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An Enhanced HL-RF Method for the Computation of Structural Failure Probability Based On Relaxed Approach

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ABSTRACT: The computation of structural failure probability is vital importance in the reliability analysis and may be carried out on the basis of the first-order reliability method using various mathematical iterative approaches such as Hasofer-Lind and Rackwitz-Fiessler (HL-RF). This method may not converge in complicated problems and nonlinear limit state functions, which usually shows itself in the form of periodic, bifurcation and chaos solution. In this paper, the HL-RF method has been improved based on the relaxed method to overcome these numerical instabilities. An appropriate relaxed coefficient has been defined, ranging between 0 and 1, to enhance the HL-RF method. This coefficient can be computed using the information from the new and previous iterations of the HL-RF algorithm based on second-order fitting. Capability, robustness and efficiency of the proposed algorithm have been studied by results of several examples compared to the HL-RF. Results illustrated that the proposed method is more efficient and robust in the computation of the failure probability compared to the HL-RF method.

Keywords: Failure Probability, First-Order Reliability Method, HL-RF Method, Relaxed Method, Reliability Index.

INTRODUCTION

In engineering practices, structural performances may exhibit uncertainties in relation to material properties, environmental loads and geometrical dimensions. These uncertainties in structural systems are modelled as design random variables; the probabilistic reliability theory can provide a powerful methodology to take into account those uncertainties in evaluating failure probability. Physical phenomena can be subjected to reliability analysis with the help of an idealized mathematical relation between resistance (R) and load (S) (Nowak and Collins, 2000). These mathematical models are known as limit state or performance functions (G=R-S). Such a function is a combination of basic random variables that may be determined based on the function itself and the failure probability can be calculated by (Luo et al., 2009):

$$P_f = P[G(U) < 0] = \int_{G(U)\le 0} f_X(X)dX \qquad (1)$$

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where, P_f is the structural failure probability, $G(U)$ is the limit state function in the normal standard space, $g(X) \leq 0$ defines the failure region of the structural components (*S*>*R*) and f_X is the Joint Probability Density Function (JPDF) for the basic random variables *X*.

In probabilistic theories, a closed form solution of the above equation is less likely since it involves multiple integrals in addition to the JPDFs of the random variables for general cases (Santosh et al., 2006). Approximate solutions of the failure probability for this type of problem are found by simulation methods such as: the Monte Carlo simulation, the importance sampling and the quasi-Monte Carlo (Latin hypercube sampling) simulation (Nowak and Collins, 2000; Choi et al., 2007). These methods are time-consuming in estimating low failure probabilities due to a high number of simulations undertaken (Naess et al., 2009). Moment methods such as: the First-Order Second-Moment (FOSM), the Second-Order Second-Moment (SOSM) and the Mean Value First-Order Second-Moment method (MVFOSM) are for the estimation of failure probability of the explicit limit state functions based on the reliability index (Choi et al., 2007; Naess et al., 2009).

$$P_f = 1 - \Phi(\beta) = \Phi(-\beta) \quad , \quad \beta = \frac{\mu_G}{\sigma_G} \qquad (2)$$

where, Φ is the standard normal cumulative distribution function, β is the reliability index, μ_G and σ_G are mean and standard deviation of the limit state function G, respectively. The moment methods do not consider an appropriate probability density function for the basic random variables (Nowak and Collins, 2000). Mathematical iterative methods e.g. the First-order reliability method (FORM) have been established for estimation of failure probability (Choi et al., 2007; Hasofer and Lind, 1974). The main idea of the FORM method was established by Hasofer and Lind (HL), who proposed an appropriate method for the determination of the safety index. In this method, the safety index is the objective of a constrained optimization problem in the standard normal space, which is calculate by the following equation (Nowak and Collins, 2000; Elegbede, 2005).

$$\begin{aligned} &\beta = \min(U^T.U)^{1/2} \\ &\textit{subjected to} \quad U \in G(U) = 0 \end{aligned} \qquad (3)$$

where, U is the standard normal variable with the mean equal to zero and the standard deviation equal to one.

$$U = \frac{(X - \mu_X)}{\sigma_X} \qquad (4)$$

where, μ_X and σ_X are the mean and the standard deviation of the basic random variable X respectively.

According to Equation 3, the acceptable probabilistic determination of the safety index is obtained when a point can be approximated on the limit state surface that has the maximum likelihood probability of failure (most probable point) (Yang, 2010; Santosh et al., 2006). This point specifies the minimum distance from the origin in the standard normal space to the limit state surface; this distance defines the reliability index and it is computed as:

$$\beta = (U^{*T}.U^*)^{1/2} \qquad (5)$$

in which U^* is the design point of the problem considering Eq. 3. The difficulty with the HL-RF method for the reliability analysis of the FORM in some nonlinear and complex performance functions is the numerical instability such as; periodic, bifurcation and chaos (Yang, 2010).

In order to improve the efficiency and robustness of the HL–RF method, certain modifications have been suggested by several researchers (Luo et al., 2009; Liu and Der Kiureghian, 1991; Lee et al., 2002; Yang, 2010). Liu and Kiureghian (1991) introduced a merit function, which monitors the convergence of the sequence and at each step and the new iteration point is selected by a line search along the direction vector (Liu and Der Kiureghian, 1991; Yang, 2010; Santosh et al., 2006). Recently, Santosh et al. (2006) have improved the modified HL-RF method by selecting an appropriate stepsize based on Armijo rules. Wang and Grandhi (1994, 1996) improved the known HL-RF method based on intervening variables and considering the adaptive nonlinear two-point approximation of the limit state function. Their algorithm yields stable results in problems with high nonlinearity (Wang and Grandhi, 1996), but if the performance function has several local minimum points, this method may converge to the local minimum point (Liu and Der Kiureghian, 1991; Yang, 2010). Lee et al. (2002) enhanced the HL-RF algorithm by eliminating its iteration zigzags in each step, based on the results of the new and previous design vector. Elegbede (2005) established the particle swarm optimization algorithm to compute the safety index. He considered random variables with a normal distribution density function in his proposed approach. Yang et al. (2006) has analyzed the FORM algorithm with the help of a nonlinear transformation in the form of a discrete dynamic system using the chaos theory. Yang (2010) has performed the reliability analysis of several examples using a stability transformation approach. Their algorithm converges very slowly in complicated problems with high-order nonlinearity.

In summary, all these efforts have been made to develop an efficient and robust algorithm for structural reliability analysis. In this paper, the HL-RF method has been introduced first. Then, according to the presented HL-RF algorithm, a new method has been developed to enhance it using the relaxed approach. An appropriate relaxed coefficient was then established on the basis of a quadratic fitting, using two successive design points information. The efficiency, robustness and capability of the new proposed algorithm have been evaluated using a number of examples.

THE HL-RF ALGORITHM

Hasofer and Lind (1974) proposed a new iterative approach for the first-order reliability analysis which approximates the hypersurface using the linearization of the LSF at the most probable failure point on the failure surface. It is based on Taylor's series expansion of the limit state function at the design point.

$$G(U) \approx G(U_k^*) + \nabla^T G(U_k^*)(U_{k+1}^* - U_k^*) = 0 \qquad (6)$$

where, $\nabla G(U_k)$ is the gradient vector of the limit state function in the normal standard space at the design point $\nabla G(U) = [\partial G/\partial u_1, \partial G/\partial u_2, ..., \partial G/\partial u_n]^T$.

Rearranging the above relation, the new design vector will be computed as:

$$U_{k+1}^* = \frac{\nabla^T G(U_k^*)\ U_k^* - G(U_k^*)}{\nabla^T G(U_k^*)\ \nabla G(U_k^*)} \nabla G(U_k^*) \qquad (7)$$

The HL algorithm could consider normal random variables for the determination of the MPP point. However, many structural reliability problems involve non-normal random variables. There are many methods available for conducting the transformations, such as Rosenblatt (Rackwitz and Fiessler, 1978; Santosh et al., 2006). Rackwitz and Fiessler (1978) extended algorithm developed by the Hasofer and Lind algorithm in order to include the distribution information of the basic random variables. The extended algorithm could consider non-normal basic random variables with a simple and approximate transformation. Hence, Eq. 4 can be written as follows (Rackwitz and Fiessler, 1978; Santosh et al., 2006):

$$U = \frac{(X - \mu_X^e)}{\sigma_X^e} \tag{8}$$

where, μ_x^e and σ_x^e are the equivalent mean and the standard deviation respectively, which are given as (Rackwitz and Fiessler, 1978; Santosh et al., 2006; Choi et al., 2007):

$$\sigma_x^e = \frac{1}{f_X(x^*)}\phi[\Phi^{-1}\{F_X(x^*)\}] \tag{9}$$

$$\mu_x^e = x^* - \sigma_x^e \Phi^{-1}[F_X(x^*)] \tag{10}$$

Based on the above approximation equations, the computational steps can be easily implemented for the safety index calculation into a computer program as summarized below:

1. Define the limit state function $G(X) = 0$, choose a feasible initial design point $X_o = \{\mu_1, \mu_2, \mu_3, ..., \mu_n\}^T$ and set k=0
2. Normalize the basic random variables (the mean and standard deviation of the variable are zero and unit, respectively) using the transformation relations given in Eqs. (8-10).
3. Calculate the gradient vector of the limit-state function ($\nabla G(U_k^*) = \frac{\partial G(U)}{\partial u_i}\Big|_{U_k^*}$) and compute the corresponding value of the limit state function at the design point ($G(U_k^*)$).
4. Determine the new value of the design vector according to Eq. (7).
5. Compute the value of the reliability index in the form $\beta_{k+1} = (U_{k+1}^T \times U_{k+1})^{1/2}$
6. If $|\beta_{k+1} - \beta_k| \le 10^{-6}$ then stop.
7. Set $k=k+1$ and return to step 2.

This method may not converge in some nonlinear problems. Therefore, some modifications have been suggested to improve its stability in the following section.

ENHANCED HL-RF (EHL-RF) ALGORITHM

The Lagrangian associated with the optimization relation, Eq. (3), is defined by:

$$f(U) = \frac{U^T.U}{2} + \lambda G(U) \tag{11}$$

where, f(U) is the unconstrained reliability function and λ is the penalty coefficient. The penalty coefficient for a local minimum point of the LSF must satisfy $U + \lambda G(U) = 0$ and $G(U) = 0$. Therefore the penalty coefficient is computed from the following equation:

$$\lambda = -\frac{\nabla^T G(U).\, U}{|\nabla G(U)|^2} \tag{12}$$

The calculation of λ has been developed by Liu and Kiureghian which further details on that can be found in (Liu and Der Kiureghian, 1991). According to the deviational methods and the HL-RF method, the differential design vector is written as follows:

$$\Delta U_{k+1}^* = U_{k+1}^* - U_k^* = d_k = \frac{\nabla^T G(U_k^*)\, U_k^* - G(U_k^*)}{\nabla^T G(U_k^*)\, \nabla G(U_k^*)} \nabla G(U_k^*) - U_k^* \tag{13}$$

where, d_k is the search direction vector in optimization problems and its value is the counterpart of the reliability function gradient vector of Eq. (11) ($d_k = -\nabla f(U_{k+1}^*)$) (Rao, 1996).

The new design point can be iteratively computed considering the attained design vector from the HL-RF method and its variations (based on the search direction vector and calculating the relaxed coefficient (α_k)) as:

$$U_{k+1} = U_k + \alpha_k d_k = U_k^* + \alpha_k [\frac{\nabla^T G(U_k^*)\ U_k^* - G(U_k^*)}{\nabla^T G(U_k^*)\ \nabla G(U_k^*)} \nabla G(U_k^*) - U_k^*] \quad (14)$$

in which, k is the iteration number and α_k is known as the relaxed coefficient at the k[th] iteration (a real and positive number selected between zero and one). If the relaxed coefficient is selected equal to one, the obtained algorithm will conform to the HL-RF algorithm.

The determination of the relaxed coefficient is vital in reliability problems. So, we may first compute the value of $f(U_k^*)$ and $-d_{k-1}^T.d_k$ for $\alpha_k = 0$, and the value of $f(U_{k+1}^*)$ for $\alpha_k = 1$. Then, a second-order function may be fitted for the computation of the optimum relaxed coefficient as follows:

$$f(\alpha) = A + B\alpha + C\alpha^2 \quad (15)$$

By expanding Taylor's series of the objective function, considering Eqs. (11) and (13), we will have

$$f(U^*_{k+1}) = f(U^*_k) + \nabla^T f(U^*_k).\alpha_k d_k + 1/2\alpha_k d_k^T [H] \alpha_k d_k \quad (16)$$

in which, $[H]$ is the Hessian matrix ($H_{i,j} = \frac{\partial^2 f}{\partial u_i \partial u_j}$) (Rao, 1996; Yang et al. 2006). Assuming $\alpha = 0$ Eqs. (15) and (16), parameter A can be found as follows:

$$f(\alpha = 0) = f(U_k^*) = A \quad (17)$$

Nevertheless, since the relaxed coefficient has been optimized for the previous iteration, the differentiation of Eq. (16) with respect to α should satisfy the differential of Eq. (15). This is as follows:

$$\left.\frac{df(U_{k+1}^*)}{d\alpha}\right|_{\alpha=0} = (\nabla^T f(U_k^*).d_k + d_k^T [H] \alpha d_k)\,|_{\alpha=0} = \nabla^T f(U_k^*).d_k = -d_{k-1}^T.d_k \quad (18)$$

$$\left.\frac{df(\alpha)}{d\alpha}\right|_{\alpha=0} = B + 2C\alpha|_{\alpha=0} = B \quad (19)$$

By Equating Eqs. (18) and (19), parameter B is computed as follows:

$$B = -d_{k-1}^T.d_k \quad (20)$$

Eq. (14) shows that for $\alpha = 1$ the new value of the design point is written as $U^*_k + d_k$, so Eq. (15) can be written as:

$$f(\alpha = 1) = A + B + C = f(U^*_k + d_k) \quad (21)$$

Substituting Eqs. (17) and (20) in Eq. (21), parameter C can be computed as

$$C = f(U^*_k + d_k) - f(U^*_k) + d_{k-1}^T.d_k \quad (22)$$

By inserting A, B and C into Eq. (15), the following equation is obtained:

$$f(\alpha) = f(U^*_k) - d_{k-1}^T.d_k \alpha + [f(U^*_k + d_k) - f(U^*_k) + d_{k-1}^T.d_k]\alpha^2 \quad (23)$$

The optimum relaxed coefficient in the proposed method is the extermum of Eq. (15), so it can be computed as:

$$\frac{df(\alpha)}{d\alpha} = B + 2\alpha C = 0 \Rightarrow \alpha = -\frac{B}{2C}$$
$$\alpha_k = \frac{d_{k-1}^T . d_k}{2[f(U^*{}_k + d_k) - f(U^*{}_k) + d_{k-1}^T . d_k]} \quad (24)$$

where, d_k is the search direction vector (Eq. (13)) and $f(U^*{}_k)$ is the value of the reliability function at the design point. According to Eq. (11) it can be computed from:

$$f(U_k^*) = \frac{|U_k^*|^2}{2} - \frac{\nabla^T G(U_k^*).\ U_k^*}{|\nabla G(U_k^*)|^2} G(U_k^*) \quad (25)$$

Furthermore, the value of $f(U^*{}_k + d_k)$ is proposed by:

$$f(U^*{}_k + d_k) = \frac{|U_{k+1}^*|^2}{2} - \frac{\nabla^T G(U_k^*).\ U_{k+1}^*}{|\nabla G(U_k^*)|^2} G(U_{k+1}^*) \quad (26)$$

in which U_{k+1}^* is the new design point, which is calculated using the HL-RF algorithm. Limiting the relaxed coefficient between zero and one ($0 \le \alpha_k \le 1$) prevents it from getting large and negative amounts, therefore the stable results can be obtained using the proposed method. In this state, the periodic-oscillation convergence of the iterative procedure can be prevented. According to the above, the new proposed algorithm is written as follows:

1. Define the limit state function $G(X) = 0$, choose parameter $\alpha_0 = 1$, $\varepsilon > 0$ and set $k=0$; given the initial design point.
2. Normalize the basic random variables using the transformation relations given in Eqs. (8-10). (mean and standard deviation of variable are zero and unit respectively).
3. Calculate the gradient vector of the limit-state function ($\nabla G(U_k^*) = \frac{\partial G(U)}{\partial u_i}\Big|_{U_k^*}$) and compute the corresponding value of the limit state function at the design point ($G(U_k^*)$).
4. Determine the new value of the design vector in terms of Eq. (7).
5. Compute the search direction vector (d_k) according to Eq. (13).
6. If $k>1$ then compute the relaxed coefficient based on Eqs. (24 – 26).
7. Determine the new design vector on the basis of Eq. (14).
8. Calculate the reliability index in the form $\beta_{k+1} = (U_{k+1}^T \times U_{k+1})^{1/2}$
9. If $|\beta_{k+1} - \beta_k| \le 10^{-6}$ then stop.
10. Set $k=k+1$ and return to Step 2.

ILLUSTRATIVE EXAMPLES

Considering the previous sections, the possibility of applying the HL-RF and the new proposed algorithms (EHL-RF) in the reliability analysis of structures has been investigated. These two algorithms are coded into a computer program so that their performances can be further investigated through numerical examples. Five numerical examples including the mathematical problems with nonlinear performance functions and the structural problems with complex LSF are selected from the literature to demonstrate the efficiency and robustness of the EHL-RF method. Numerical examples which are widely used to investigate the performance of a new algorithm in reliability analysis were selected with a wide range of variables and limit state function to cover most area of the problems. The numbers of $\nabla G(U)$ computations (i.e. central finite difference), CPU run time required by each method to converge and the reliability index are selected as measures for comparison in these algorithms.

Example 1

A high nonlinear mathematical function

A high nonlinear performance function has been selected in this example with the following limit state function (Wang and Grandhi, 1994):

$$G_1 = x_1^4 + 2x_2^4 - 20 \qquad (27)$$

where, x_1 and x_2 are the normally-distributed independent random variables whose means and standard deviations are $\mu_1 = \mu_2 = 10$ and $\sigma_1 = \sigma_2 = 5$, respectively. The reliability index is equal to 2.3654 for this example, which is extracted from (Wang and Grandhi, 1994). Figure 1 shows the iterative history of the proposed and HL-RF algorithms. This example has periodically converged to two- design points using the HL-RF algorithm and yielded a safety index, (β), of 2.365373 after 67 iterations through the reliability analysis of the proposed algorithm. This result shows a very close agreement with the result found in (Wang and Grandhi, 1994).

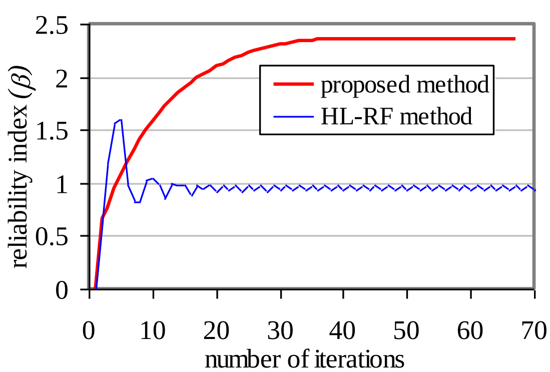

Fig. 1. Iterative history comparison of Example 1.

Example 2

A Nonlinear Mathematical Function

The nonlinear limit state function presented for this example is in the following form (Wang and Grandhi, 1996):

$$G_2 = x_1^3 + x_1^2 x_2 + x_2^3 - 18 \qquad (28)$$

where, x_1 and x_2 are the normally-distributed independent random variables with means $\mu_1 = 10$ and $\mu_2 = 9.9$, respectively and standard deviations $\sigma_1 = \sigma_2 = 5$.

In this example, the reliability index is extracted from (Wang and Grandhi, 1996) with a value of 2.2983. Figure 2 illustrates the convergence history of the EHL-RF and HL-RF method for this example. As seen, the HL-RF algorithm is not a suitable and stable solution, but the proposed algorithm needs 38 iterations to attain the stable solution. In this state, the reliability index equals 2.298243, which is basically in agreement with the figure presented in (Wang and Grandhi, 1994). This example clearly indicates the robustness of the new proposed algorithm in the reliability analysis with high nonlinearity compared with the HL-RF algorithm.

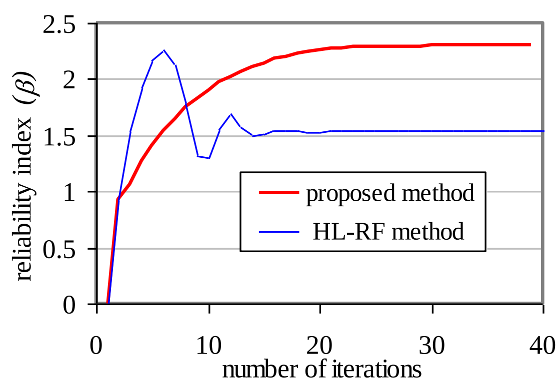

Fig. 2. Iterative history comparison of Example 2.

Example 3

A Complex Non-Linear Function

A complex problem which is highly non-linear performance function and has non-normal variables is taken from (Liu and Der Kiureghian, 1991; Yang, 2010). This problem is taken from the reliability analysis of a pipeline where the limit state surface was generated by response-surface fitting.

The performance function takes the following form:

$$G_3 = 1.1 - 0.00115x_1x_2 + 0.00157x_2^2 + 0.00117x_1^2 + 0.0135x_2x_3 - 0.0705x_2 - 0.00534x_1 - 0.0149x_1x_3 - 0.0611x_2x_4 + 0.0717x_1x_4 - 0.226x_3 + 0.0333x_3^2 - 0.558x_3x_4 + 0.998x_4 - 1.339x_4^2 \quad (29)$$

where, x_1 to x_4 are statistically independent basic random variables. The random variable x_1 has the type-II largest value distribution with a mean of 10 and standard deviation of 5. x_2 and x_3 are the normally-distributed random variables with the means of 25 and 0.8, and standard deviations of 5 and 0.2, respectively. The random variable x_4 follows a lognormal distribution density function with a mean of 0.0625 and standard deviation of 0.0625.

Based on the results extracted from (Liu and Der Kiureghian, 1991) for this example, the reliability index is equal to 1.36 and the design vector is $\mathbf{X}^*$= [15.09, 25.027, 0.8653, 0.03582]. This example has been recently analyzed by Yang (2010) so that the converged results, after 285 iterations, have given a safety index equal of 1.3304 and a design vector $\mathbf{X}^*$= [14.906, 25.067, 0.8995, 0.04606]. Using the new proposed algorithm after 33 iterations, the converged values of the reliability index and the design vector have respectively been β=1.33053 and $\mathbf{X}^*$= [14.905, 25.067, 0.85956, 0.046061]. The convergence history of this example has shown in Figure 3. In this example, it is evident that the HL-RF algorithm has yielded the periodic-2 solutions i.e. {1.04958, 1.15364}, but the reliability analysis of the new proposed algorithm converged to a stable solution of 1.33053, which is in a good agreement with the results obtained by (Yang, 2010).

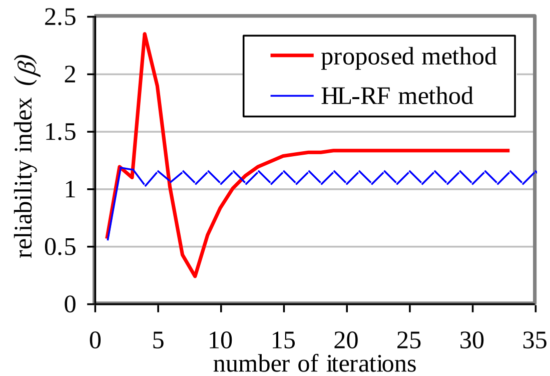

Fig. 3. Iterative history comparison of Example 3.

Example 4
A Conical Structure

Figure 4 shows the geometrical and mechanical features of a conical structure under a compressive axial load P, and a bending moment M. The main mechanisms of the structure failure are the loss of strength and buckling due to instability. Hence, the buckling criterion is considered as the failure mode for this structure as shown in the following mathematical equation (Elegbede, 2005):

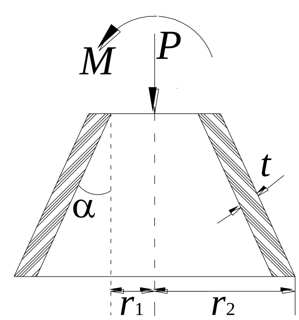

Fig. 4. The conical structure of Example 4 (Elegbede, 2005).

$$\frac{P}{P_{cr}} + \frac{M}{M_{cr}} > 1 \quad (30)$$

where, P and M are the compressive axial load and bending moment due to external loading and P_{cr} and M_{cr} are the critical axial load and bending moment (for buckling) respectively (Elegbede, 2005).

$$P_{cr} = \gamma \frac{2\pi E t^2 \cos^2 \alpha}{\sqrt{3(1-\upsilon^2)}} \quad (31)$$

$$M_{cr} = \eta \frac{\pi E t^2 r_1 \cos^2 \alpha}{\sqrt{3(1-\upsilon^2)}} \quad (32)$$

where, γ and η are coefficients to correlate between theoretical and experimental results of *P* and *M*, which are considered to be 0.33 and 0.41, respectively in this example (Elegbede, 2005), and v is Poisson's ratio, which is 0.3.

Based on Eqs. (33-35), the limit state function for this structure can be presented in the following form (Elegbede, 2005):

$$G_4 = 1 - \frac{\sqrt{3(1-v^2)}}{\pi E t^2 \cos^2 \alpha} \times (\frac{P}{2\gamma} + \frac{M}{\eta r_1}) \quad (33)$$

This problem consists of six normal independent basic random variables whose statistical characteristics are listed in Table 1.

Extracted from (Elegbede, 2005), the reliability index for this problem is about 4.883. Based on the reliability analysis undertaken using the proposed method, the converged safety index and design vector are estimated to be $\beta = 4.79429$ and $\mathbf{X}^*$= [63697, 0.002, 0.8281, 0.8874, 90064, 74247], respectively. Figure 5 shows a comparison on the iterative history. It also indicates that the HL-RF algorithm has not converged to a final solution given the tolerance level defined for convergence in this example (i.e. $|\beta_{k+1} - \beta_k| \leq 10^{-6}$). The proposed algorithm indicates a stable and accurate solution compared with the HL-RF method.

Table. 1. Basic random variables for Example 4.

Variables	Description	Distribution	Mean	Coefficient of Variation
E	Young's modulus (MPa)	normal	70000	0.05
t	Thickness(m)	normal	0.0025	0.05
α	Slop angle (rad)	normal	0.524	0.02
r_1	Internal radius(m)	normal	0.9	0.025
M	Bending moment(N-m)	normal	80000	0.08
P	Axial load(N)	normal	70000	0.08

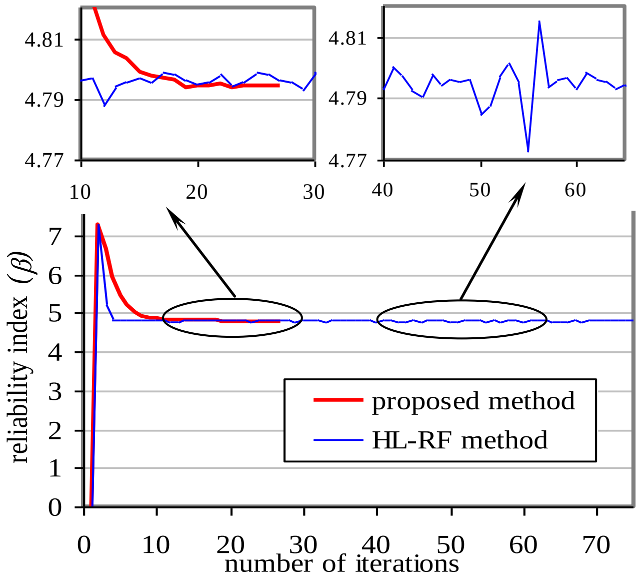

Fig. 5. Iterative history comparison of Example 4.

Example 5
A 10-Bar Truss

This example studies a 10-bar truss structure as shown in Figure 6. The vertical, horizontal and diagonal truss members are aluminum rods with three different cross-sectional areas A_1, A_2 and A_3, respectively. In this structure, the diagonal members do not intersect at the intersection point. The structure is subjected to an external load, P, as shown in Figure 6. The performance function for the vertical displacement at a specified point (see Figure 6 for the location of the point) on the truss structure with allowable displacement can be written (Choi et al., 2007; Naess et al., 2009):

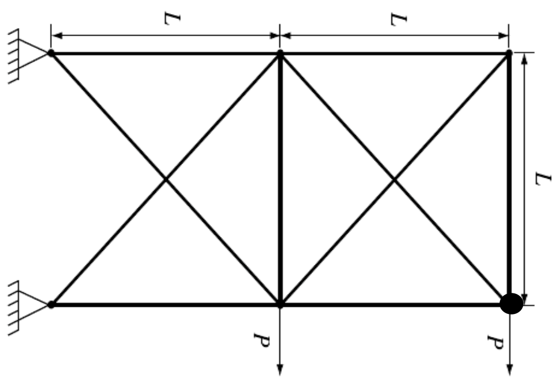

Fig. 6. The 10-bar truss structure (Naess et al., 2009).

$$G_5 = d_0 - D$$

$$D = \frac{P.L.B}{A_1 A_3 E}\left\{\begin{array}{l} \frac{4\sqrt{2}A_1^3(24A_1^2 + A_3^2) + A_3^3(7A_1^2 + 26A_2^2)}{D_T} + \\ 4A_1 A_2 A_3 \frac{20A_1^2 + 76A_1 A_2 + 10A_3^2}{D_T} + \\ 4\sqrt{2}A_1 A_2 A_3^2 \frac{25A_1 + 29A_2}{D_T} \end{array}\right\} \quad (34)$$

where,

$$D_T = 4A_2^2(8A_1^2 + A_3^2) + 4\sqrt{2}A_1 A_2 A_3(3A_1 + 4A_2) + A_1 A_3^2(A_1 + 6A_2)$$

and E is the Young's module.

The random variable B has been introduced to account for model uncertainties in this example. The basic random variables A_1, A_2, A_3, B, P and E have been assumed to be independent and their statistical properties are summarized in Table 2. L and d_0 are deterministic variables having values equal to 0.1 and 9 meter respectively. The failure probability calculated using the reliability analysis was 3.5×10^{-6} (Naess et al., 2009); and the reliability index was estimated to be about 4.4937 using Monte Carlo simulation with 1.7×10^9 samples.

Table. 2. Basic random variables for Example 5.

Variable	Distribution	Mean	Coefficient of Variation
A_1	normal	m^2 10^{-2}	0.05
A_2	normal	m^2 $10^{-3}\times1.5$	0.05
A_3	normal	m^2 $10^{-3}\times6$	0.05
B	normal	1	0.10
P	Gumbel	N $10^5\times2.5$	0.10
E	Log-normal	MPa $10^4\times6.9$	0.05

Figure 7 shows the comparison of iterative history of converged reliability indices for both methods in this example. Both methods, converged to the reliability indices of 4.30064 after 28 iterations and 4.29946 after 20 iterations using the HL-RF and EHL-RF algorithms, respectively. This implies that the new method is more efficient than the HL-RF method.

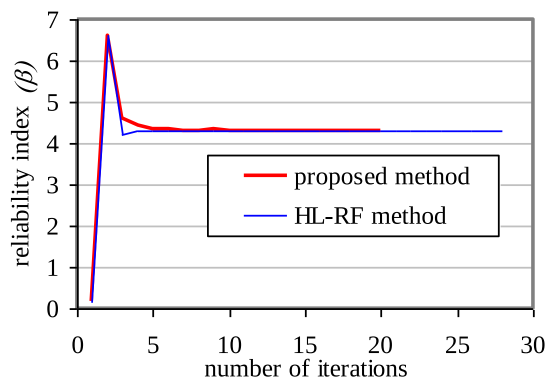

Fig. 7. Iterative history comparison of Example 5.

DISCUSSION

The results of this study revealed that an appropriate relaxed coefficient in the EHL-RF method not only could improve the efficiency of the HL-RF algorithm but also made the proposed algorithm more robust and stable. In this study, five examples were solved using the EHL-RF and the HL-RF algorithm. The results obtained using both algorithms for all examples along with the CPU time required to converge have been presented in Table 3. The results showed that the HL–RF iterative algorithm is diverged in most engineering nonlinear problems, while the EHL-RF method can offer a robust, efficient solution so that the proposed relaxed method can control the numerical instability of the HL-RF iterative algorithm. From a practical point of view, the proposed method does not require a prior knowledge of the LSF value and gradient vector of the HL-RF results at the new and the previous iterations.

Table. 3. Comparison of the convergence for the EHL-RF and HL-RF.

Example	HL-RF			EHL-RF			Reference Reliability Index
	Reliability index	Run time (Sec)	No. of Iterations	Reliability index	Run time (Sec)	No. of Iterations	
1	not converged	failed	----	2.36537	2.36	67	2.3654 (Wang and Grandhi, 1994)
2	not converged	failed	----	2.29824	0.58	38	2.2983 (Wang and Grandhi, 1996)
3	not converged	failed	----	1.33053	0.24	33	1.36 (Liu and Der Kiureghian, 1991) 1.3304 (Yang, 2010)
4	not converged	failed	----	4.79429	0.58	27	4.883 (Elegbede, 2005)
5	4.30064	1.415	28	4.29946	0.602	20	4.4937 (Naess et al., 2009)

CONCLUSIONS

In this paper, a new algorithm has been proposed to enhance the capability of the HL-RF method for the determination of the reliability index. This new algorithm is based on a combination of the HL-RF algorithm and the relaxed method that defines an appropriate relaxed coefficient. The results showed that it works in such a way that it not only guarantees the numerical stability, but also, in some cases, it decreases the number of iterations. In addition, it is more efficient, robust and stable than the HL-RF method and has acceptable convergence for complex limit state functions and complicated high nonlinear performance functions in structural reliability analyses. In such problems, the proposed method outperforms the HL-RF method and it can be recommended as a general method for reliability analysis of structures as well as a vast range of reliability problems. This method is similar to other common reliability that methods with an exception that can include any type of the modified HL-RF methods. The main difference between this and the other modified HL-RF methods is that it establishes an appropriate relaxed coefficient obtained from the results of the new and the previous iterations. The choice of an appropriate relaxed coefficient in the proposed algorithm considerably increased its robustness compared with the HL-RF method.

REFERENCES

Choi, S.K., Grandhi, R.V. and Canfield, R.A. (2007). *Reliability-based structural design*, Springer-Verlag, London.

Elegbede, C. (2005). "Structural reliability assessment based on particles swarm optimization", *Structural Safety*, 27(2), 171–186.

Hasofer, A.M. and Lind, N.C. (1974). "Exact and invariant second moment code format", *Journal of the Engineering Mechanics Division*, 111(21), 111-121.

Lee, J.O., Yang, Y.S. and Ruy, W.S. (2002). "A comparative study on reliability-index and target-performance-based probabilistic structural design optimization", *Computers and Structures*, 80(3-4), 257–269.

Liu, P.L. and Kiureghian, A.D. (1991). "Optimization algorithms for structural reliability", *Structural Safety*, 9(3), 161–178.

Luo, Y., Zhan, K. and Li, A. (2009). "Structural reliability assessment based on probability and convex set mixed model", *Computers and Structures*, 87(21), 1408–1415.

Naess, A., Leira, B.J. and Batsevych, O. (2009). "System reliability analysis by enhanced Monte Carlo simulation", *Structural Safety*, 31(5), 349–355.

Nowak, A.S. and Collins, K.R. (2000). *Reliability of Structures*, McGraw-Hill, New York.

Rackwitz, R. and Fiessler, B. (1978). "Structural reliability under combined load sequences", *Computers and Structures*, 9(8), 489–494.

Rao, S.S. (1996). "*Engineering optimization, theory and practice*", Wiley, New York.

Santosh, T.V., Saraf, R.K., Ghosh, A.K. and Kushwaha, H.S. (2006). "Optimum step length selection rule in modified HL-RF method for structural reliability", *International journal of Pressure Vessels Piping*, 83(10), 742–748.

Wang, L.P. and Grandhi, R.V. (1994). "Efficient safety index calculation for structural reliability analysis", *Computers and Structures*, 52(1), 103–111.

Wang, L.P. and Grandhi, R.V. (1996). "Safety index calculation using intervening variables for structural reliability analysis", *Computers and Structures*, 59(6), 1139–1148.

Yang, D. (2010). "Chaos control for numerical instability of first order reliability method", *Communications in Nonlinear Science and Numerical Simulation*, 15(10), 3131–3141.

Yang, D., Li, G. and Cheng, G. (2006). "Convergence analysis of first order reliability method using chaos theory", *Computers and Structures*, 84(8-9), 563–571.