{ex}}$
Arithmetic Mean *m*	0.84	0.67	0.95	0.76	1.26	0.86	0.75	0.68	1.05
Standard Deviation *s*	0.19	0.13	0.16	0.14	0.24	0.13	0.15	0.09	0.14
Slope of regression	0.80	0.66	0.96	0.76	1.21	0.86	0.74	0.68	1.06
$\overline{Er}$ *(%)* Eq. (1)	20	34	14	25	29	16	26	32	13
R-squared value	0.81	0.85	0.92	0.91	0.88	0.90	0.83	0.94	0.92
RMSE Eq. (2)	24.07	33.29	13.23	24.22	26.87	17.66	27.71	30.71	14.77

Arithmetic mean is a measure of the bias of each model towards underestimating or overestimating. Values larger than one represent a tendency to overestimate patch load resistance and values smaller than one correspond to a tendency to underestimate the patch load resistance.

The results presented in Table 2 indicate that the model P_{u9} with the m = 1.05, s = 0.14, $\overline{Er}$ = 13%, r^2 = 0.92, *rmse* = 14.77 and *slope* = 1.06 predicts the patch load resistance of plate girders with more accuracy than the other models. However, this comparison does not show that anything about the safety resulting from the model selection.

MODEL CALIBRATION

Conventionally, calibration involves adjusting model parameters in order to closely match model output to some observed system behavior, with the aim of reducing parameter uncertainty and increasing the accuracy of state variable characterization. Because most models were developed based on a limited range of experimental conditions, the value of the coefficients in these models may not be optimal for a wider range (Rattanapitikon, 2007; McCabe et al., 2005). This is also observed in references presented by Kuhlmann et al. (2012) for other statistical calibrations related to patch loading. Therefore, the errors in Table 2 should not be used to judge the applicability of the selected models. The coefficients in all models should be calibrated before comparing the applicability of the models.

The data shown in Table 2 are used to calibrate the models using a simple multiplying factor. The linear regression model can be written as follows:

$$P_{u-cal} = k_{cal} \times P_u \qquad (3)$$

where P_{u-cal} is the calibrated patch load resistance model, P_u is the existing patch load resistance model and k_{cal} is the calibration factor.

Calibration can be carried out using arithmetic mean $(k_{cal} = \frac{1}{m})$ (method 1) or slope of regression line obtained based on the *R*-squared value $(k_{cal} = \frac{1}{slope})$ (method 2) for each model. These two options lead to very close results. For the best calibrated model (obtained with either method 1 or method 2), arithmetic mean, standard deviation, average relative error, *R*-squared value and root mean square error are computed and shown in Table 3.

The results can be summarized as follows:

(a) After calibration, the accuracy of most models has been improved significantly whereas the accuracy of models 3, 6 and 9 was slightly improved. This confirms that the coefficients in the existing models were not of the optimal values.

(b) The overall accuracy of the calibrated models in decreasing order are the models 9, 8, 6, 3, 4, 2, 5, 7 and 1. This order is more or less is the inverse of that of the number of parameters.

(c) Considering the overall accuracy of all models in Table 3, it can be concluded that most of all calibrated models can be used for practical work. The model that gives the best prediction (with $\overline{Er}$ of 11%) is Model 9.

Table 3. Statistical parameters of the calibrated models (using method 1).

Model ratio	$\frac{P_{u1-cal}}{P_{ex}}$	$\frac{P_{u2-cal}}{P_{ex}}$	$\frac{P_{u3-cal}}{P_{ex}}$	$\frac{P_{u4-cal}}{P_{ex}}$	$\frac{P_{u5-cal}}{P_{ex}}$	$\frac{P_{u6-cal}}{P_{ex}}$	$\frac{P_{u7-cal}}{P_{ex}}$	$\frac{P_{u8-cal}}{P_{ex}}$	$\frac{P_{u9-cal}}{P_{ex}}$
Calibrated factor Method 1	1.19	1.50	1.05	1.32	0.79	1.16	1.33	1.48	0.95
Calibrated factor Method 2	1.25	1.50	1.04	1.31	0.82	1.16	1.35	1.47	0.95
Arithmetic mean	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Standard deviation	0.23	0.21	0.17	0.18	0.22	0.16	0.20	0.14	0.14
$\overline{Er}$ (%) Eq. (1)	18	16	14	15	16	13	17	12	11
R-squared Value (*)	0.81	0.85	0.92	0.91	0.88	0.90	0.83	0.94	0.92
RMSE Eq. (2)	19.20	18.16	13.13	13.50	14.87	13.98	18.84	11.42	13.07

(*) unchanged by calibration

SAFETY ANALYSIS AND DESIGN MODELS

Even when the accuracy is "good" (*i.e.* low value for *s*, $\overline{Er}$, *RMSE* and high value for R^2), about half of all predictions overestimate the resistance. In some cases, the difference between experiment and model may be large. The need to use a model, in design standards, with safety requirements is the reason why a careful attention must be paid to the statistical distribution of $r_{j,i}$ values.

The statistical distribution of the ratio $\frac{P_{u-cal}}{P_{ex}}$ and the relative error $(\frac{P_{u-cal}}{P_{ex}}-1)$ for all models are investigated. The suitability of the normal distribution to represent the ratio $\frac{P_{u-cal}}{P_{ex}}$ and the relative error is checked using Kolmogorov-Smirnov and χ^2 (chi-square) tests. All calibrated models can be fitted with the Gaussian distribution. For instance, the cumulative distribution function of ratio $\frac{P_{u-cal}}{P_{ex}}$ and relative error for the calibrated model 9 are shown in Figures 3 and 4. As it can be seen, the empirical CDF shows perfect agreement with normal CDF.

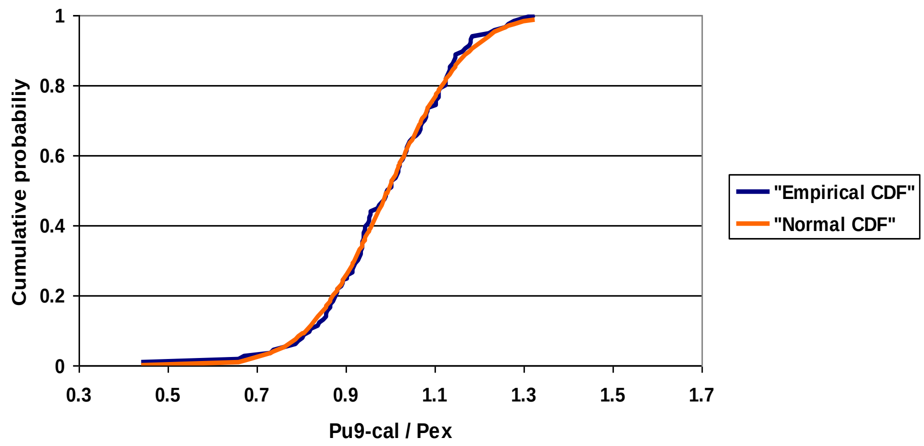

Fig. 3. Cumulative distribution function of the $\frac{P_{u9-cal}}{P_{ex}}$ ratio.

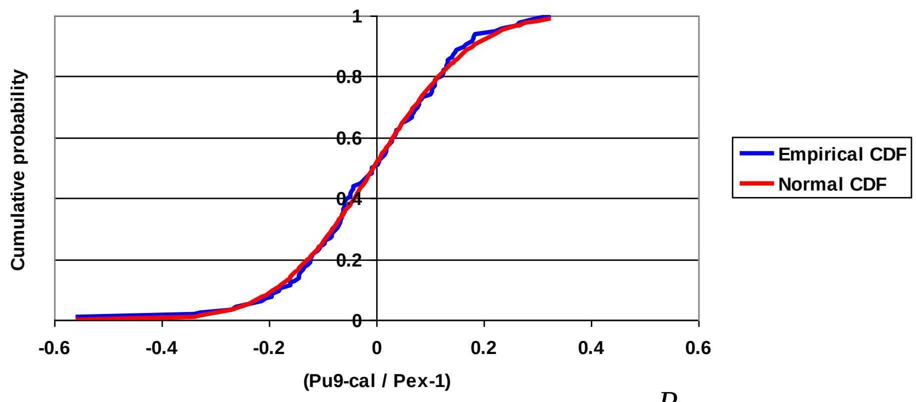

Fig. 4. Cumulative distribution function of the relative error $(\frac{P_{u9-cal}}{P_{ex}}-1)$.

Another way to assess the performance of calibrated models is by computing design factors (γ_d) associated with specific percentiles. A percentile corresponds to a $(\frac{P_{u-des}}{P_{ex}})$ value that corresponds to a specific risk level, *i.e.* to a specified probability of exceedance.

For satisfying safety requirements, one has to check that the value predicted by the "model for design" is resulted from a specified (usually reasonably low) target probability, p_{target}, larger than the experimental value:

$$p(P_{u-des} \succ P_{ex}) \leq p_{target} \tag{4}$$

The cumulative distribution F for random variable X is defined as follows:

$$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)dx \tag{5}$$

For example, Figure 5 shows the cumulative distribution of $(\frac{P_{u-cal}}{P_{ex}})$ of Model 9, with $p_{target} = 1 - F_P(\frac{P_{u-des}}{P_{ex}})$. If one chooses a given p_{target} value, for instance 5%, the identification of the relevant P_{u-cal} is therefore straightforward. The design factor is the value of the P_{u-cal} / P_{ex} variable corresponding to this target probability. The design factor is the scalar γ_d (larger than 1, since all models had been calibrated in the first step, the model value must be divided in order to fulfill the target probability.

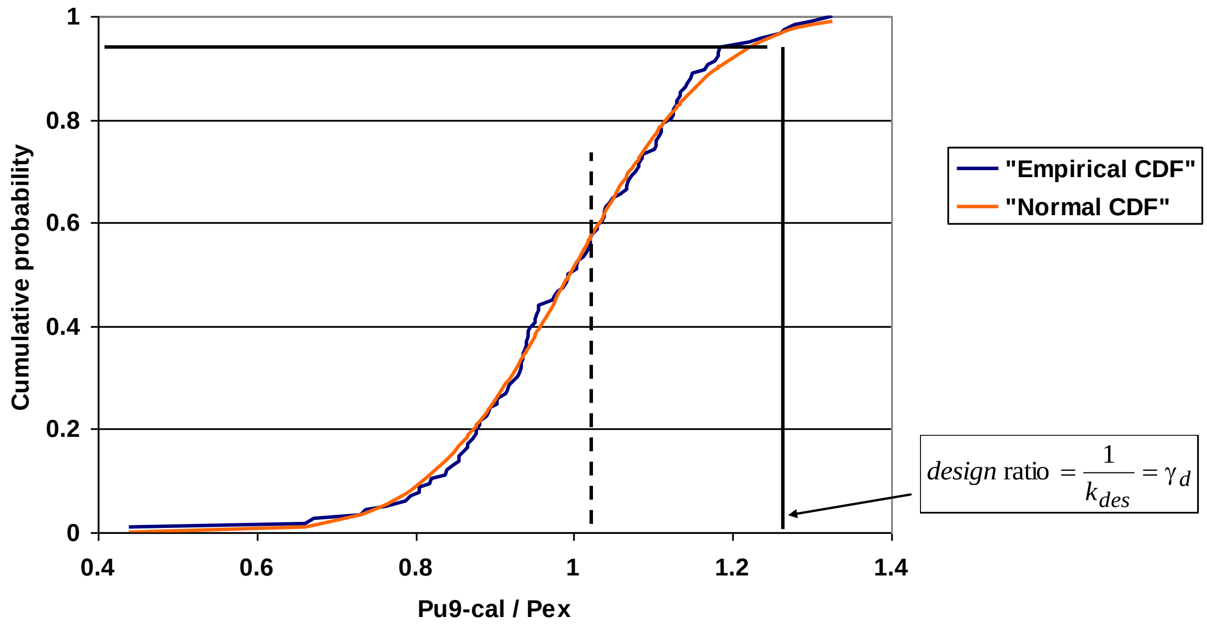

Fig. 5. Design factor for the calibrated model 9.

Using the Gaussian distribution, design ratios ($\frac{P_{u-des}}{P_{ex}}$) and design factors (γ_d) are computed for a 5% probability of exceedance. The design model P_{u-des} is related to the calibrated model P_{u-cal} as follows:

$$p_{u-des} = \frac{p_{u-cal}}{\gamma_d} \quad (6)$$

One can also write

$$k_{des} = \frac{1}{\gamma_d} = \frac{1}{F_p^{-1}(1-p_{t\arg et})} = \frac{1}{F_p^{-1}(0.95)} \quad (7)$$

From Eqs. (4), (6) and (7), it comes $p(\,p_{u-cal} \times k_{des} \succ P_{ex}\,) \le p_{t\,arg\,et}$ and

$$p(\frac{P_{u-cal}}{P_{ex}} \succ \frac{1}{k_{des}} = \gamma_d\,) \le p_{t\,arg\,et} \quad (8)$$

as illustrated in Figure 5.

The design patch load resistance is finally written as follows:

$$P_{u-des} = k_{cal} \times k_{des} \times P_u \quad (9)$$

The computed design factors for the nine models are presented in Table 4. It can be seen that the calibrated models 8 and 9 have lower factors. This means that they are "more reliable" than other models for estimating the patch load resistance of plate girders. For practical purposes, it must be pointed that the smaller the γ_d value, the lighter the structure will be for a target safety level. The quality of the model as a direct impact on construction costs.

EFFECT OF VARIABILITY ON RESISTANCE VARIABLES

Sensitivity analysis has been widely applied in engineering design to explore the model response behavior, to evaluate the accuracy of a model, to test the validity of the assumptions made, etc. In deterministic design, sensitivity analysis is used to find the rate of change in a model output due to changes in the model inputs. That is usually performed by varying input variables one at a time near a given central point, which involves partial derivatives and is often called deterministic sensitivity analysis.

When uncertainty is considered, sensitivity analysis has different meanings. We assume that the uncertainty in a design performance can be described probabilistically by its mean (μ), variance (σ^2), or, more generally by the probability density function (PDF) or the cumulative distribution function (CDF), etc. Correspondingly, the sensitivity analysis under uncertainty needs to be performed on the stochastic characteristics of a model response with respect to the stochastic characteristics of model inputs (Liu and Chen, 2004).

Table 4. Design factor γ_d for the calibrated models.

Model ratio	$\frac{P_{u1-cal}}{P_{ex}}$	$\frac{P_{u2-cal}}{P_{ex}}$	$\frac{P_{u3-cal}}{P_{ex}}$	$\frac{P_{u4-cal}}{P_{ex}}$	$\frac{P_{u5-cal}}{P_{ex}}$	$\frac{P_{u6-cal}}{P_{ex}}$	$\frac{P_{u7-cal}}{P_{ex}}$	$\frac{P_{u8-cal}}{P_{ex}}$	$\frac{P_{u9-cal}}{P_{ex}}$
Design factor	1.37	1.34	1.28	1.30	1.37	1.26	1.33	1.23	1.23

Table 5. Statistics of random resistance variables.

Variable	Symbol	Distribution	Coefficient of variations (COV)
Web thickness	t_w	Normal	0.03
Web depth	d_w	Normal	0.03
Web width	b_w	Normal	0.03
Flange thickness	t_f	Normal	0.03
Flange width	b_f	Normal	0.03
Load length	c	Normal	0.03
Young's modulus	E	Log-normal	0.03
Web yield stress	σ_w	Log-normal	0.07
Flange yield stress	σ_f	Log-normal	0.07

Safety in construction standards, e.g. Eurocodes, is accounted for by recognizing three main sources of uncertainty and errors, respectively those in the load definition, in the material properties and in modelling. In previous sections, the focus was given on model uncertainties. Each model requires input data relative to geometrical and material properties. Thus, for a given model, any uncertainty on these data will have additional effects on the distribution of P_u and then $P_{u\text{-}cal}$.

The uncertainty in material properties can be represented by means of random variables. This includes the assumption of a particular probability distribution model. In general, it is the response to static and time dependent material loading that matters for structural design (Chaves et al., 2010). Table 5 shows the parameters and distributions of random resistance variables considered in this paper (JCSS, 2001-2).

To investigate the performance of the calibrated models to predict the patch load resistance of plate girders, the Monte Carlo simulation method is used in which over 1000 samples are generated for each set of experimental result Pex_i (i = 1, 2, 3, …, 116) and each model Pu_j (j = 1, 2, …, 9).

The probabilistic distribution of $\frac{P_{u-cal-st}}{P_{ex}}$ ratio for all models (1000 simulations for each of 9 models and each of 116 experiments) can be built. The suitability of the normal distribution to represent $\frac{P_{u-cal-st}}{P_{ex}}$ ratio is checked: all calibrated models can be represented by the Gaussian distribution. For instance, the cumulative distribution function of $\frac{P_{u-cal-st}}{P_{ex}}$ ratio for the calibrated model 9 is shown in Figure 6.

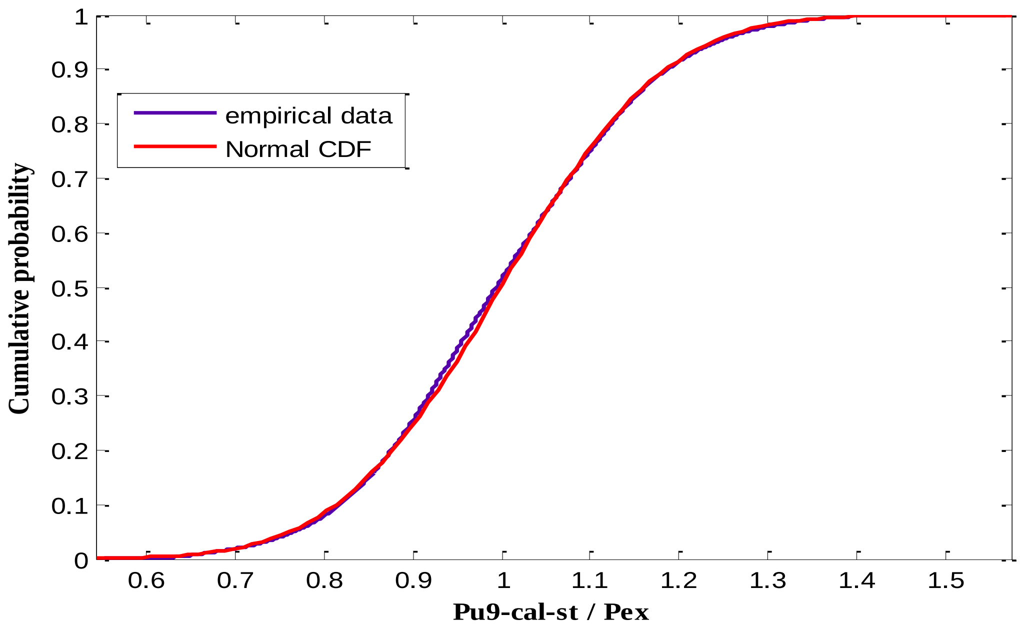

Fig. 6. Cumulative distribution function of the ratio $\dfrac{P_{u9-cal-st}}{P_{ex}}$ (based on 1000 simulations for each of 116 experiments).

The average error ($\overline{Er}$) for each model is calculated as follows:

$$\overline{Er} = \frac{1}{116}\sum_{i=1}^{116} Er_i \tag{10}$$

in which

$$Er_i = \frac{1}{1000}\sum_{k=1}^{1000}\left(\frac{\left|P_{ex_i} - P_{u-cal-st_{j,k}}\right|}{P_{ex_i}}\right)\times 100 \quad (j = 1, 2, \ldots, 9) \tag{11}$$

where P_{u_i} is the estimated resistance by each model for the *i*-th sample of the generated random variables and $P_{ex\,j}$ is the experimental result for the *j*-th number of experiment.

A summary of stochastic results for the models is shown in Table 6. It is interesting to note, by comparing Table 6 with Table 3, that the results are quite similar. Thus, accounting for material variability in addition to model error has only few consequences, the latter being predominant.

Regarding safety requirements, the same method was followed in order to identify design factors. Using the Gaussian distribution, design factors (γ_d) for the calibrated models are computed for a risk level of 5% probability of exceedance. Figure 7 presents the design factor for Model 9.

Table 6. Summary of stochastic results for the calibrated models.

Model ratio	$\frac{P_{u1-cal-st}}{P_{ex}}$	$\frac{P_{u2-cal-st}}{P_{ex}}$	$\frac{P_{u3-cal-st}}{P_{ex}}$	$\frac{P_{u4-cal-st}}{P_{ex}}$	$\frac{P_{u5-cal-st}}{P_{ex}}$	$\frac{P_{u6-cal-st}}{P_{ex}}$	$\frac{P_{u7-cal-st}}{P_{ex}}$	$\frac{P_{u8-cal-st}}{P_{ex}}$	$\frac{P_{u9-cal-st}}{P_{ex}}$
Average error	19	16	14	15	16	13	17	12	12
Standard deviation	0.23	0.21	0.17	0.18	0.22	0.16	0.20	0.14	0.14
Mean standard deviation (material variability)	0.06	0.06	0.05	0.05	0.06	0.05	0.04	0.05	0.05

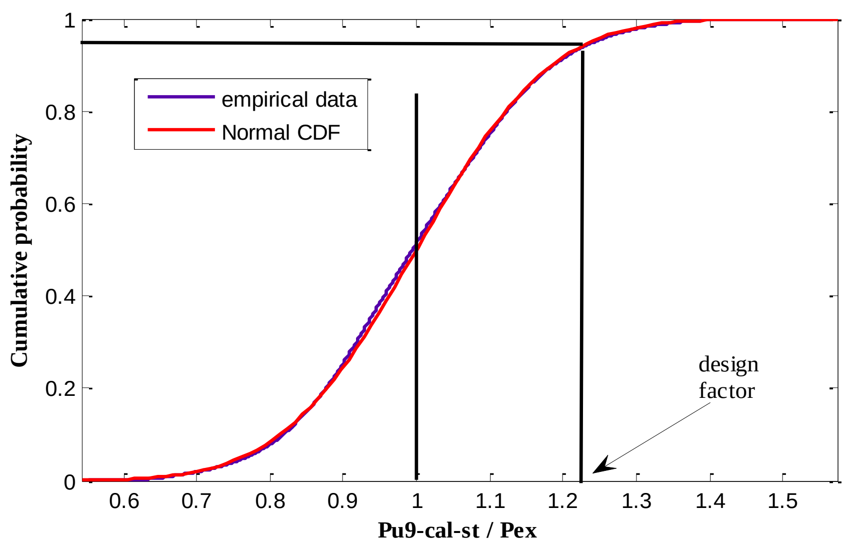

Fig. 7. Design factor for the calibrated model 9 considering uncertainties.

The computed design factors for all calibrated models are presented in Table 7. It can be seen that the amount of COVs (3-7%) does not affect the results significantly and that the results are quite similar to those in Table 4.

Table 7. Design factors for the calibrated models considering uncertainties.

Model ratio	$\frac{P_{u1-cal-st}}{P_{ex}}$	$\frac{P_{u2-cal-st}}{P_{ex}}$	$\frac{P_{u3-cal-st}}{P_{ex}}$	$\frac{P_{u4-cal-st}}{P_{ex}}$	$\frac{P_{u5-cal-st}}{P_{ex}}$	$\frac{P_{u6-cal-st}}{P_{ex}}$	$\frac{P_{u7-cal-st}}{P_{ex}}$	$\frac{P_{u8-cal-st}}{P_{ex}}$	$\frac{P_{u9-cal-st}}{P_{ex}}$
Design factor	1.38	1.34	1.28	1.31	1.33	1.26	1.33	1.24	1.24

Table 8. Probabilistic results for the calibrated models for "COV = 20%" without any correlation between the random variables.

Model ratio	$\frac{P_{u1-cal-st}}{P_{ex}}$	$\frac{P_{u2-cal-st}}{P_{ex}}$	$\frac{P_{u3-cal-st}}{P_{ex}}$	$\frac{P_{u4-cal-st}}{P_{ex}}$	$\frac{P_{u5-cal-st}}{P_{ex}}$	$\frac{P_{u6-cal-st}}{P_{ex}}$	$\frac{P_{u7-cal-st}}{P_{ex}}$	$\frac{P_{u8-cal-st}}{P_{ex}}$	$\frac{P_{u9-cal-st}}{P_{ex}}$
Average error	37	35	28	29	36	27	27	28	30
Standard deviation	0.47	0.45	0.35	0.37	0.61	0.34	0.34	0.36	0.39
Design factor	1.82	1.76	1.57	1.61	2.08	1.55	1.54	1.59	1.64

In order to further assess the performance of the calibrated models in the stochastic field, the probabilistic parameters of each model have also been computed considering a very high level of variability for "COV = 20%". The results are presented in Table 8.

As it can be seen in Table 8, there is a significant difference between the results for the variables with "COV = 20%" and those obtained with a deterministic model (Table 6). In practice, such a high level of COV is not encountered that is to quality control and the amounts of variability of 3-7% are not sufficient to have a significant influence on ultimate load, because of yet existing model errors.

CONCLUSIONS

The behavior of plate girders subjected to patch loading represents complex stability and elastoplastic problems. Some empirical and semi-empirical formulas were established, but they still present significant errors when compared to experimental results.

Uncertainties are usually incorporated in an engineering design; particularly uncertainties resulting from the selection of the analytical-physical model or those inherent to geometry and material properties. A stochastic-based comparison has been carried out between plate girders subjected to patch loading and a number of considering geometrical and material properties randomness.

The accuracy of nine models in estimating the patch load resistance of plate girders has been evaluated. The verified results have been presented in terms of arithmetic mean, standard deviation, average relative error, $\overline{Er}$, *R*-squared value and root mean square error. Because most of the existing models were developed based on the limited experimental conditions, the models might not be the optimal ones. Therefore, all models have been calibrated for comparing the applicability of the models. The comparison has shown that most of the calibrated models can be used for computing the average patch load resistance, with an average relative error of about 11-16%. Model 9 provided the best predictions (with $\overline{Er}$ of 11%).

In order to evaluate the safety performance of the calibrated models, a target risk level of 5% probability of exceedance has been chosen. The design factor γ_d by which the model value must be divided in order to fulfill the target probability has been computed for all nine models. The comparison of the computed design factors has shown that calibrated models 8 and 9 are more reliable than the other models with a partial safety factor equal to 1.24.

In a separate step, uncertainties existed in geometry and material properties have been considered. To investigate the performance of the existing plate girders resistance models in this situation, the Monte Carlo simulation method has been used, in which over 1000 samples were generated for each set of models and experiment results. The average relative error, standard deviation and the design factor (i.e. the partial safety factor) have been calculated for all models and compared. Stochastic results showed that the material variability, in addition to model error, has only few consequences: there is no any significant difference between the stochastic results and the deterministic ones. It is only when a very large degree of material/geometrical variability is considered that the stochastic results differ from the deterministic ones. However, since these levels do not correspond to those encountered in practice, this has no real consequence on the design process.

This study confirms that attention should be paid when using empirical models that estimate patch load resistance of plate girders from a deterministic point of view. It is however possible, after calibration and analysis of model error, to calculate the partial design factor corresponding to the model error and to compute the design value in order to obtain the required level of safety, in agreement with semi probabilistic design codes.

This study furnishes a ground to consider uncertainties to be incorporated in an engineering design and particularly uncertainties resulting from selection of the analytical-physical models or those inherent to geometry and material properties. The method proposed herein could be used by the Civil Infrastructures Community to evaluate the available models as well as the present world-wide design codes, which are of great interest for practice engineers, constructors, researchers and bridge owners.

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